1. A particle *P* is projected vertically upwards and reaches its greatest height 0.5 s after the instant of projection. Calculate

i. the speed of projection of *P*,

[2]

ii. the greatest height of *P* above the point of projection.

[3]

It is given that the point of projection is 0.539 m above the ground.

iii. Find the speed of *P* immediately before it strikes the ground.

[3]

2. A particle *P* is projected vertically downwards with initial speed 3.5 ms^{-1} from a point *A* which is 5 m above horizontal ground.

i. Find the speed of *P* immediately before it strikes the ground.

[2]

After striking the ground, *P* rebounds and moves vertically upwards and 0.87 s after leaving the ground *P* passes through *A*.

ii. Calculate the speed of *P* immediately after it leaves the ground.

[3]

It is given that the mass of *P* is 0.2 kg.

iii. Calculate the change in the momentum of *P* as a result of its collision with the ground.

[2]

3. A particle P is projected vertically downwards with speed 14 m s^{-1} from a point 30 m above the ground.

i. Calculate the speed of P when it reaches the ground.

[2]

ii. Find the distance travelled by P in the first 0.4 s of its motion.

[2]

iii. Calculate the time taken for P to travel the final 15 m of its descent.

[3]

4. A stone is released from rest on a bridge and falls vertically into a lake. The stone has velocity 14 m s^{-1} when it enters the lake.

i. Calculate the distance the stone falls before it enters the lake, and the time after its release when it enters the lake.

[4]

The lake is 15 m deep and the stone has velocity 20 m s^{-1} immediately before it reaches the bed of the lake.

ii. Given that there is no sudden change in the velocity of the stone when it enters the lake, find the acceleration of the stone while it is falling through the lake.

[3]

5. A particle is projected with speed u ms^{-1} at an angle of θ above the horizontal from a point O. At time t s after projection, the horizontal and vertically upwards displacements of the particle from O are x m and y m respectively.

i. Express x and y in terms of t and θ and hence obtain the equation of trajectory

$$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}.$$

[4]

In a shot put competition, a shot is thrown from a height of 2.1 m above horizontal ground. It has initial velocity of 14 ms^{-1} at an angle of θ above the horizontal. The shot travels a horizontal distance of 22 m before hitting the ground.

ii. Show that $12.1\tan^2\theta - 22\tan\theta + 10 = 0$, and find the value of θ.

[5]

iii. Find the time of flight of the shot.

[2]

6. A boy kicks a ball from a point O on horizontal ground. The ball first hits the ground at a distance of 60 m from O and the time of flight is 4 seconds. This motion of the ball is modelled as that of a particle moving freely under gravity.

(a) Find the horizontal and vertical components of the initial velocity of the ball. [3]

The ball just clears a vertical post, of height h m, at a horizontal distance of 15 m from O.

(b) Show that $h = 14.7$. [2]

(c) Find the speed of the ball as it passes over the post. [4]

Measurements show that the speed of the ball as it passes over the post is in fact not equal to the value found in part (c).

(d) State a deficiency of the model that might account for this. [1]

(e) Explain whether an improved model would require a larger or smaller initial speed for the ball. [1]

7. A child is trying to throw a small stone to hit a target painted on a vertical wall. The child and the wall are on horizontal ground. The child is standing a horizontal distance of 8 m from the base of the wall. The child throws the stone from a height of 1 m with speed 12 m s^{-1} at an angle of 20° above the horizontal.

i. Find the direction of motion of the stone when it hits the wall.

[6]

The child now throws the stone with a speed of *V* m s^{-1} from the same initial position and still at an angle of 20° above the horizontal. This time the stone hits the target which is 2.5 m above the ground.

ii. Find *V*.

[6]

8. A football is kicked from horizontal ground with speed 20 m s^{-1} at an angle of θ° above the horizontal. The greatest height the football reaches above ground level is 2.44 m. By modelling the football as a particle and ignoring air resistance, find

i. the value of θ,

[2]

ii. the range of the football.

[2]

9. A particle is projected with speed v ms^{-1} from a point *O* on horizontal ground. The angle of projection is θ° above the horizontal. At time t seconds after the instant of projection the horizontal displacement of the particle from *O* is x m and the upward vertical displacement from *O* is y m.

i. Show that

$$y = x\tan\theta - \frac{4.9x^2}{v^2\cos^2\theta}.$$

[4]

A stone is thrown from the top of a vertical cliff 100 m high. The initial speed of the stone is 16 ms^{-1} and the angle of projection is θ° to the horizontal. The stone hits the sea 40 m from the foot of the cliff.

ii. Find the two possible values of θ.

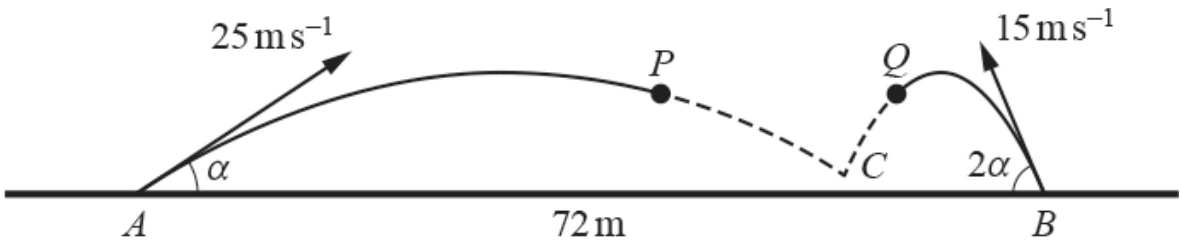
[6]

10. A golfer hits a ball from a point O on horizontal ground with a velocity of 55 m s^{-1} at an angle of 20° above the horizontal. The ball first hits the ground at a point A and the time of flight is t seconds. Assuming that there is no air resistance, calculate

(i) the value of t and the distance OA, [4]

(ii) the speed and direction of motion of the ball 2.6 s after the golfer hits the ball. [5]

11. **In this question you must show detailed reasoning.**



A football P is kicked with speed 25 m s^{-1} at an angle of elevation 2α from a point A on horizontal ground. At the same instant a second football Q is kicked with speed 15 m s^{-1} at an angle of elevation 2α from a point B on the same horizontal ground, where AB = 72 m. The footballs are modelled as particles moving freely under gravity in the same vertical plane and they collide with each other at the point C (see diagram).

(a) Calculate the height of C above the ground. [7]

(b) Find the direction of motion of P at the moment of impact. [4]

(c) Suggest one improvement that could be made to the model. [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$U = 0.5g$ *OR* $U - 0.5g = 0$	M1	Consider descent *OR* ascent. $v = u + at$ with consistent signs for non-zero terms. $U + 0.5g = 0$ is M0 hence A0.	
		i	$U = 4.9\ \text{m s}^{-1}$	A1	Allow use of 4.9 without penalty in (ii) and (iii) even if 0/2 here.	
		ii	$U^2 = \pm 2gs$	M1	$v^2 = u^2 + 2as$	
		ii	$4.9^2 = \pm 2 \times 9.8 \times s$	A1		
		ii	$s = 1.225$ m	A1	+ve, 49/40, 1.22 or 1.23 BoD loss of – sign in final answer	
		ii	*OR*			
		ii	$s = \pm(ut \pm gt^2/2)$ *OR* $s = \pm gt^2/2$	M1	Rise to / fall from greatest height. $S = \pm(vt \pm g\frac{t^2}{2})$ is similar.	
		ii	$s = \pm(4.9 \times 0.5 - g \times 0.5^2/2)$ *OR* $s = \pm g \times 0.5^2/2$	A1		
		ii	$s = 1.225$ m	A1	+ve, 1.22 or 1.23 BoD loss of – sign in final answer	
		ii	*OR*			
		ii	$s = \pm Ut/2$	M1	$s = (u + v)t/2$	
		ii	$s = \pm 4.9 \times 0.5/2$	A1		
		ii	$s = 1.225$ m	A1	+ve, 1.22 or 1.23 BoD loss of - sign in final answer	
		iii	$v^2 = 2g(s \pm 0.539)$	M1	Overall descent, zero initial speed	
		iii	$v^2 = 2 \times 9.8 \times (0.539 + 1.225)$	A1ft	ft cv (1.225), tolerate sign change from (ii)	
		iii	$v = 5.88\ \text{ms}^{-1}$	A1	Exact, isw rounding of 5.88 to 5.9 if 5.88 seen	

		iii	*OR*			
		iii	$v^2 = u^2 \pm 2g \times 0.539$	M1	Motion from projection level down, non-zero initial speed	
		iii	$v^2 = 4.9^2 + 2g \times 0.539$	A1ft	ft cv (4.9), tolerate sign change from (i) Exact, isw rounding of 5.88 to 5.9 if 5.88 seen	
		iii	$v = 5.88\ \text{ms}^{-1}$	A1	**Examiner's Comments** A few candidates had a clearly expressed sense of which direction was positive for velocity, acceleration and displacement. Each part could be tackled either from the position of projection or from the top of the motion. Which was the candidate's intention was sometimes unclear and might change from part to part. In part (iii) the answer required was exactly 5.88. Candidates who evaluated 1.23 + 0.539 (and calculated the speed of the particle after it had fallen from a position of rest at its greatest height) would get 5.89, and so lost an accuracy mark because of their premature approximation. If using $g = 9.81$ the answers are: (i) $u = 4.905$, so accept 4.9(0) or 4.91. (ii) $s = 1.226..$, so accept 1.23 but not 1.2. (iii) $v = 5.8851..$, so accept 5.89 but not 5.9.	
			Total	8		
2		i	$v^2 = 3.5^2 + 2g \times 5$	M1	Uses $v^2 = 3.5^2 +/- 2g5$	Accept -3.5^2 for $(-3.5)^2$ etc
		i	$v = 10.5\ \text{ms}^{-1}$	A1	**Examiner's Comments** Was almost always answered correctly.	
		ii		M1	$+/-5 = 0.87u +/- g0.87^2/2$	May come from $s = vt - gt^2/2$

		ii	$5 = 0.87u - g \times 0.87^2/2$	A1		
		ii	$u = 10.0$ ms^{-1}	A1	**Examiner's Comments** This part was almost always answered correctly, save for a significant minority of candidates who had the wrong sign before the term involving *g*. One unusual feature was the high proportion of candidates who rearranged the standard *suvat* equation into a form which had *u* as its subject.	
		iii	Change = 0.2 × 10.5 + 0.2 × 10	M1	Or +/– 0.2(Ans(i) +/– Ans(ii)) It is OK get −4.1 from correct work	
		iii	Change = 4.1(0) kg ms^{-1}	A1	**Examiner's Comments** Was nearly always answered by subtracting the magnitudes of the momentum on landing and on lift-off. A minority of candidates used the initial speed of 3.5 m s^{-1} in their calculations.	
			Total	**7**		
3		i	$v^2 = 14^2 + 2g \times 30$	M1	$v^2 = u^2 +/- 2gs$	Using $v^2 = u^2 + 2as$
		i	$v = 28$ m s^{-1}	A1	**Examiner's Comments** Parts (i) and (ii) were almost always correct although a small minority of candidates took the initial velocity to be upwards or zero and a few were confused about the sign required with g.	
		ii	$s = 14 \times 0.4 + g \times 0.4^2/2$	M1		
		ii	$s = 6.384$ m	A1	Accept 6.38	

					Examiner's Comments Parts (i) and (ii) were almost always correct although a small minority of candidates took the initial velocity to be upwards or zero and a few were confused about the sign required with g. Part (ii) had the exact answer 6.384 but 6.38 was accepted.	
		iii	$15 = 28t - gt^2/2$	M1*	Uses $s = vt +/- gt^2/2$	Accept cv(28) but not v = 0
		iii	$4.9t^2 - 28t + 15 = 0$	D*M1	Attempts to solve 3 term QE	
		iii	t = (5.12) 0.598s	A1	Ignore 5.12 if seen	
		iii	*OR*			
		iii	$28^2 = u^2 + 2g \times 15$	M1*	$v^2 = 14^2 + 2g \times 15$	Accept cv(28) but not $v = 0$
		iii	$28 = \sqrt{(490)} + gt$	D*M1		
		iii	t = 0.598 s	A1		
		iii	*OR*			
		iii	$15 = 14t + gt^2/2$	M1*	Attempts to solve 3 term QE	
		iii	$30 = (14 + 28)t/2$	D*M1	Finding total time.	Accept cv(28) but not v = 0
		iii	t = 0.598 s	A1	**Examiner's Comments** This part could be solved in a variety of ways, the simplest being to use $s = vt - gt^2/2$ which seems to be the least familiar of the suvat equations. Any fully complete method was acceptable but rounding errors often accumulated in the multi-stage methods with 0.6 being a common incorrect answer. A diagram of the situation might have helped the few who only found the time for the first 15m or only the	

					total time. A minority were unable to solve the quadratic equation in t they had obtained.	
			Total	7		
4		i	$14^2 = 2gh$	M1	$v^2 = u^2$ +/−$2gs$ with $u = 0$	
		i	$h = 10$ m	A1	−ve final answer A0	
		i	$14 = gt$	M1	$v = u + gt$ with $u = 0$	
		i	$t = 1.43$ s	A1	Accept 10/7	
		i	*OR* $14 = gt$	M1	There are many alternatives, but following through of	
		i	$t = 1.43$ s	A1	wrong answer is allowed only for method marks as the	
		i	$h = 0 \times 1.43 + 9.8 \times 1.43^2 /2$	M1	h and t values can be found independently.	
		i	$h = 10(.0)$ m	A1	**Examiner's Comments** Most candidates scored full marks for this part, and only a few gave only one of distance and time.	
		ii		M1	$v^2 = 14^2$ +/− $2as$, $a \neq g$	
		ii	$20^2 = 14^2 + 2a15$	A1		
		ii	$a = 6.8$ m s^{-2}	A1	**Examiner's Comments** Again most candidates scored full marks.	
			Total	7		

5		i	$x = u\cos\theta t$	B1		
		i	$y = u\sin\theta t - ½gt^2$	B1		
		i	Eliminate t	M1		
		i	Get $y = x\tan\theta - gx^2\sec^2\theta/2u^2$ [AG]	A1	www **Examiner's Comments** Many good solutions were seen. There was very little evidence of attempts to 'fudge' the given answer.	
		ii	Substitute x = 22, y = −2.1 and u = 14	M1	May start again of course	
		ii	Use $\sec^2\theta = 1 + \tan^2\theta$	B1		
		ii	Tidy to $12.1\tan^2\theta - 22\tan\theta + 10 = 0$ [AG]	A1	www	
		ii	Solve QE for $\tan\theta$	M1	allow in radians (0.738)	
		ii	$\theta = 42.3$	A1	**Examiner's Comments** The connection between this and the previous part was not always appreciated and quite a few candidates started again. The most common error was to take $y = 2.1$ and some thought the trig identity to be used was $\sec^2\theta = 1 - \tan^2\theta$ but nevertheless still, wrongly, obtained the required result! Some candidates who could not show the given result were sensible enough to use it to find the angle.	
		iii	$t = 22/14\cos\theta$	M1	May work vertically, but must solve for t to get M1	
		iii	t = 2.12s	A1	**Examiner's Comments**	

					Although the simple method using the expression for the *x* displacement was often seen, many chose to work vertically, often unsuccessfully since they failed to consider all stages of the motion. Some correctly used their expression for *y* displacement, a few simply used 22/14.	
			Total	11		
6		a	$u_h = 15$ ms $^{-1}$ $0 = 4u_v + \frac{1}{2}(-9.8) \times 4^2$ $u_v = 19.6$ ms $^{-1}$	B1(AO1.1) M1(AO3.3) A1(AO1.1) [3]	Use of $s = ut + \frac{1}{2}at^2$ vertically, with $s = 0$ / Or $v = u + at$, eg	
		b	$u_h = 15 \Rightarrow t = 1$, so $h = 19.6 \times 1 + \frac{1}{2}(-9.8) \times 1^2$ $h = 14.7$	M1(AO3.4) A1(AO1.1) [2]	Finding *t* and using in $s = ut + \frac{1}{2}at^2$ AG	
		c	$v_v^2 = 19.6^2 + 2(-9.8) \times 14.7 \quad (= 96.04)$ $v_h = 15$ $v = \sqrt{96.04 + 15^2}$	M1(AO3.3) B1ft(AO1.1) M1(AO1.1) A1(AO1.1)	Use of $v^2 = u^2 + 2as$ vertically with their value of u_v from (a) Their value of u_h from (a)	

			17.9 ms $^{-1}$	[4]	Use of Pythagoras to find the speed / 17.917589…	
		d	Model takes no account of air resistance	E1(AO3.5b) [1]	Or any other reasonable comment, e.g. wind or rotation of the ball could affect the motion	
		e	State larger, with suitable explanation	E1(AO3.5a) [1]	E.g. air resistance will slow the ball down so to achieve the given range (or time of flight, or height at the post) the initial speed would have to be higher	
			Total	11		
7		i	$v_x = 12\cos20$	*B1	11.27631…..	
		i	$8 = 12t\cos20$	B1	Using suvat to find expression in t only. ($t = 0.70945…$)	

		i		*M1	Attempt at use of $v = u + at$	
		i	$v_y = 12\sin20 - g\text{cv}(t)$	A1	-2.84838……	
		i	$\tan\theta = v_y / v_x$	Dep**M1	Use trig to find a relevant angle	
					14.1763… (75.8° downward vertical)	
					Examiner's Comments	
		i	14.2° below horizontal	A1	Many good solutions were seen to this question. Although candidates are getting better at describing the direction relative to a fixed direction, there is still room for improvement. A simple 'below the horizontal' accompanying the angle would have been sufficient. A few candidates lost marks because they were unable to rearrange $8 = 12t\cos20$ correctly to obtain the value of t. A more common error was to use $v^2 = u^2 + 2as$ instead of $v = u + at$ to find the vertical component of velocity without justifying the sign taken when square rooting to find v.	
		ii	$8 = Vt\cos20$	B1		
		ii		*M1	Attempt at use of $s = ut + ½ at^2$	
		ii	$1.5 = Vt\sin20 - gt^2/2$	A1		
		ii	Eliminate t	dep*M1	OR Eliminate V and solve for t	
		ii	Attempt to solve a quadratic for V	dep*M1	AND Sub value for t and solve for V	
		ii	$V = 15.9$	A1	$V = 15.8606$…	
		ii	OR $y = x\tan\theta - gx^2\sec^2\theta/2u^2$	*B1	Use equation of trajectory	
		ii	Substitute values for y, x, θ	dep*M1		
		ii	$1.5 = 8\tan20 - g8^2\sec^2 20/2V^2$	A1		

		ii	Attempt to solve a quadratic for V	dep*M2	SC M1 for solving for V^2 $V = 15.8606\ldots$ **Examiner's Comments**	
		ii	$V = 15.9$	A1	This was well done in terms of the candidates knowing what was required, but in some cases the algebra wasn't always equal to the task. A small minority of candidates made the unfortunate assumption that the target was hit at the highest point of the trajectory.	
			Total	12		
8		i	$(20\sin\theta)^2 - 2g(2.44) = 0$	M1	Use $v^2 = u^2 + 2as$ vertically with $v = 0$ $\theta = 20.22908\ldots$	
		i	$\theta = 20.2$	A1	**Examiner's Comments** This question was generally well answered by the majority of candidates.	
		ii	$20\sin cv(\theta)t - 1/2gt^2 = 0$ AND range = $20\,cv(t)\cos cv(\theta)$	M1	Use $s = ut + ½at^2$ vertically with $s = 0$ OR use $v = u + at$ and doubles t AND horizontally with time found from vertical. (t = 1.4113… s or 1.4093…s (from 20.2))	
		ii	Range = 26.5 m	A1	Range = 26.48541… m or 26.45387….m (from 20.2)	
		ii	OR $\dfrac{20^2\sin(2\times cv(\theta))}{g}$	M1	Use of range formula Range = 26.48541… m or 26.45387….m (from 20.2)	
		ii	Range = 26.5 m	A1	**Examiner's Comments** There were two common approaches to the solution to this	

					question. Candidates either found the time of flight first and then the horizontal distance, or used the range formula, although this is not a requirement of the specification. The most common error was for those found the time to the greatest height first but not doubling this time when finding the range.	
			Total	4		
9		i	$x = vt\cos\theta$	B1	aef	
		i	$y = vt\sin\theta - ½gt^2$	B1	aef; may see this with t already eliminated	
		i		M1	Eliminate t	
		i	$y = x\tan\theta - \dfrac{4.9x^2}{v^2\cos^2\theta}$	A1	www; AG **Examiner's Comments** This question was answered correctly by nearly all candidates. A common mistake made was for a sign error in $s = ut + ½at^2$ using 9.8 instead of −9.8.	
		ii		M1	Attempt to substitute values into trajectory equation	
		ii	$-100 = 40\tan\theta - 4.9 \times 40^2/(16^2 \times \cos^2\theta)$	A1		
		ii	$(30.625\tan^2\theta - 40\tan\theta - 69.375 = 0)$	A1	aef; obtain correct quadratic in $\tan\theta$, may be unsimplified	
		ii	$(\tan\theta = 2.2937…$ or $-0.9876…)$	M1	Attempt to solve quadratic in $\tan\theta$	
		ii	$\theta = 66.4$	A1		
		ii	$\theta = -44.6$	A1	Allow 44.6 below the horizontal **Examiner's Comments**	

					A minority of candidates started again but most realised they simply had to substitute into the trajectory equation given in (i). There were a few cases where y = 0 was used; the more common error was to use 100 instead of -100. A necessary substitution was widely known. Some sign errors occurred when expanding -k(1 + tan²θ). The second solution of -44.6 was frequently converted to 135 but usually ISW could be applied and the mark awarded.	
			Total	10		
10		i	$0=(55\sin 20)t-\frac{1}{2}(9.8)t^2$ t = 3.84s OA = (55cos20)t = 198m	M1 A1 M1 A1 [4]	Use of $s=ut+\frac{1}{2}at^2$ vert. with s = 0 or use of $v = u + at$ with v = 0 and double t found or T = (2u sin θ) / g or use symmetry and $v = u + at$ or use OA found from equation of trajectory 3.83900… Use of $s = ut$ horizontally with cv(t) or R = (u^2 sin 2θ) / g or use equation of trajectory with y = 0 198.4114… **Examiner's Comments** This proved a good source of marks for the majority of candidates. The majority either used standard constant acceleration equations, or quoted and used standard results for time of flight and range of a projectile. The common error was to find only the time for the projectile to reach its maximum height.	

		ii	$v_x = 55 \cos 20$ $v_y = 55 \sin 20 - 9.8(2.6)$ $v = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$ or $\tan\theta = \frac{v_y}{v_x}$ $v = 52.1\text{m s}^{-1}$ $\theta = 7.35°$ below horizontal	B1 B1 M1 A1 A1 [5]	51.68309… ±6.668892… Use of Pythagoras or relevant trig on cv(v_x) and cv(v_y) 52.11157… AEF; 7.352494…; direction may be shown on diagram with minimum of arrow on resultant or arrows on both components Examiner's Comments The vast majority of candidates were able to pick up 4 of the 5 marks in this question. A few candidates failed to include the 'below' required for the direction and not all of these were able to pick up the mark by having a suitable diagram but on the whole the need to include 'below' now seems to be recognised.	
			Total	9		
11		a	DR $(25 \sin \alpha)t - 4.9t^2 = (15 \sin 2\alpha)t - 4.9t^2$ $25 \sin \alpha = 15 \sin 2\alpha$ $25 \sin \alpha = 30 \sin \alpha \cos \alpha \Rightarrow \cos \alpha = \ldots$	M1 (AO 3.3) A1 (AO 1.1) M1 (AO 2.1)	Use $s = ut + \frac{1}{2}at^2$ for both, and equate	

			$\cos\alpha = \frac{5}{6}$ $\left(\text{and } \sin\alpha = \frac{1}{6}\sqrt{11}\right)$ $(25\cos\alpha)t + (15\cos 2\alpha)t = 72 \Rightarrow t = \ldots$ $t = 2.7$ Height of C is $(25\sin\alpha)t - 4.9t^2 = 1.59$ m	A1 (AO 1.1) M1 (AO 3.3) A1 (AO 2.2a) A1 (AO 3.4) [7]	Correct use of double angle formula and attempt to solve for cos α / $\alpha = 33.557\ldots°$ Use $s = ut$ for both, equate total to 72 and attempt to solve for t 1.5910288…	
		b	DR $v_h = 25\cos\alpha$ $v_v = 25\sin\alpha - 9.8t$	B1ft (AO 3.4) B1ft (AO 3.4) M1 (AO 3.1a)	With their value of cos α / $v_h = 20.833\ldots$ With their values of sin α and t / $v_v = \pm 12.640\ldots$	

			$\tan\theta = \frac{v_v}{v_h}$ Direction is 31.2° below the horizontal	A1 (AO 3.2a) [4]	θ is angle with horizontal; condone sign error/ambiguity for this mark 31.24739…	
		c	e.g. include the dimensions of the footballs in the model of the motion e.g. use a more accurate value of g in the model of the motion e.g. include air resistance in the model of the motion	B1 (AO 3.5c) [1]	DR	
			Total	12		