1. Ali is throwing flat stones onto water, hoping that they will bounce, as illustrated in Fig. 5.

Ali throws one stone from a height of 1.225 m above the water with initial speed 20 ms^{-1} in a horizontal direction. Air resistance should be neglected.

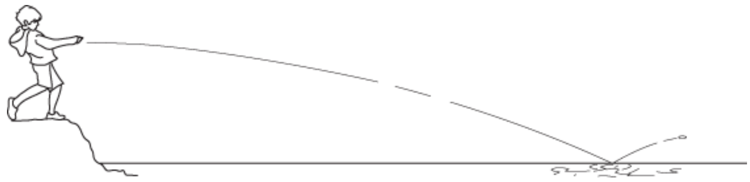

Fig. 5

i. Find the time it takes for the stone to reach the water.

[2]

ii. Find the speed of the stone when it reaches the water and the angle its trajectory makes with the horizontal at this time.

[5]

2. Fig. 4 illustrates a situation in which a film is being made. A cannon is fired from the top of a vertical cliff towards a ship out at sea. The director wants the cannon ball to fall just short of the ship so that it appears to be a near-miss. There are actors on the ship so it is important that it is not hit by mistake.

The cannon ball is fired from a height 75 m above the sea with an initial velocity of 20ms^{-1} at an angle of 30° above the horizontal. The ship is 90 m from the bottom of the cliff.

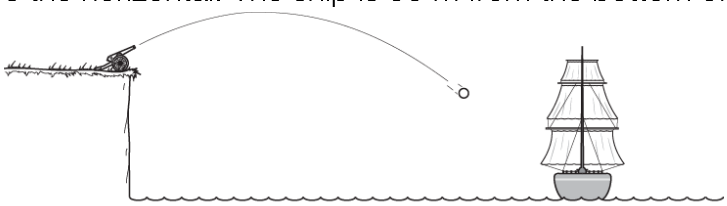

Fig. 4

i. The director calculates where the cannon ball will hit the sea, using the standard projectile model and taking the value of g to be 10ms^{-2}.

Verify that according to this model the cannon ball is in the air for 5 seconds.Show that it hits the water less than 5 m from the ship.

[6]

ii. Without doing any further calculations state, with a brief reason, whether the cannon ball would be predicted to travel further from the cliff if the value of g were taken to be 9.8 ms^{-2}.

[1]

3. In this question, air resistance should be neglected.

Fig. 2 illustrates the flight of a golf ball. The golf ball is initially on the ground, which is horizontal.

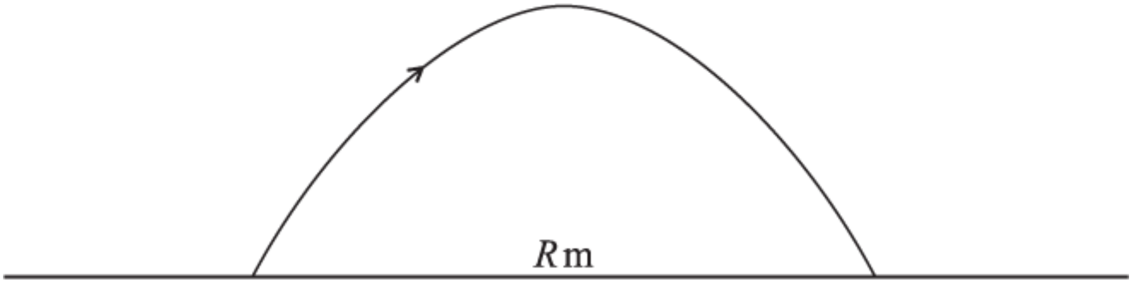

Fig.2

It is hit and given an initial velocity with components of 15 ms^{-1} in the horizontal direction and 20 ms^{-1} in the vertical direction.

i. Find its initial speed.

[1]

ii. Find the ball's flight time and range, R m.

[4]

iii.

A. Show that the range is the same if the components of the initial velocity of the ball are 20 ms^{-1} in the horizontal direction and 15 ms^{-1} in the vertical direction.

[1]

B. State, justifying your answer, whether the range is the same whenever the ball is hit with the same initial speed.

[2]

4. A golf ball is hit at an angle of 60° to the horizontal from a point, O, on level horizontal ground. Its initial speed is 20 m s^{-1}. The standard projectile model, in which air resistance is neglected, is used to describe the subsequent motion of the golf ball. At time *t* s the horizontal and vertical components of its displacement from O are denoted by *x* m and *y* m.

i. Write down equations for *x* and *y* in terms of *t*.

[2]

ii. Hence show that the equation of the trajectory is

$$y = \sqrt{3}x - 0.049x^2.$$

[2]

iii. Find the range of the golf ball.

[2]

iv. A bird is hovering at position (20,16).

Find whether the golf ball passes above it, passes below it or hits it.

[2]

5. Mr McGregor is a keen vegetable gardener. A pigeon that eats his vegetables is his great enemy.

One day he sees the pigeon sitting on a small branch of a tree. He takes a stone from the ground and throws it. The trajectory of the stone is in a vertical plane that contains the pigeon. The same vertical plane intersects the window of his house. The situation is illustrated in Fig. 5.

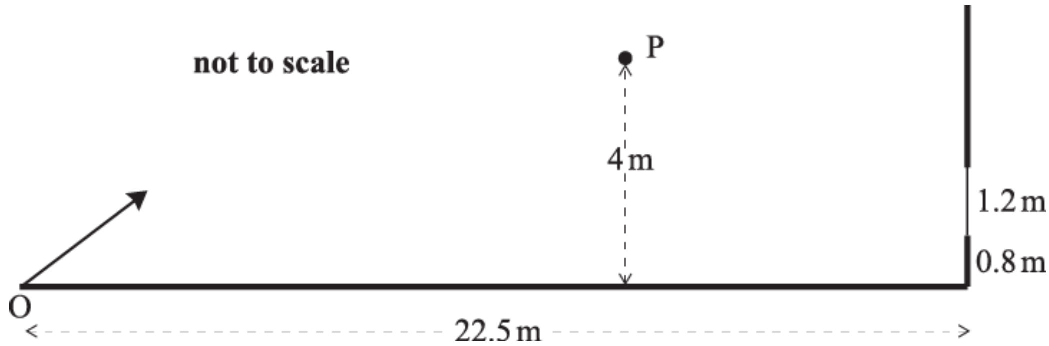

Fig. 5

- The stone is thrown from point O on level ground. Its initial velocity is 15 ms^{-1} in the horizontal direction and 8 ms^{-1} in the vertical direction.
- The pigeon is at point P which is 4 m above the ground.
- The house is 22.5 m from O.
- The bottom of the window is 0.8 m above the ground and the window is 1.2 m high.

Show that the stone does not reach the height of the pigeon.

Determine whether the stone hits the window.

[7]

6. **In this question take $g = 10$.**
A small stone is projected from a point O with a speed of 26 ms $^{-1}$ at an angle θ above the horizontal. The initial velocity and part of the path of the stone are shown in Fig. 7. You are given that $\sin\theta = \frac{12}{13}$. After t seconds the horizontal displacement of the stone from O is x metres and the vertical displacement is y metres.

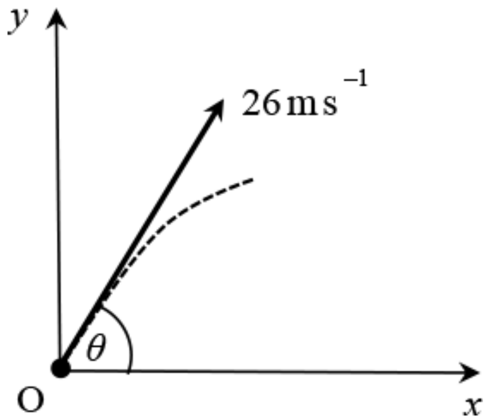

Fig.7

(a) Using the standard model for projectile motion,

- show that $y = 24t - 5t^2$,
- find an expression for x in terms of t.

[4]

The stone passes through point A. Point A is 16m above the level of O.
(b) Find the two possible horizontal distances of A from O. [4]

A toy balloon is projected from O with the same initial velocity as the small stone.
(c) Suggest two ways in which the standard model could be adapted. [2]

7. Arjun is trying to hit a can with a stone. The can is standing on a narrow wall 4 m away from him. The can is
10 cm tall and its base is 1.9 m above the ground, which is level. Arjun throws the stone at the can with a speed of 8 ms^{-1} at an angle of 35° above the horizontal. The point of projection is 1 m above the ground.

Determine whether the stone hits the can. [7]

8. **In this question you should use the standard projectile model with $g = 9.8$ ms^{-2}.**

Fig. 7 illustrates the trajectory of a tennis ball which has been served by a player. It is not drawn to scale.

- The ball must pass over the net and land in the service court.
- The player hits the ball at an angle of α above the horizontal.

Three junior members of a tennis club take turns to serve a tennis ball. They are Hamish (a beginner), Oscar (of medium standard) and Tara (a good player). They each stand at the same point and hit the ball in the same vertical plane at the same point P. The following figures apply to their serves.

- The player hits the ball from a height of 2.22 m.
- The height of the net is 0.995 m.
- The player is 12.5 m from the net.
- The ball must bounce within 6.5 m of the net.

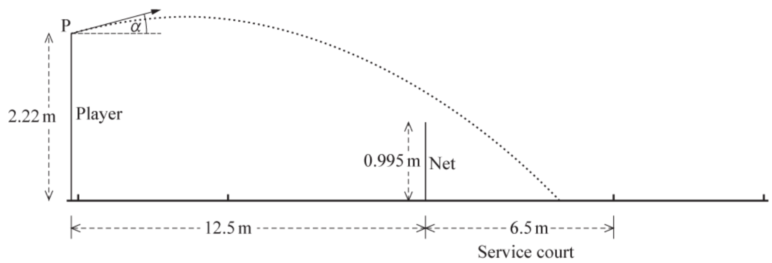

Fig. 7

Hamish serves the ball with components of velocity 10 m s^{-1} horizontally and 5.5 m s^{-1} vertically upwards.

(i) Find the speed of Hamish's serve and the value of α. **[2]**

(ii) Show that Hamish's serve passes over the net. **[3]**

(iii) Find the time at which Hamish's serve hits the ground.

Does it land in the service court? **[4]**

Oscar hits the ball horizontally, so $\alpha = 0$. The initial speed of the ball is u m s^{-1}.

(iv) Find the range of possible values of u for which the ball lands in the service court. **[6]**

Tara serves the ball at an angle of 2° below the horizontal. The ball clears the net and bounces after 0.57 seconds.

(v) Find the initial speed of Tara's serve. **[3]**

9. In this question, **i** is a horizontal unit vector and **j** is a unit vector directed vertically upwards.

A particle is projected from the origin with an initial velocity of $(u_1\mathbf{i} + u_2\mathbf{j})\text{ms}^{-1}$, and moves freely under gravity. Its position vector **r** m at time t s is given by

$$\mathbf{r} = (u_1\mathbf{i} + u_2\mathbf{j})\ t - 5t^2\mathbf{j}.$$

(a) Write down the value of g used in this model. [1]

(b) Explain what is meant by the statement that g is not a universal constant. [1]

The position vector of the particle when it reaches its maximum height is (14**i** + 20**j**) m.

(c) Determine the initial velocity of the particle, giving your answer as a vector. [7]

(d) The particle hits a building which is 21 m away from the origin in the **i** direction. Calculate the height above the level of the origin at which the particle hits the building. [3]

10. A pebble is thrown horizontally at 14 m s^{-1} from a window which is 5 m above horizontal ground. The pebble goes over a fence 2 m high d m away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the *x*-axis horizontal in the direction in which the pebble is thrown and the *y*-axis vertically upwards.

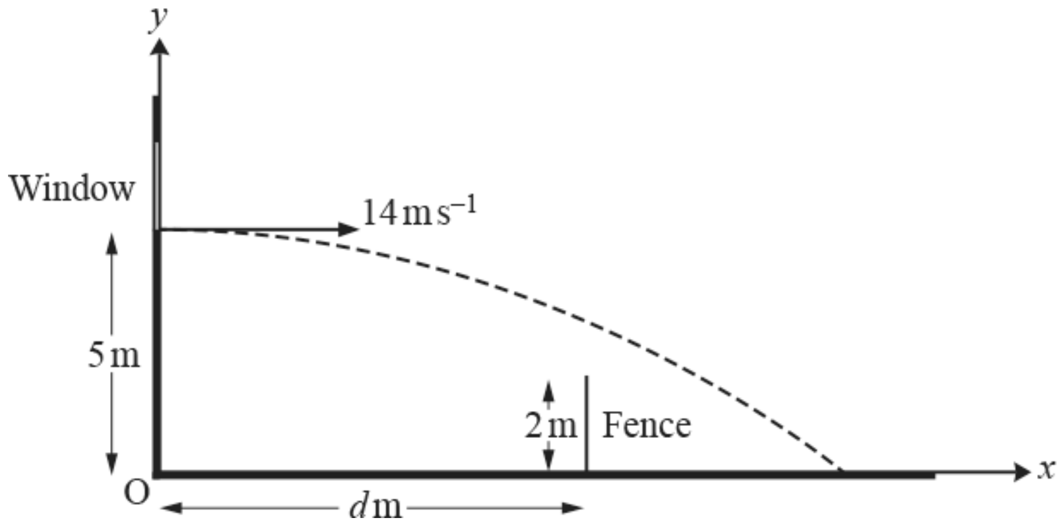

Fig. 9

(a) Find the time the pebble takes to reach the ground. [3]

(b) Find the cartesian equation of the trajectory of the pebble. [4]

(c) Find the range of possible values for *d*. [3]

11. A pebble is thrown horizontally at 21 m s^{-1} from a point 1.6 m above level ground. Calculate the horizontal distance travelled by the pebble before it hits the ground.

[4]

12. A goalkeeper kicks a football from ground level on a level playing field. The ball is in the air for 3.5 s.

(a) State a modelling assumption in the standard projectile model. [1]

(b) Calculate the vertical component of the initial velocity of the ball. [2]

(c) Calculate the maximum height of the ball. [2]

(d) The ball lands 42 m from its original position. Calculate

[3]

(i) the initial speed of the ball,

(ii) the angle that the initial velocity makes with the ground. [2]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		i	Vertical motion: $s = ut + \frac{1}{2}at^2$	2	
		i	At water: $-1.225 = 0 \times t + \frac{1}{2} \times (-9.8) \times t^2$	M1	Condone sign errors
		i	$\Rightarrow t = 0.5$s	A1	Signs must be consistent
		ii	Horizontal component of velocity = 20 m s^{-1}	B1	
		ii	Vertical component = 0.5 × 9.8 = 4.9 m s^{-1}	B1	Follow through for "their t × 9.8"
		ii	Speed $= \sqrt{20^2 + 4.9^2} = 20.6$	M1	Use of Pythagoras on previous two answers
		ii	$\tan\alpha = \frac{4.9}{20}$	M1	Use of an appropriate trig ratio with their figures for v. Must be explicit if final answer is incorrect.
		ii	α = 13.8°	A1	Coa **Examiner's Comments** In this question a model was presented for the familiar game of "ducks and drakes", skimming a stone along the surface of some water. The stone's initial velocity was in the horizontal direction and this presented a difficulty for the many candidates who did not infer that the vertical component of the initial velocity was zero; it was common to give it the value of the horizontal component (20 m s-1) instead. Consequently, although there were many fully correct answers to this question, there were also many that were worth few marks, if any.
			Total	7	
2		i	Vertical component of initial velocity= 20sin 30° (=10)	B1	

		i	Vertical motion $s = s_0 + ut + \frac{1}{2}at^2$	M1	Substitution required. The sign of g must be correct. Condone no s_0
		i	When it hits the sea $0 = 75 + 10t - 5t^2$	A1	
		i	$75 + 10 \times 5 - 5 \times 5^2 = 0$ As required		Or equivalent, eg solving the quadratic equation.
		i	This is satisfied when $t = 5$	E1	
		i	**Alternative**		
		i	Vertical component of initial velocity $= 20\sin 30°$ (=10)	B1	
		i	Vertical motion $v = u + at$ At the top $0 = 10 - 10t \Rightarrow t = 1$ It takes another 1 second to reach the level of the cliff top At that point its speed is 10 ms^{-1} downwards	M1	Complete method for finding $t = 5$ required.
		i	When it hits the sea $-75 = -10t - 5t^2$		Or equivalent finding the time (4 seconds) from the top (height 80 m) to hitting the sea
		i	$t^2 - 2t - 15 = 0 \Rightarrow t = 3$	A1	
		i	Total time = 1 + 1 + 3 = 5 seconds	E1	
		i	Horizontal motion $x = 20 \times \cos 30° \times t$ $t = 5 \Rightarrow 86.6$	M1	
		i	It is 3.4 m from the ship so within 5 m	E1	Condone 3.5 m
		ii	It is longer in the air so it goes further	B1	Justification for travelling further is required for this mark. **Examiner's Comments**

					This question was about projectiles and was well answered with many candidates gaining all the marks. Virtually all candidates knew what they were trying to do but many made sign errors in the vertical motion equation. The most straightforward approach to this question involved treating the motion in a single stage. A few candidates considered it in two, or even three, stages; this increased the scope for errors and consequently most such responses were less than perfect. The question ended by asking candidates to comment on the effect of taking a different value for g. This produced a pleasing number of highly articulate responses.
			Total	7	
3		i	Initial speed is 25 ms^{-1}	B1	<u>Examiner's Comments</u> This question was about a projectile (a golf ball). The horizontal and vertical components of its initial velocity were given. Nearly all candidates were able to find the initial speed and the flight time and range.
		ii	Vertical motion: $y = 20t - 4.9t^2$	M1	Forming an equation or expression for vertical motion
		ii	When $y = 0$,	M1	Finding t when the height is 0
		ii	$T = (0 \text{ or}) \ \dfrac{20}{4.9} = 4.08 \text{ s}$	A1	
		ii	$R = 15 \times 4.08\ldots = 61.22$	F1	Allow 15 × their T Note If horizontal and vertical components of the initial velocity are interchanged treat it as a misread; if no other errors are present this gives 3 marks.
		ii	Alternative Using time to maximum height		
		ii	Vertical motion: $v = 20 - 9.8t$	M1	Forming an equation or expression for vertical motion
		ii	Flight time = 2 × Time to top	M1	Using flight time is twice time to maximum height or equivalent for range.

		ii	$T = 2\times\frac{20}{9.8} = 4.08$ s	A1	
		ii	$R = 15 \times 4.08\ldots = 61.22$	F1	Allow 15 × their T
		ii	Alternative Using formulae		
		ii	Finding angle of projection		
		ii	$\alpha = \arctan\left(\frac{20}{15}\right) = 53.1°$	M1	Only award this mark if there is a clear intention to use this method
		ii	$R = \frac{2u^2 \sin\alpha\cos\alpha}{g} = \frac{2\times 25^2 \times \sin 53.1° \times \cos 53.1°}{9.8}$	M1	Allow the alternative form $R = \frac{u^2 \sin 2\alpha}{g}$ with substitution
		ii	$R = 61.2$	A1	
		ii	$T = \frac{2u\sin\alpha}{g} = 4.08$	A1	**Examiner's Comments** Common mistakes were to interchange the vertical and horizontal components, and, for those who used the method of finding the time to maximum height, to fail to double it for the flight time.
		iii	(*A*) $\text{Flight time } = \frac{15}{4.9}$		
		iii	$\text{Range } = 20\times\frac{15}{4.9} = 61.22$	B1	Allow FT from part (ii) for a correct argument that they should be the same **Examiner's Comments** In part (iii) (A), candidates were asked to show that the range was the same if the components of the initial speed were interchanged; most did this by repeating the calculation from part (ii) but a few saw that this result could be deduced from the form of the expression for the range.
		iii	(*B*) No	M1	Attempt at disproof or counter-example. There must be some reference to the angle.

		iii	eg angle of projection 45°	A1	Complete argument **Examiner's Comments** Candidates went into part (iii) (B) having just met an example where the same initial speed but a different angle of projection produced the same range; they were asked whether this was generally true. Many candidates saw the point of the question and gave a counter-example (commonly the ball being projected vertically upwards). However, others incorrectly thought that the statement was generally true. There were also many answers which gave an inadequate explanation of the correct result.
			Total	8	
4		i	$x = 10t$	B1	Allow $x = 20\cos 60°\ t$
		i	$y = 10\sqrt{3}t - 4.9t^2$	B1	Allow $y = 20\sin 60°\ t - \frac{g}{2}t^2$ or $y = 17.3\ t - \frac{9.8}{2}t^2$
		ii	Substitute $t = \frac{x}{10}$ n equation for y	M1	Substitution of a correct expression for t.
		ii	$\Rightarrow y = \sqrt{3}x - 0.049x^2$	A1	Notice that this is a given result
		iii	When $y = 0$, $x = \frac{1.732}{0.049}$ (or 0)	M1	Use of $y = 0$, or 2 × Time to maximum height
		iii	The range is 35.3 m	A1	
		iv	When $x = 20$, $y = 1.732 \times 20 - 0.049 \times 20^2$	M1	Use of equation of trajectory
		iv	Height is 15.04 m so passes below the bird whose height is 16 m	A1	
		iv			**Special Case** Allow **SC2** for substituting $y = 16$ in the trajectory, showing the equation for x has no real roots and concluding the height of the ball is always less than16 m. This can also be done with the equation for vertical motion.
		iv	Alternative: Using time		

		iv	When $x = 20$, $t = 2$		
		iv	$y = 10\sqrt{3} \times 2 - 4.9 \times 2^2$	M1	Use of equation of trajectory
		iv	Height is 15.04 m so passes below the bird whose height is 16 m	A1	
		iv	**Alternative: Maximum height**		
		iv	The maximum height of the ball (is 15.3 m)	M1	A valid method for finding the maximum height
		iv	Since 15.3 < 16, it is always below the bird	A1	**Examiner's Comments** This question started with finding the equation of the trajectory of a projectile and then went on to apply it to the flight of a golf ball. It was very well answered. It was particularly pleasing to see that almost all candidates were able to handle the algebra in the first two parts. In the final part candidates had to investigate whether the ball passed above, below or hit a hovering bird. This was easily done using the equation of the trajectory but some candidates found other interesting and valid methods.
			Total	8	
5			At maximum height	M1	For considering maximum height
			$v^2 - u^2 = 2as \Rightarrow 0^2 - 8^2 = 2 \times (-9.8) \times h$	M1	Use of suitable *suvat* equation(s) eg finding and using *t* formaximum height (0.816 s). Allow for use of calculus.
			$h = 3.265...$	A1	CAO but allow 3.26 as well as 3.27
			(3.265... <4) so the stone misses the pigeon	A1	Dependent on previous mark
			Alternative Substitute $y = 4$ in $y = 8t - 4.9t^2$	M1	
			Attempt to solve $4.9t^2 - 8t + 4 = 0$	M1	

			Discriminant (= 64 − 4 × 4.9 × 4 = − 14.4) < 0	A1	
			No value of t so the stone does not reach height 4 m	A1	
			Time to house is $\frac{22.5}{15}$ = 1.5 s	B1	
			Height at house = $8\times1.5-\frac{1}{2}\times9.8\times1.5^2$ = 0.975 m	B1	Allow answers from essentially correct working that round to 0.96, 0.97 or 0.98, eg 0.96375 from g = 9.81
			0.8 < 0.975 < 2.0 so it hits the window.	B1	A 2-sided inequality must be given, either in figures or in words.Condone 0.8 < 0.975 < 1.2 Dependent on previous mark **Examiner's Comments** This question was on projectiles. It involved Mr McGreggor throwing a stone at a pigeon,missing it and hitting the window of his house instead. It was extremely well answered. Although presented as a single question for 7 marks, it actually broke down into two parts: showingthat the stone did not go high enough to hit the pigeon and then showing that it did hit the window.Most candidates found the maximum height of the stone and showed that it was less than theheight of the pigeon. However, a considerable number substituted the height of the pigeon in thequadratic equation for the height of the stone at time t and then showed that this equation had noreal roots; this showed considerable mathematical understanding. Full marks were available foreither method and for any correct variant on them, for example working with the equation of thestone's trajectory. Most candidates found the correct height of the stone when it reached the house but many lost amark by failing to give a convincing argument that this height was within the interval for the window. Answers Max height of stone = 3.27 m, Height at the house = 0.975 m
			Total	**7**	
6		a	$y = ut\sin\theta - \frac{1}{2}gt^2$ stated and used	M1(AO3.3) E1(AO2.1)	

			$y = 26 \times \frac{12}{13}t - 5t^2$ $= 24t - 5t^2$ $x = 26 \times \frac{5}{13}t$ $= 10t$	M1(AO3.4) A1(AO1.1) [4]	AG Use of $\frac{5}{13}$ Accept any form	Given answer must be seen to score E1
		b	We require $16 = 24t - 5t^2$ Solving $5t^2 - 24t + 16 = 0$ $((5t - 4)(t - 4) = 0$ or ...) $t = 0.8$ or 4 Distances are $10 \times 0.8 = 8$ m and $10 \times 4 = 40$ m.	M1(AO3.4) M1(AO1.1) A1(AO1.1) B1FT(AO3.2a) [4]	Equating their y expression to 16 Method that could give 2 correct roots for their quadratic. Implied by 2 correct roots for their quadratic Cao FT only their t	
		c	E.g. Air resistance should be included E.g. The balloon should not be treated as a particle E.g. Horizontal force due to wind should be considered	B1(AO3.5c) B1(AO3.5c)	Any two appropriate factors that would have an impact on the model.	

				[2]	
			Total	10	
7			Horizontal motion: $x = (8\cos35)t$	B1(AO 3.3)	
			Vertical motion: $y = (8\sin 35)t - 4.9t^2 + 1$	M1(AO 3.3)	
				M1(AO 3.1b)	Allow for RHS with first two terms only
			Time to wall: $(8\cos35)t = 4$	A1(AO 1.1b)	
				M1(AO 1.1a)	Attempt to find t when $x = 4$
			$t = 0.6104$	A1(AO 1.1b)	Allow 0.61 or better
			Height at this time: $(8\sin 35)0.6104 - 4.9(0.6104)^2 + 1$	E1(AO 3.2a)	Substitute their value of t in their y
			$y = 1.975$		
				B1	Allow 0.975 only if compared with 0.9
			This is between 1.9 and 2.0 so the stone hits the can	M1	Comment must be supported by evidence
			Alternative method Horizontal motion: $x = (8\cos35)t$	M1	May be implied if trajectory eqn is quoted
			Vertical motion: $y = (8\sin 35)t - 4.9t^2 + 1$	A1	May be implied if trajectory eqn is quoted
			Trajectory: $y = (8\sin 35)\left(\frac{x}{8\cos 35}\right) - 4.9\left(\frac{x}{8\cos 35}\right)^2 + 1$	M1	Allow without the '+1' if subsequent work is all consistent with origin at height

			Answer	Marks	Guidance	
			Height at $x=4:\ (8\sin 35)\left(\frac{4}{8\cos 35}\right)-4.9\left(\frac{4}{8\cos 35}\right)^2+1$ = 1.975 This is between 1.9 and 2.0 so the stone hits the can	A1 E1 [7]	1 Comment must be supported by evidence	
			Total	7		
8		i	Initial speed $=\sqrt{10^2+5.5^2}=11.412\ldots$ so 11.4 m s-1 (to 1 dp) $\alpha=\arctan\left(\frac{5.5}{10}\right)=28.810\ldots$ so 28.8° (to the nearest 0.1°)	B1 B1 [2]		
		ii	Horizontal motion: Time to net 10t = 12.5 so 1.25 s Vertical motion $s=s_0+ut+\frac{1}{2}at^2$ y = 2.22 + 5.5 × 1.25 − 4.9 × 1.25^2 = 1.43875	B1 M1	A complete method for finding the height of the ball when it crosses the net. With the point of projection as the origin for vertical motion, the distance fallen in 1.25 s is 0.78125 m and 2.22 − 0.78125 =1.43875	

			Answer	Marks	Guidance
			This is greater than 0.995 so the ball goes over the net.	A1	Conclusion stated
				[3]	
			Alternative using time to net level and horizontal distance		
			$0.995 = 2.22 + 5.5t - 4.9t^2 \Rightarrow t = 1.313$		
			Horizontal distance $10 \times 1.313 = 13.13$	B1	A complete method for finding the position of the ball when it is at the height of the top of the net.
				M1	Conclusion stated.
			13.13 > 12.5 so the ball passes over the net	A1	
			$$y = y_0 + x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$		
			$y = 2.22 + 0.55x - 0.049x^2$		
			$x = 12.25$	B1	
			$\Rightarrow y = 1.438\ldots > 0.995$	M1	
				A1	
				[3]	
		iii	Vertical motion $s = s_0 + ut + \frac{1}{2}at^2$	M1	
			$0 = 2.22 + 5.5t - 4.9t^2$		Setting up an equation for vertical motion containing the right elements. (Vertical velocity on landing = 8.59

					m s−1)
			$t=\dfrac{5.5\pm\sqrt{5.5^2+4\times4.9\times2.22}}{2\times4.9}=1.437...\ (\text{or }-0.315...)$	A1	
			Horizontal motion x = 10×1.437… = 14.37…	M1	Allow for 10 × their time. This may be implied.
			14.37… < 19 so the ball does land in the service court	A1	Conclusion stated. FT for their value of t.
				[4]	
			$y = 2.22 + 0.55x - 0.049x^2$	M1	
			$x = 19$ ⇒ $y = -5.019$ | Or $y = 0$: $x = 14.376…$ (or $-3.151…$)	M1	Dependent on both M marks
				A1	
			So the ball lands in the service court	A1	
				[4]	
		iv	**Clearing the net**		
			The ball falls 2.22 − 0.995 = 1.225 m to the height of the net		
			Time taken is given by $1.225 = 4.9t^2$	M1	

			So $t = 0.5$ Speed must be greater than $\dfrac{12.5}{0.5} = 25\ \text{m s}^{-1}$ **Not going too far** Time to fall to the ground is given by $2.22 = 4.9t^2$ So $t = 0.673\ldots$ Horizontal distance must not exceed 19 m Maximum speed $= \dfrac{19}{0.673\ldots} = 28.227\ldots\ \text{m s}^{-1}$ **(Overall)** (So the ball's speed must be between 25 and 28.2 m s^{-1}.) $y = 2.22 - 0.049\left(\dfrac{x}{u}\right)^2$ **Clearing the net** To clear the net $2.22 - 4.9\left(\dfrac{12.5}{u}\right)^2 > 0.995$	A1 A1 M1 A1 A1 [6]	The value of t can be implied and need not be seen. The value of t can be implied and need not be seen.

			$\Rightarrow \left(\frac{u}{12.5}\right)^2 > \left(\frac{4.9}{1.225}\right)$ Speed must be greater than $\frac{12.5}{0.5} = 25 \text{ m s}^{-1}$ **Not going too far** To land inside the service court, horizontal distance must not exceed 19 m $\Rightarrow 2.22 - 4.9 \times \left(\frac{19}{u}\right)^2 < 0$ $\frac{u}{19} < \sqrt{\frac{4.9}{2.22}}$ $u < 28.227$ Maximum speed = 28.227… ms^{-1} (So the ball's speed must be between 25 and 28.2 m s^{-1}.)	M1 A1 B1 M1 A1 A1 [6]	
		v	Vertical motion $s = s_0 + ut + \frac{1}{2}at^2$ **Using *u* to be the initial speed**	M1	An equation for vertical motion which could be used to find *u*. It must contain all three elements. No sin-cos

			Answer	Marks	Guidance
			$0 = 2.22 - u \times \sin 2° \times 0.57 - 4.9 \times 0.57^2$		interchange. If sin2° is not seen use the alternative method. The equation must be correct including signs.
			$u = \dfrac{2.22 - 4.9 \times 0.57^2}{0.57 \times \sin 2°}$	A1	CAO
			u = 31.568… so the speed of Tara's serve is 31.6 m s^{-1}	A1 [3]	Or equivalent for vertical motion upwards
			Alternative Using *U* as the initial vertical component downwards $0 = 2.22 - U \times 0.57 - 4.9 \times 0.57^2$	M1	
			$U = \dfrac{2.22 - 4.9 \times 0.57^2}{0.57} = 1.10173...$	A1	The value of *U* is calculated correctly. It should be negative if the direction of U is upwards.
			$\text{Speed} = \dfrac{U}{\sin 2°} = 31.568...$ So the speed of Tara's serve is 31.6 m s^{-1}	A1	**Examiner's Comments** This was the second of the two long questions. It was based on the context of tennis players serving. There were several points that candidates had to take into account: the ball was served from a given height; it had to pass over the net; it had to land in the service court. In addition the three players served with different speeds and at different angles to the horizontal. All of this meant that a significant amount of analysis was required and as a result some candidates were not successful on the later parts of the question. In parts (i) to (iii) the serve was modelled as a projectile with given horizontal and vertical speeds. Most candidates were reasonably successful on all three parts: finding the speed and angle of projection in part (i), showing the ball passed over the net in part (ii) and finding out whether the ball landed in the service court in part (iii). The most common mistake was confusing horizontal and vertical components of the motion; there were also many sign errors.

					In part (iv) a different player was serving, this time horizontally. The question asked candidates to find the range of possible values of the initial speed for the serve to land in the service court. This involved essentially the same work as parts (i) to (iii) although the situation was actually simpler with no vertical component of the initial velocity. However, no guidance was given and so candidates were required to analyse the situation; a substantial minority of candidates failed to do so and scored no marks. Among those candidates who did come to terms with the situation, some obtained both limits for the initial speed but many made a mistake with the lower limit, finding the minimum initial speed for the ball to reach the net without bouncing rather than to pass over the net. In part (v) a third player served with initial direction below the horizontal. Only a minority of candidates scored any marks on this question and, among those who did, sign errors were quite common.
			Total	18	
9		a	10 (m s^{-2})	B1(AO 3.4) [1]	
		b	*g* varies according to location	B1(AO 1.2) [1]	
		c	$v = (u_1 i + u_2 j) - 10tj$ Maximum height when $u_2 - 10t = 0$ $t = \frac{u_2}{10}$ $(u_1\mathbf{i} + u_2\mathbf{j})\frac{u_2}{10} - 5\mathbf{j}\left(\frac{u_2}{10}\right)^2 = 14\mathbf{i} + 20\mathbf{j}$	M1(AO3.1b) M1(AO3.1b) A1(AO1.1b) M1(AO1.1a) A1(AO1.1b) M1(AO3.1b)	Differentiation of r to find v Equating their j component to zero Equate r with their *t* to given vector cao, from equating j

			Answer	Marks	Guidance	
			$\frac{u_2^2}{10} - \frac{5u_2^2}{100} = 20 \Rightarrow u_2 = 20$	A1(AO2.5)	components	
			$u_1 \times \frac{20}{10} = 14 \Rightarrow u_1 = 7$	M1	Equating i components to find u_x	
				M1	Must be in vector form	
			Initial velocity is (7i + 20j) m s^{-1}			
			Alternative solution	A1		Accept $\begin{pmatrix} 7 \\ 20 \end{pmatrix}$
			Vertical motion has $u = u_2$, $v = 0$, $a = -10$, $s = 20$	M1	Use of *suvat* equation(s) leading to u_2	
			$0 = u_2^2 + 2 \times (-10) \times 20$	A1	cao	
				M1	Use of *suvat* equation(s) leading to *t*	
			$u_2 = 20$			
			$0 = 20 - 10t$	A1	Use of their *t* in constant speed eqn	
				[7]	Must be in vector form	
			$t = 2$			
			Horizontal motion: $14 = u_1 \times 2$ &⇒ $u_1 = 7$			
			Initial velocity is (7i + 20j) m s^{-1}			

						Accept $\begin{pmatrix} 7 \\ 20 \end{pmatrix}$
		d	$21 = 7t$ &⇒ $t = 3$ Height at $t = 3$ is $20 \times 3 - 5 \times 3^2$ = 15 m	M1(AO3.1b) M1(AO1.1a) A1(AO3.2a) [3]	Finding t from horizontal motion Use of $s = ut + \frac{1}{2}at^2$ with their t	
			Total	12		
10		a	Vertical motion $u = 0$ $s = ut + \frac{1}{2}at^2$ $-5 = 0 - \frac{9.8}{2}t^2$ $t = \sqrt{\frac{10}{9.8}} = 1.01\text{ s}$	B1 (AO 3.3) M1 (AO 3.4) A1 (AO 1.1b) [3]	Using $u = 0$ in the vertical direction to model horizontal motion soi Using suvat equation(s) to find t. Allow sign errors and incorrect value for u. Must follow from working where the signs are consistent. Examiner's Comments Most candidates realised that the initial velocity in the vertical direction was zero and successfully completed this question.	

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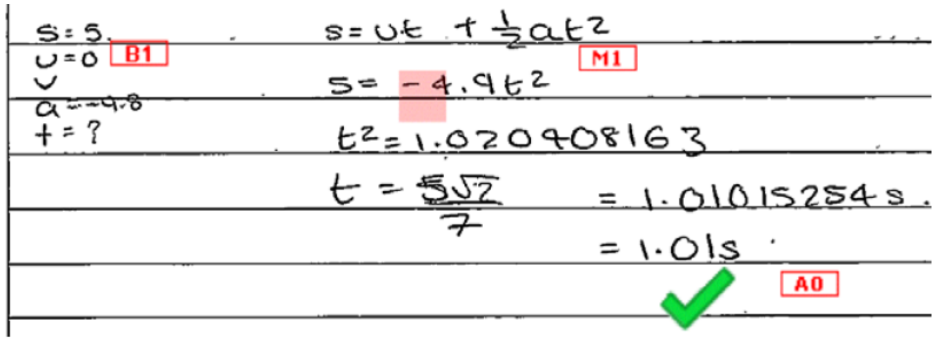
<td>Exemplar 3


This candidate was credited B1 using u = 0 and M1 for the equation with a sign error. Although the correct answer is seen, it comes from incorrect working and was therefore not credited the final mark. This candidate obviously recognised that there was an issue with the working and should have gone back to identify and correct their mistake ($s = -5$ or a = + 9.8).

AfL It is important to make use of a consistent sign convention, with acceleration and displacement either both positive or both negative. A clear statement of the positive direction at the start of the answer helps avoid problems.</td>
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<tr>
<td></td><td></td><td>b</td>
<td>$x = 14t$

$y = 5 - 4.9t^2$

So cartesian equation is

$$y = 5 - 4.9\left(\frac{x}{14}\right)^2 \left[= 5 - \frac{x^2}{40}\right]$$</td>
<td>B1 (AO 3.3)
B1 (AO 3.3)

M1 (AO 1.1a)
A1 (AO 1.1b)
[4]</td>
<td><table><tr><td>May be implied

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Attempt to eliminate t
Any form</td><td></td></tr></table>
<u>Examiner's Comments</u></td>
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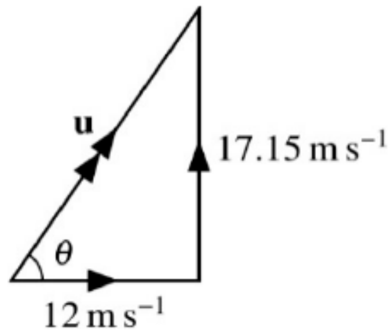
			Answer	Marks	Guidance	
					Many correct answers were seen. The most common error was to omit the initial height of the pebble. The origin is given in the question, so the correct equation is $y = 5 - 4.9t^2$. AfL Take careful note of the origin and remember to include the initial position in the equations.	
		c	EITHER When $y = 2$: $y = 5 - \frac{x^2}{40} = 2\text{ m}$ $\frac{x^2}{40} = 3$ $x = \sqrt{120} = 10.9544...$ [0 <]d<11.0 m	M1 (AO3.4) A1 (AO1.1b) E1 (AO3.2a) [3]	Using their equation of trajectory and $y = 2$ Must be 11.0 or better Allow "Fence must be less than 10.95 m from the origin." FT their value	SC 2 for $d < \sqrt{80}\ [= 8.94]$ SC 2 for $d = \sqrt{80}\ [= 8.94]$
			OR When $y = 2$ $2 = 5 - 4.9t^2$ $t = 0.782$ When $t = 0.782$ $x = 14 \times 0.782 = 10.95$	M1 A1 A1 [3]	Both steps required for M1 Must be 11.0 or better Allow "Fence must be less than 10.95 m from the origin."	Allow if the origin is taken to be at window height and the top of the wall is 3m below the window. Signs must be consistent for A1

			[0 <]d<11.0 m		**Examiner's Comments** The question was designed so that the simplest way to answer this was to substitute $y = 2$ in the equation of the trajectory leading to $x = 10.95$. Common sense was enough to use this as a boundary value for the inequality – the pebble would go over the wall if it were nearer the window than that value. Many candidates went back to the original model, found the time to drop to the height of the wall, used that to work out the boundary value for d, and received full credit.
			Total	10	
11			Vertical motion using $u = 0$ Using $s = ut + \frac{1}{2}at^2$ with $s = -1.6$ to find t $-1.6 = -4.9t^2$ $\Rightarrow t = \frac{4}{7}$ Horizontal distance is $21 \times \frac{4}{7} = 12$ m	M1 (AO 3.3) M1 (AO 3.1b) A1 (AO 1.1) B1 (AO 1.1) [4]	May be implied For complete method to find t Allow for 0.571 or better FT their t; dependent on 2nd M mark Allow $+s$ with $+a$
			Total	4	
12		a	Resistance to the motion is modelled as being negligible	B1 (AO 3.3)	Or similar Or any correct assumption e.g. motion is in a vertical plane; football is modelled as a

				[1]	particle; acceleration is constant…
		b	Vertical motion: $0 = 3.5u - \frac{1}{2} \times 9.8 \times 3.5^2$ $u = 17.15$, so velocity component is 17.2 m s^{-1} (to 3sf)	M1 (AO 3.1a) A1 (AO 1.1b) [2]	Use of *suvat* equation(s) to find *u*. using either $s = 0$, $g = -9.8$ and $t = 3.5$ or $v = 0$, $g = -9.8$ and $t = 1.75$
		c	$s = 17.15 \times 1.75 - \frac{1}{2} \times 9.8 \times 1.75^2$ Maximum height is 15.0 m	M1 (AO 1.1a) A1 (AO 1.1b) [2]	Use of *suvat* equation(s) to find *s*, using any of $u =$ (their) 17.15, $v = 0$, $t = 1.75$, $g = -9.8$ Allow for 15 or better
		d	(i) Horizontal velocity component is $\frac{42}{3.5} = 12$ Initial speed $= \lvert \mathbf{u} \rvert = \sqrt{17.15^2 + 12^2}$ $= 20.9$ m s^{-1}	B1 (AO 3.1a) M1 (AO 1.1a) A1 (AO 1.1b) [3] M1 (AO 1.1a) A1 (AO 1.1b)	Use of Pythagoras FT their components

			(ii) Angle θ with horizontal is given by $$\tan\theta = \frac{17.15}{12}$$ Angle with horizontal is 55.0°	[2]	Allow reciprocal for M mark only FT their components; allow 55° or better 
			Total	10	