1.

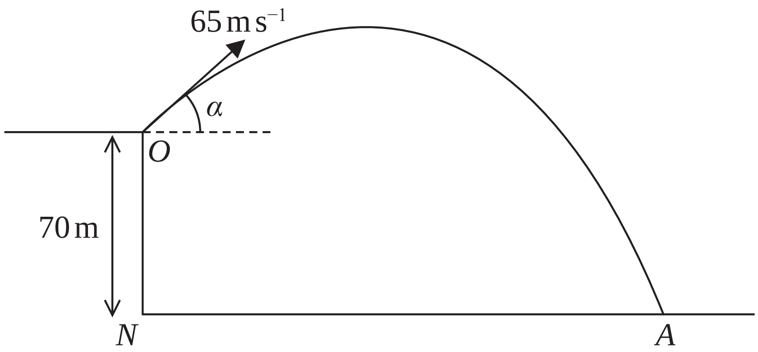


Figure 3

A small stone is projected with speed 65 m s^{-1} from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude 10 m s^{-2}

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

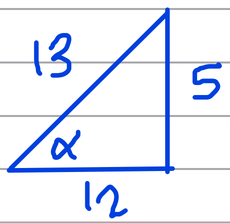
(5)

One limitation of the model is that it ignores air resistance.

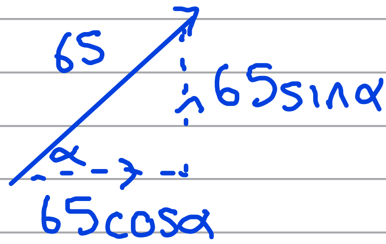
(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

a) $\tan\alpha = 5/12$



$\therefore \sin\alpha = 5/13$
$\cos\alpha = 12/13$



motion from O to A

vertical $(\uparrow +)$

$s = -70$

$u = 65\sin\alpha = 25$

$v =$

$a = -10$

$t = t$

using $s = ut + \frac{1}{2}at^2$ 1

$-70 = 25t - 5t^2$ 1

$5t^2 - 25t - 70 = 0$

$t^2 - 5t - 14 = 0$

$(t-7)(t+2) = 0$

$t = 7$ or $t = -2$

$t > 0 \therefore t = 7$ 1

b) speed $= \sqrt{(\text{horiz. velocity})^2 + (\text{vert. velocity})^2}$

vertical $(\uparrow +)$

$s = -70$

$u = 25$

$v = v$

$a = -10$

$t = 7$

$v = u + at$

$= 25 - 10 \times 7$ 1

$= -45$

$= 45\,\text{ms}^{-1}$ downwards 1

horizontal $(\rightarrow +)$

$s =$

$u = 65\cos\alpha = 60$ 1

$v = v$

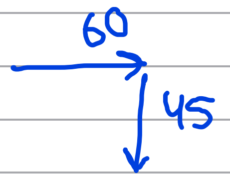
$a = 0$

$t = 7$

$v = u + at$

$v = 60$

$= 60\,\text{ms}^{-1}$ to the right



1

speed $= \sqrt{60^2 + 45^2} = 75\,\text{ms}^{-1}$ 1

c) $g = 10$ instead of $g = 9.8$ has been used 1

2.

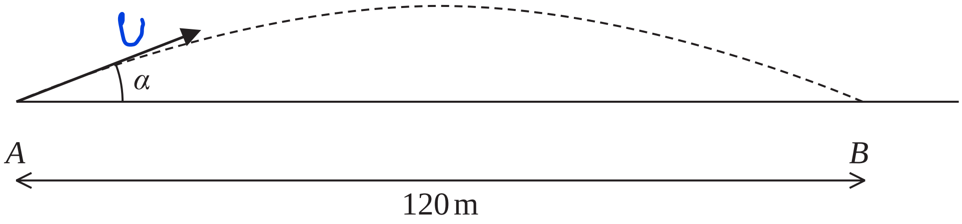


Figure 3

A golf ball is at rest at the point A on horizontal ground.

The ball is hit and initially moves at an angle α to the ground.

The ball first hits the ground at the point B, where $AB = 120\,\text{m}$, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is $U\,\text{m}\,\text{s}^{-1}$

Using this model,

(a) show that $U^2 \sin\alpha \cos\alpha = 588$

(6)

The ball reaches a maximum height of 10 m above the ground.

(b) Show that $U^2 = 1960$

(4)

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from A to B, is now modelled as that of a particle whose initial speed is $V\,\text{m}\,\text{s}^{-1}$

This refined model is used to calculate a value for V

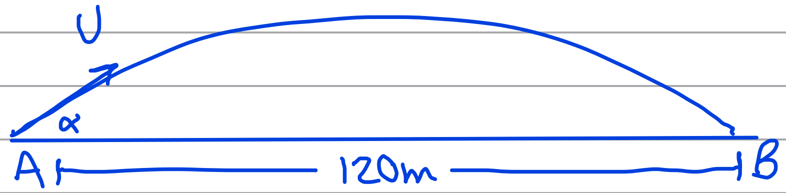
(c) State which is greater, U or V, giving a reason for your answer.

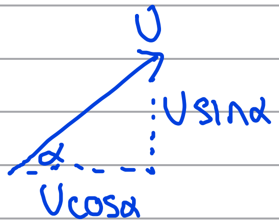
(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

a)





Motion AB: Method: eliminate t to form an equation in terms of only U and α .

Horizontal (→ +)

$s = 120$ (1)
$u = U\cos\alpha$
$v =$
$a = 0$
$t = t$

$$s = ut + \frac{1}{2}at^2$$

$$120 = Ut\cos\alpha \quad (1)$$

$$\Rightarrow t = \frac{120}{U\cos\alpha}$$

Vertical (↑ +)

$s = 0$ (1)
$u = U\sin\alpha$
$v =$
$a = -9$
$t = \frac{120}{U\cos\alpha}$ (1)

$$s = ut + \frac{1}{2}at^2$$

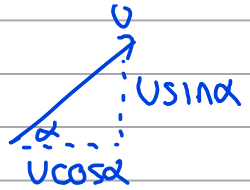
$$0 = \frac{120U\sin\alpha}{U\cos\alpha} - \frac{9}{2}\left(\frac{120}{U\cos\alpha}\right)^2 \quad (1)$$

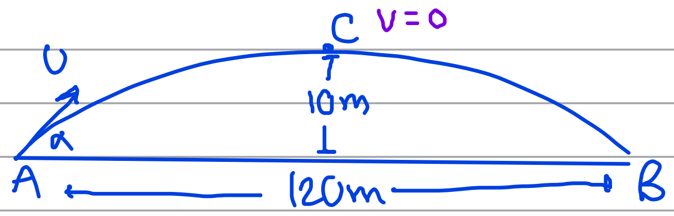
$$0 = \frac{120\sin\alpha}{\cos\alpha} - 4.9\left(\frac{14{,}400}{U^2\cos^2\alpha}\right)$$

$$0 = 120U^2\sin\alpha\cos\alpha - 70560$$

$$120U^2\sin\alpha\cos\alpha = 70560$$

$$U^2\sin\alpha\cos\alpha = 588 \quad (1)$$

b) 



Motion AC:

Vertical: (↑+) ①

$s = 10$ $\qquad v^2 = u^2 + 2as$

$u = U\sin\alpha$

$v = 0$ $\qquad 0 = U^2\sin^2\alpha - 20g$ ①

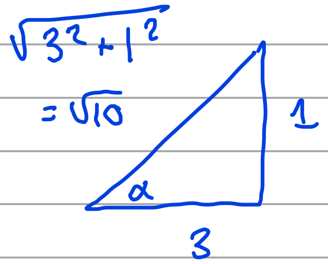
$a = -g$

$t =$ $\qquad U^2\sin^2\alpha = 196$ ①

from (a), $U^2\sin\alpha\cos\alpha = 588$ ②

①÷②: $\dfrac{U^2\sin^2\alpha}{U^2\sin\alpha\cos\alpha} = \dfrac{196}{588}$ ①

$\tan\alpha = \dfrac{1}{3}$



$\therefore \sin\alpha = \dfrac{1}{\sqrt{10}} \Rightarrow \sin^2\alpha = \dfrac{1}{10}$

①: $U^2 = \dfrac{196}{1/10} = 1960$ ①

we dont need to worry about the sign of sinα; α is acute so sinα, cosα, and tanα are all positive.

c) V must be greater as air resistance has to be overcome ①

d) consider the dimensions of the ball. ①

3.

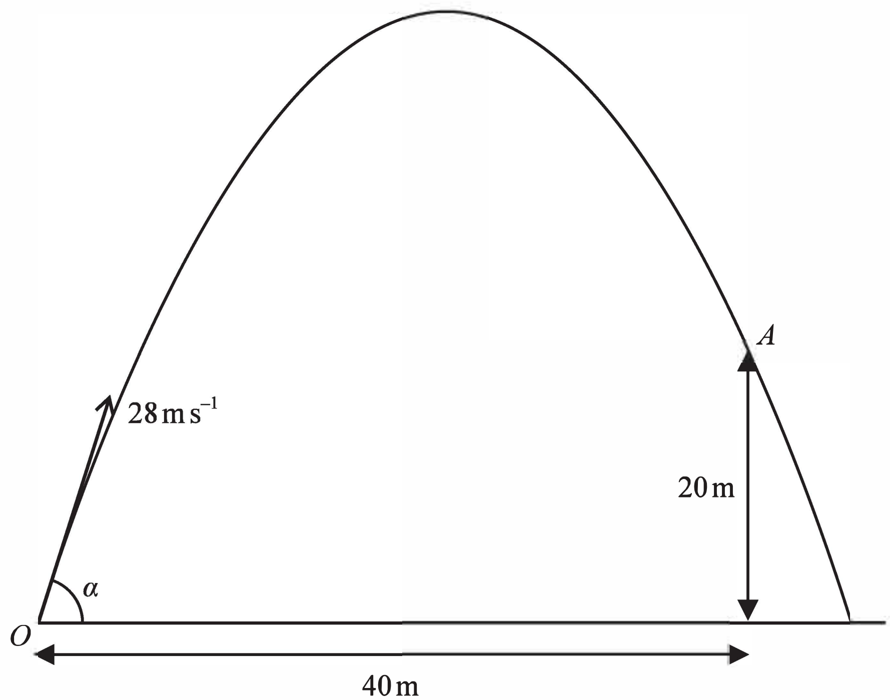


Figure 2

A small ball is projected with speed 28 m s^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

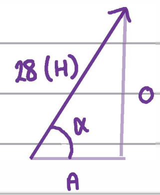
(3)

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

(a)



Initial speed (28) has horizontal and vertical components.

Horizontal (A) $= 28\cos\alpha$
Vertical (O) $= 28\sin\alpha$
} using SOHCAHTOA

Horizontally:

$$28\cos\alpha = \frac{40}{T} \quad 1 \quad \leftarrow \text{speed} = \frac{\text{distance}}{\text{time}}$$

$$28\cos\alpha \times T = 40$$

$$T = \frac{40}{28\cos\alpha}$$

$$T = \frac{10}{7\cos\alpha} \quad 1$$

(b) Vertically:

$$20 = (28\sin\alpha \times T) + \left(\tfrac{1}{2} \times -g \times T^2\right) \quad 1 \quad \leftarrow s = ut + \tfrac{1}{2}at^2$$

$$20 = (28\sin\alpha \times T) - \tfrac{1}{2}gT^2 \quad 1$$

$$20 = \left[28\sin\alpha \times \frac{10}{7\cos\alpha}\right] - \left[\frac{1}{2}g\left(\frac{10}{7\cos\alpha}\right)^2\right] \quad 1$$

$$20 = 40\frac{\sin\alpha}{\cos\alpha} - \frac{g}{2} \times \frac{100}{49\cos^2\alpha}$$

$\frac{\sin x}{\cos x} = \tan x$

$$20 = 40\tan\alpha - \frac{100g}{98} \times \frac{1}{\cos^2\alpha}$$

$\frac{1}{\cos^2 x} = \sec^2 x$

$\sec^2 x = 1 + \tan^2 x$

$$20 = 40\tan\alpha - \frac{100 \times 9.8}{98} \times (1 + \tan^2\alpha) \quad 1$$

$$20 = 40\tan\alpha - 10 - 10\tan^2\alpha$$

$$10\tan^2\alpha - 40\tan\alpha + 30 = 0$$

$$\tan^2\alpha - 4\tan\alpha + 3 = 0 \quad 1$$

(c) $\tan^2\alpha - 4\tan\alpha + 3 = 0$

$(\tan\alpha - 3)(\tan\alpha - 1) = 0$

$\therefore \tan\alpha = 3 \qquad \tan\alpha = 1$ ← select larger value of α to

$\alpha = 71.6^\circ \qquad \alpha = 45^\circ$ (1) obtain "greatest possible height"

$v^2 = u^2 + 2as$ ← at highest point, vertical velocity is 0.

$0 = (28\sin\alpha)^2 + (2 \times -g \times H)$ (1)

$0 = (28 \times \sin(71.6^\circ))^2 - 2 \times 9.8 \times H$

$0 = 26.56^2 - 19.6H$

$19.6H = 705.43$

$H = 35.99$

$H = 36.0\text{m}$ to 3.s.f (1)

(d) Ball is modelled as a particle. (1)