1. In this question, the unit vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ are in the directions east and north.

Distance is measured in metres and time, t, in seconds.

A radio-controlled toy car moves on a flat horizontal surface. A child is standing at the origin and controlling the car.

When $t = 0$, the displacement of the car from the origin is $\begin{pmatrix}0\\-2\end{pmatrix}$ m, and the car has velocity $\begin{pmatrix}2\\0\end{pmatrix}$ m s^{-1}.

The acceleration of the car is constant and is $\begin{pmatrix}-1\\1\end{pmatrix}$ m s^{-2}.

i. Find the velocity of the car at time t and its speed when $t = 8$.

[4]

ii. Find the distance of the car from the child when $t = 8$.

[4]

2. The directions of the unit vectors **i** and **j** are east and north.

The velocity of a particle, **v** ms^{-1}, at time t s is given by

$$\mathbf{v} = (16 - t^2)\mathbf{i} + (31 - 8t)\mathbf{j}.$$

Find the time at which the particle is travelling on a bearing of 045° and the speed of the particle at this time.

[6]

3. The map of a large area of open land is marked in 1 km squares and a point near the middle of the area is defined to be the origin. The vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ are in the directions east and north.

At time t hours the position vectors of two hikers, Ashok and Kumar, are given by:

Ashok $\mathbf{r}_A = \begin{pmatrix}-2\\0\end{pmatrix} + \begin{pmatrix}8\\1\end{pmatrix}t,$

Kumar $\mathbf{r}_K = \begin{pmatrix}7t\\10-4t\end{pmatrix}.$

i. Prove that the two hikers meet and give the coordinates of the point where this happens.

[4]

ii. Compare the speeds of the two hikers.

[3]

4. A particle is initially at the origin, moving with velocity **u**. Its acceleration **a** is constant. At time t its displacement from the origin is **r** = $\begin{pmatrix}x\\y\end{pmatrix}$, where $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix}t - \begin{pmatrix}0\\4\end{pmatrix}t^2.$

i. Write down **u** and **a** as column vectors.

[2]

ii. Find the speed of the particle when $t = 2$.

[3]

iii. Show that the equation of the path of the particle is $y = 3x - x^2$.

[3]

5. A model boat has velocity v = $((2t - 2)\mathbf{i} + (2t + 2)\mathbf{j})$ m s^{-1} for $t \geq 0$, where t is the time in seconds.
i is the unit vector east and **j** is the unit vector north.
When $t = 3$, the position vector of the boat is $(3\mathbf{i} + 14\mathbf{j})$ m.

(a) Show that the boat is never instantaneously at rest. [2]

(b) Determine any times at which the boat is moving directly northwards. [2]

(c) Determine any times at which the boat is north-east of the origin. [5]

6. A toy car moves on a horizontal surface. Its velocity in m s^{-1} is given by

$$\mathbf{v} = 1.5\mathbf{i} + 0.5t\mathbf{j}$$

where **i** and **j** are unit vectors east (x-direction) and north (y-direction) respectively and t is the time in seconds.

Initially the car is at the point 2 m north of the origin.

(a) Calculate the speed of the car after 3 seconds. [2]

(b) Find the position vector of the car after t seconds. [3]

(c) Show that the cartesian equation of the path of the car is $y = \frac{x^2}{9} + 2$. [3]

7. In this question, $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ are unit vectors in the x- and y-directions.

A bird is flying in the vertical plane defined by these directions.

The origin is a point on the ground.

The position vector, **r** m, of the bird at time t seconds, where $t \geq 0$, is given by

$$\mathbf{r} = \begin{pmatrix}0\\8\end{pmatrix} + \begin{pmatrix}2\\-4\end{pmatrix}t + \begin{pmatrix}0\\1\end{pmatrix}t^2.$$

(i) Find the velocity of the bird when $t = 2.5$. [3]

(ii) Find the time at which the speed of the bird is 10 m s^{-1}. [3]

(iii) Find the times at which the bird is flying at an angle of 45° to the horizontal. [2]

8. The position vector **r** metres of a particle at time t seconds is given by

$$\mathbf{r} = (1 + 12t - 2t^2)\,\mathbf{i} + (t^2 - 6t)\mathbf{j}.$$

(a) Find an expression for the velocity of the particle at time t. [2]

(b) Determine whether the particle is ever stationary. [2]

9. In this question the unit vectors **i** and **j** are directed east and north respectively.

A model boat sails from a point A with an initial velocity of $-2\mathbf{j}$ m s^{-1}. It accelerates uniformly to a velocity of
$(4\mathbf{i} + 6\mathbf{j})$ms^{-1} in 8 s.

(a) Calculate the acceleration of the boat. [2]

(b) Find the time at which the boat is sailing due east. [2]

4 s after leaving A, the boat is at point B with position vector $(5\mathbf{i} - 2\mathbf{j})$m.

(c) Find the position vector of A. [4]

10. In this question the positive x and y directions are east and north respectively.
A model boat sails from the origin with initial velocity 3ms^{-1} due west and moves with acceleration
$\begin{pmatrix} -0.1 \\ 0.2 \end{pmatrix}$ m s^{-2} for 25 s.

(a) Show that the velocity of the boat after 25 s is $\begin{pmatrix} -5.5 \\ 5 \end{pmatrix}$ m s^{-1} . [3]

(b) Find the cartesian equation of the path of the boat. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		i	v = **u** + **a**t	M1	May be implied by either of the next two answers but not the final answer. Evidence of use of vectors in question necessary.
		i	Velocity $\mathbf{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \end{pmatrix} \left(= \begin{pmatrix} 2-t \\ t \end{pmatrix} \right)$	A1	
		i	When $t = 8$, $\mathbf{v} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$	A1	May be implied by the final answer
		i	speed $\sqrt{(-6)^2 + 8^2} = 10 \text{ m s}^{-1}$	A1	Cao but condone no units Give SC2 for 10 without working
		ii	$\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	4	Use of correct equation with substitution. Condone omission of $\mathbf{r}_0$.
		ii		M1	Or equivalent equation
		ii	$\mathbf{r} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} \times 8 + \frac{1}{2} \times \begin{pmatrix} -1 \\ 1 \end{pmatrix} \times 8^2$	A1	Condone omission of $\mathbf{r}_0$. Follow through for their value of **v**
		ii	$\mathbf{r} = \begin{pmatrix} -16 \\ 30 \end{pmatrix}$	A1	Cao but may be implied by a correct final answer.
		ii	Distance = 34 m	A1	Allow for 35.77... from $\mathbf{r} = \begin{pmatrix} -16 \\ 32 \end{pmatrix}$ and 37.57... from $\mathbf{r} = \begin{pmatrix} -16 \\ 34 \end{pmatrix}$ Examiner's Comments This question was about motion in two dimensions using column vectors. It was well answered. Such marks as

					were lost were usually as a result of candidates not fully answering the questions, omitting the velocity at time t and the speed in part (i) and the distance travelled in part (ii).
			Total	8	
2			Equate **i** and **j** components of **v**	M1	The candidate recognises that the **i** and **j** components must be equal.
			$16 - t^2 = 31 - 8t$	A1	An equation is formed.
			$t^2 - 8t + 15 = 0$		
			$(t - 3)(t - 5) = 0$		
			$t = 3$ or 5	A1	May be implied by later working.
			When $t = 3$, **v** = 7**i** + 7**j**	B1	
			Speed when $t = 3$ is $7\sqrt{2} = 9.9 \text{ m s}^{-1}$	B1	
			The values of the **i** and **j** components must both be positive for the bearing to be 045°.	B1	This mark is dependent on obtaining A1 for the result $t = 3$ or 5. It is awarded if the speed for the case when $t = 5$ is not included (since $t = 5 \Rightarrow$ v = −9**i** − 9**j** and the bearing is 225°).
					Note: Candidates who obtain **r** and equate the east and north components should be awarded SC1 for the whole question.
			Alternative trial and error		
			The **i** and **j** components of **v** must be equal	M1	The candidate recognises that the **i** and **j** components must be equal.
			The **i** and **j** components of **v** must both be positive for the bearing to be 045°.	B1	This can be demonstrated during the question either by a suitable convincing diagram including 45°, or by a suitable convincing argument.
			At least one value of t is substituted	A1	Trial and error is used
			$t = 3$	A1	$t = 3$ is found by trial and error
			When $t = 3$, **v** = 7**i** + 7**j**	B1	

			Speed when $t = 3$ is $7\sqrt{2} = 9.9 \text{ m s}^{-1}$	B1	
					Note Candidates who obtain **r** and equate the east and north components should be awarded SC1 for the whole question.
					<u>Examiner's Comments</u> In this question, candidates were given the velocity of a particle using **i**, **j** notation to denote east and north, and they were asked to find when it was travelling on a compass bearing of 045° and its speed at that time. This involved equating the components of **v**; this gave a quadratic equation, leading to two possible times. Candidates then had to recognise that at one of these times the bearing was 225° not 45°. Many candidates obtained full marks on this question. A few made the mistake of trying to work with position vector instead of the velocity. A common mistake was to fail to eliminate the 225° case. A small number of candidates set out to answer this question using a trial and error method and some credit was given for this.
			Total	6	
3		i	**Either** $-2 + 8t = 7t$ **Or** $t = 10 - 4t$	M1	Forming an equation for t. Accept vector equation for this mark. May be implied by a statement that $t = 2$.
		i	$\Rightarrow t = 2$	A1	
		i	Substituting $t = 2$ in **both** expressions	B1	oe, eg showing $t = 2$ satisfies both equations or a vector equation.
		i	They meet at (14, 2)	B1	Accept $\begin{pmatrix} 14 \\ 2 \end{pmatrix}$ Examiner's Comments Candidates were asked to prove the two hikers meet and this involved showing their position vectors are the same at some time ($t = 2$). Many lost a mark by not showing that this was true for both components when the position vectors were equated.

		ii	Ashok's speed is $\sqrt{8^2+1^2}=\sqrt{65}$	B1	
		ii	Kumar's speed is $\sqrt{7^2+(-4)^2}=\sqrt{65}\text{ km h}^{-1}$	B1	
		ii	They both walk at the same speed	B1	CAO from correct speeds SC1 for finding both velocities correctly but neither speed **Examiner's Comments** Candidates were asked to compare the speeds of the two hikers and most got this right. A few compared their velocities. Others did not know how to obtain the velocities from the given expressions in terms of *t* for the position vectors.
			Total	7	
4		i	**u** = $\begin{pmatrix}2\\6\end{pmatrix}$	B1	
		i	**a** = $\begin{pmatrix}0\\-8\end{pmatrix}$	B1	**Examiner's Comments** In this question the position of a particle at time *t* was given as a column vector. In part (i)candidates were asked to write down **u** and **a** as column vectors. Most were successful in this but a common mistake was to give **v** instead of **u**. Answer $\begin{pmatrix}2\\6\end{pmatrix}, \begin{pmatrix}0\\-8\end{pmatrix}$
		ii	v = **u** + **a***t*		
		ii	$t=2 \Rightarrow \mathbf{v}=\begin{pmatrix}2\\6\end{pmatrix}+\begin{pmatrix}0\\-8\end{pmatrix}\times 2$	M1	Or equivalent. FT for their **u** and **a**

		ii	$= \begin{pmatrix} 2 \\ -10 \end{pmatrix}$	A1	Continue the FT for this mark
		ii	speed = $\sqrt{2^2 + (-10)^2}$ = 10.2 ms^{-1} (to 3sf)	B1	FT from their v **Examiner's Comments** In the next part candidates were asked to find the speed at a certain time and this was wellanswered with many recovering from errors in part (i). Follow through was allowed for the values ofu and a that they found in part (i). Common mistakes were sign errors and not distinguishingbetween speed and velocity. Answer $\begin{pmatrix} 2 \\ -10 \end{pmatrix}$, 10.2 m s^{-2}
		iii	$x = ut \Rightarrow x = 2t \Rightarrow t = \frac{x}{2}$	M1	This mark may also be obtained for substituting x for $2t$ in the expression for y.
		iii	$y = 6t - 4t^2$	B1	
		iii	$y = 6 \times \frac{x}{2} - 4 \times \left(\frac{x}{2}\right)^2 = 3x - x^2$	A1	**Examiner's Comments** In the final part candidates were asked to show that the position vector at time t led to agiven cartesian equation for the path of the particle. This was answered confidently and almostentirely successfully.
		iii	**Alternative** $x = 2t$		
		iii	Substitute for x in given answer	M1	
		iii	$y = 3x - x^2 \Rightarrow y = 6t - 4t^2$	A1	
		iii	This is the given expression for y	B1	
			Total	8	

5		a	Require both components zero at the same *time* **i** component zero only when $t = 1$ and **j** component only when $t = -1$ so there are no such times	M1(AO3.1b) A1(AO2.4) [2]	May be implied but must be clear Or say **j** component ≥ 2 since $t \geq 0$	
		b	This requires use of the velocity vector Travelling due north means that the **i** component is zero and the **j** component +ve So we need $2t - 2 = 0$ for **i** component, giving $t = 1$. This gives **j** component $4 > 0$ so yes at $t = 1$.	M1(AO3.3) A1(AO2.4) [2]	Recognise velocity vector required Must test **j** component	
		c	This requires use of the position vector **either** $\mathbf{r} = \int \mathbf{v}\,dt$ so $\mathbf{r} = \int((2t-2)\mathbf{i} + (2t+2)\mathbf{j})dt =$ $(t^2 - 2t + C)\mathbf{i} + (t^2 + 2t + D)\mathbf{j}$ $\mathbf{r} = 3\mathbf{i} + 14\mathbf{j}$ when $t = 3$ so $C = 0$ and $D = -1$ so $\mathbf{r} = (t^2 - 2t)\mathbf{i} + (t^2 + 2t - 1)\mathbf{j}$ **Or** $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j}$ when $t = 3$ $\mathbf{v} = 4\mathbf{i} + 8\mathbf{j}$, $\mathbf{r} = (4\mathbf{i} + 8\mathbf{j})(t-3) + \frac{1}{2}(2\mathbf{i} + 2\mathbf{j})(t-3)^2 + 3\mathbf{i} + 14\mathbf{j}$ and so $\mathbf{r} = (t^2 - 2t)\mathbf{i} + (t^2 + 2t - 1)\mathbf{j}$ the boat is NE of O when the **i and j** components are equal and +ve	M1(AO3.1b) M1(AO1.1) A1(AO1.1) M1(AO1.1) A1(AO1.1) M1(AO3.2b) A1(AO2.1) [5]	Recognise position vector required May use + **C** instead Must find a but may omit 3**i** + 14**j**	

			we require $t^2 - 2t = t^2 + 2t - 1$ so $t = 0.25$ this gives components of − 0.4375 so no.		Award even if +ve not mentioned Must be complete argument
			Total	9	
6		a	When $t = 3$, $\|\mathbf{v}\| = \|1.5\mathbf{i} + 1.5\mathbf{j}\| = \sqrt{1.5^2 + 1.5^2}$ Speed is $\frac{3}{2}\sqrt{2} = 2.12\ \mathrm{m\,s^{-1}}$	M1(AO 1.1b) A1(AO 1.1b) [3]	Substitute for t and attempt modulus
		b	$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t = 1.5t\mathbf{i} + 0.5 \times \frac{t^2}{2}\mathbf{j} + \mathbf{c} = 1.5t\mathbf{i} + 0.25t^2\mathbf{j} + \mathbf{c}$ $t = 0$ and $\mathbf{r} = 2\mathbf{j} \Rightarrow \mathbf{c} = 2\mathbf{j}$ $\mathbf{r} = 1.5t\,\mathbf{i} + (0.25t^2 + 2)\mathbf{j}$	M1(AO 1.1a) A1(AO 1.1b) A1(AO 2.1) [3]	Attempt to integrate both terms Allow without + **c** Terms need not be collected
		c	$x = 1.5t$ and $y = 0.25t^2 + 2$ $y = 0.25\left(\frac{x}{1.5}\right)^2 + 2$	M1(AO 2.1) M1(AO 2.1) A1(AO 1.1b)	Allow for either equation Eliminate t

			$y=\frac{x^2}{9}+2$ **AG**	[3]	Must be fully justified
			Total	8	
7		i	Differentiating **r** $\mathbf{v}=\begin{pmatrix}2\\-4\end{pmatrix}+\begin{pmatrix}0\\2\end{pmatrix}t$ $\mathbf{v}=\begin{pmatrix}2\\1\end{pmatrix}$	M1 A1 B1 [3]	Attempt at differentiation must be seen Apply ISW for speed $=\sqrt{5}$ providing $\begin{pmatrix}2\\1\end{pmatrix}$ is seen. **Examiner's Comments** This question was about vectors, using the context of the flight of a bird. The position vector of the bird was given in term of the time. In part (i) candidates were asked about the velocity of the bird and this was well answered using vectors.
		ii	$\sqrt{2^2+(-4+2t)^2}=10$ $t^2-4t-20=0$ $t=\frac{4\pm\sqrt{4^2-4\times1\times-20}}{2}$ $(=6.898...\text{ or }-2.898...)$	M1 M1	Attempt at formulation of the given information using their vector **v** from part (i). Must involve both components. e.g. $-4+2t=\sqrt{96}$ Accept drawing triangle of velocities Attempted solution of an equation for *t*. Dependent on previous M mark Allow FT from their vector expression for v in part (i). Else CAO. Condone not giving the negative value of *t* as well as the correct value. Dependent on both M marks.

			$t = 6.9$ (or -2.9) (to 2 sf)	A1 [3]	**Examiner's Comments** In part (ii), the time was to be found at which the bird had a given speed. This involved using a vector expression to form a scalar equation. Many candidates did not know how to go about this. Others obtained the right answer but their explanations were not always the most elegant. However there were very good answers written by some candidates.
		iii	Either $2 = -4 + 2t \Rightarrow t = 3$ Or $-2 = -4 + 2t \Rightarrow t = 1$	B1 B1 [2]	FT from their vector expression for v in part (i). FT from their vector expression for v in part (i). **Examiner's Comments** In part (iii), candidates were asked to find the times when the bird was flying at an angle of 45° to the horizontal. Correct answers to this were somewhat uncommon. One common mistake was to equate the components of the position vector rather than the velocity; most of those who did use the velocity considered only the case when the bird was flying above the horizon and not when it was flying below it.
			Total	8	
8		a	$\mathbf{v} = \frac{d\mathbf{r}}{dt} = (12 - 2 \times 2t)\mathbf{i} + (2t - 6)\mathbf{j}$	M1 (AO 1.1a) A1 (AO 2.5) [2]	Attempt to differentiate at least one coefficient Must use vector notation **Examiner's Comments** Most candidates used vector notation accurately and successfully differentiated to obtain a correct expression for velocity.
		b	When $t = 3$ both components of velocity are zero,	M1 (AO 3.1a)	Equating at least one component of their vector velocity to zero Do not allow M1 for solving $12 - 4t = 2t - 6$ unless at least one zero subsequently established

			so the particle is stationary at $t = 3$.	E1 (AO 2.2a) [2]	Must be argued from two zero components **Examiner's Comments** This was typically well answered with most candidates realising the requirement for both components to be zero at the same time. **Misconception** It is not sufficient to equate the components and solve to find $t = 3$. From this starting point, candidates would need to check that the components were zero to achieve the method mark.
			Total	4	
9		a	$4\mathbf{i} + 6\mathbf{j} = -2\mathbf{j} + 8\mathbf{a}$ $\mathbf{a} = 0.5\mathbf{i} + \mathbf{j}$ m s^{-2}	M1 (AO 1.1a) A1 (AO 2.5) [2]	For use of vector equation $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, or alternatively for finding both acceleration components separately Must be vector form; do not allow final answer as separate components Do not allow for the magnitude of the acceleration unless final answer is labelled as such
		b	$\mathbf{v} = -2\mathbf{j} + (0.5\mathbf{i} + \mathbf{j})t$ is directed east when $-2 + t = 0$	M1 (AO 3.1b)	For equating **j** component of velocity to zero, giving equation

			$t = 2$, so boat sails east at time 2 s	A1 (AO 3.2a) [2]	for t cao	
		c	Displacement in 4 s is $(-2\mathbf{j})\times 4+\frac{1}{2}(0.5\mathbf{i}+\mathbf{j})\times 4^2$ $= 4\mathbf{i}\ (+\ 0\mathbf{j})$ Position at time $t = 4$ gives $\mathbf{r}_A + 4\mathbf{i} = 5\mathbf{i} - 2\mathbf{j}$ $\mathbf{r}_A = \mathbf{i} - 2\mathbf{j}$	M1 (AO 3.1b) A1 (AO 1.1) M1 (AO 3.1b) A1 (AO 1.1) [4]	For any use of $\mathbf{s}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with $t = 4$ and their **a** even if no clarity about displacement/ position For correct use of position vectors Must be in vector form	Equation may include initial position term $\mathbf{r}_A$ May be earned earlier, if original equation is $\mathbf{r}=\mathbf{r}_0+\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$
			Total	8		
10		a	Initial velocity is $\begin{pmatrix}-3\\0\end{pmatrix}$ $\mathbf{v}=\mathbf{u}+\mathbf{a}t=\begin{pmatrix}-3\\0\end{pmatrix}+\begin{pmatrix}-0.1\\0.2\end{pmatrix}25$	B1 (AO 2.1) M1 (AO 2.1) A1 (AO 2.5) [3]	For correct vector seen FT their initial velocity as long as it is a vector AG; must be in vector form	Allow full credit for an answer where the two components are considered separately only if the final answer is given as a vector

			$=\begin{pmatrix}-5.5\\5\end{pmatrix}$		
		b	$\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2=\begin{pmatrix}-3\\0\end{pmatrix}t+\frac{1}{2}\begin{pmatrix}-0.1\\0.2\end{pmatrix}t^2$ $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-3t-0.05t^2\\0.1t^2\end{pmatrix}$ Substitute $t^2=10y$ into equation for x $x=-3\sqrt{10y}-0.5y$	M1 (AO 3.1a) M1 (AO 3.1a) A1 (AO 1.1b) [3]	FT their **u** Attempt to eliminate t; FT their **r** Allow full credit for both components considered separately
			Total	6	