Questions

Q1.

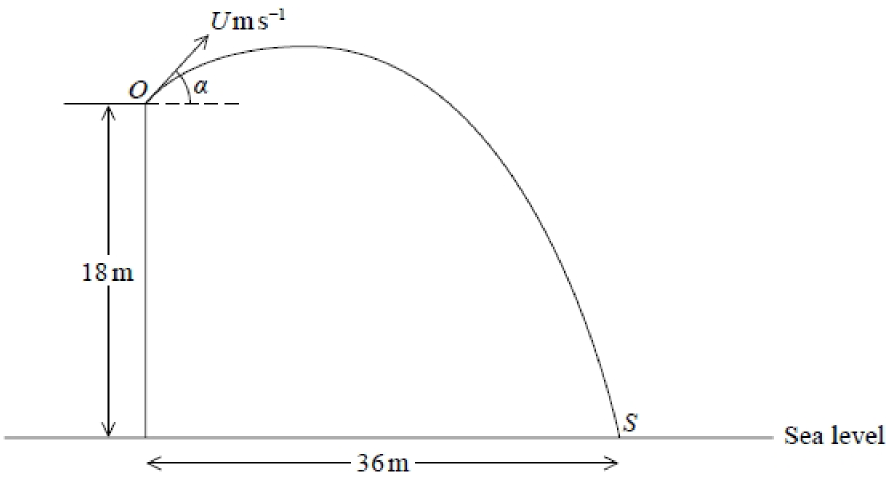

Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10$ m s^{-2}

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

(Total for question = 13 marks)

Q2.

Unless otherwise stated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

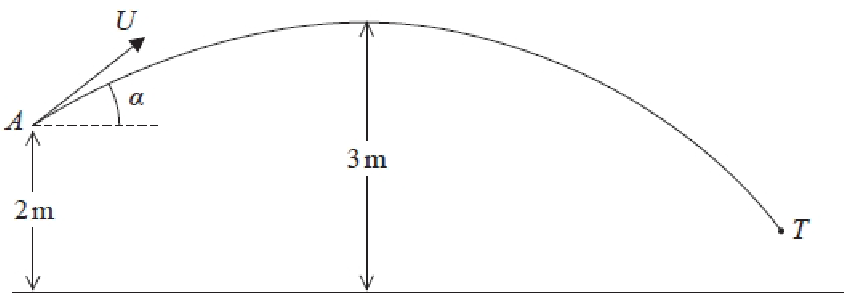

Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point *A*, the ball is 2 m above horizontal ground and is moving with speed *U* at an angle *α* above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground.
The target is modelled as being the point *T*, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

The point *T* is at a horizontal distance of 20 m from *A* and is at a height of 0.75 m above the ground. The ball reaches *T* without hitting the ground.

(b) Find the size of the angle *α*

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from *A* to *T*.

(3)

(Total for question = 15 marks)

Q3.

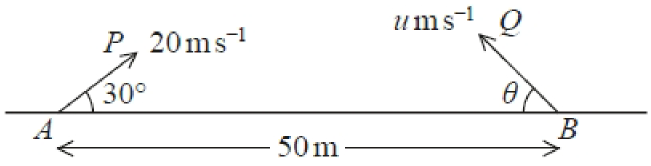

Figure 3

The points *A* and *B* lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, *P* and *Q*, are projected in the vertical plane containing *AB*.

Ball *P* is projected from *A* with speed 20 m s^{-1} at 30° to *AB*.

Ball *Q* is projected from *B* with speed u m s^{-1} at angle θ to *BA*, as shown in Figure 3.

At time $t = 2$ seconds, *P* and *Q* collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of *P* at the instant before it collides with *Q*.

(6)

(b) Find

(i) the size of angle θ ,
(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

(Total for question = 13 marks)

Q4.

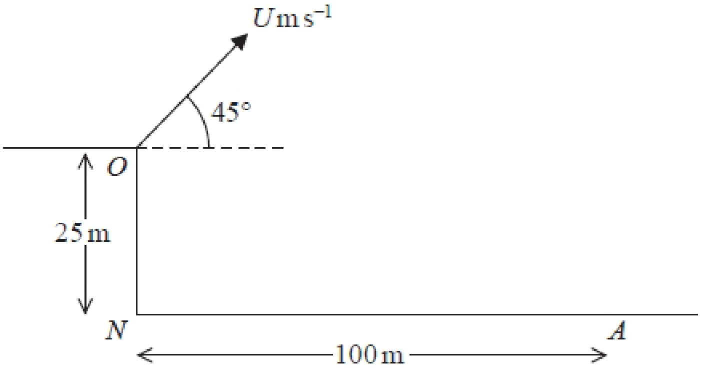

Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where AN = 100 m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that U = 28

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

(Total for question = 11 marks)

Q5.

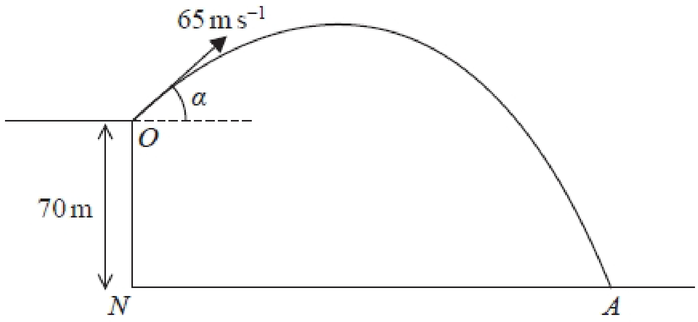

Figure 3

A small stone is projected with speed 65 m s^{-1} from a point *O* at the top of a vertical cliff.

Point *O* is 70 m vertically above the point *N*.

Point *N* is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan \alpha = \frac{5}{12}$

The stone hits the ground at the point *A*, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude 10 m s^{-2}

Using the model,

(a) find the time taken for the stone to travel from *O* to *A*,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at *A*.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

(Total for question = 10 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$36 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-18 = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	Correct strategy for solving the problem by setting up two equations in t and U and solving for U	M1	3.1b
	$U = 15$	A1	1.1b
		(6)	
(b)	Using the model and horizontal motion: $U\cos\alpha$ (12)	B1	3.4
	Using the model and vertical motion: $v^2 = (U\sin\alpha)^2 + 2(-10)(-7.2)$	M1	3.4
	$v = 15$	A1	1.1b
	Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1	3.1b
	$\sqrt{369} = 19$ m s^{-1} (2sf)	A1 ft	1.1b
		(5)	
(c)	Possible improvement (see below in notes)	B1	3.5c
	Possible improvement (see below in notes)	B1	3.5c
		(2)	
			(13 marks)

Notes:
(a) **1st M1**: for use of $s = ut$ horizontally **1st A1**: for a correct equation **2nd M1**: for use of $s = ut + \frac{1}{2}at^2$ vertically **2nd A1**: for a correct equation **3rd M1**: for correct strategy (need both equations) **2nd A1**: for $U = 15$
(b) **B1**: for $U\cos\alpha$ used as horizontal velocity component **1st M1**: for attempt to find vertical component **1st A1**: for 15 **2nd M1**: for correct strategy (need both components) **2nd A1ft**: for 19 m s^{-1} (2sf) following through on incorrect component(s)
(c) **B1, B1**: for any two of e.g. Include air resistance in the model of the motion e.g. Use a more accurate value for g in the model of the motion e.g. Include wind effects in the model of the motion e.g. Include the dimensions of the stone in the model of the motion

Q2.

Question	Scheme	Marks	AOs
(a)	Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g \leftrightarrow (3-2)$	M1	3.3
	$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4} = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\frac{5}{4} = U\sin\alpha\left(\dfrac{20}{U\cos\alpha}\right) - \frac{1}{2}g\left(\dfrac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
	$\tan\alpha = \frac{1}{4} \Rightarrow \alpha = 14^0$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t, by substituting for $U\sin\alpha$ first, then solve the quadratic to find the value of t, then use $20 = Ut\cos\alpha$ to find α. The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		

	Sub for $U\sin\alpha$ to give equation in t only	M1(II)	
	$-\frac{5}{4} = \sqrt{2g}t - \frac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut\cos\alpha$	M1(I)	
	$\alpha = 14^0$ or better	A1(II)	
(b)	**ALTERNATIVE**		
	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$		
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	M1	3.4
	$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$	A1	1.1b
	$20 = U\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for t)	M1	3.1b
	$20 = \sqrt{\frac{2g}{\sin^2\alpha}}\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for U)	M1	3.1b
	$\tan\alpha = \frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	$\Rightarrow \alpha = 14^0$ or better	A1	2.2a
		(9)	

(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U = \sqrt{\dfrac{2g}{\sin^2 \alpha}}$	M1	3.1b
	Use $20 = Ut\cos\alpha$ (or use vertical motion equation)	A1 M1	1.1b
	$t = \dfrac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 A1	1.1b
		(3)	
(d)	**ALTERNATIVE**		
	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$	M1	3.1b
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 M1	1.1b
	$= = \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $\left(= \frac{5}{\sqrt{2g}}\right)$ = 1.1 or 1.13 (s)	B1 A1	1.1b
		(3)	
			(15 marks)

Notes:
(a) **M1:** Or any other complete method to obtain an equation in U, g and α **only** **A1*:** Correct GIVEN ANSWER
(b) **M1:** Using horizontal motion **A1:** Correct equation **M1:** Using vertical motion . N.B. M0 if they use $s = \pm 2$ or ± 3, but allow $s = \pm 1.25$ or ± 0.75 or ± 2.25 or ± 2.75 **A1:** Correct equation **M1:** Using $20 = Ut\cos\alpha$ to sub. for t **M1:** Substituting for U^2 using (a) **A1:** Correct quadratic equation (in tan α **or cot** α) **M1:** Solve a 3 term quadratic, either by factorisation or formula (or by calculator (implied) if answer is correct) **and find** α **A1:** $\alpha = 14°$ or better (No restriction on accuracy since g's cancel) N.B. If answer is correct, previous M mark can be implied, but if answer is incorrect, an explicit attempt to solve must be seen to earn the previous M mark. **(b) ALTERNATIVE** **M1:** Using the model with the usual rules applying to the equation **A1:** Correct equation **M1:** Using the model to obtain the **total** time from A to T **A1:** Correct **total** time t **M1:** Substitute for t in $20 = Ut\cos\alpha$ **M1:** Substitute for U in $20 = Ut\cos\alpha$, using part (a) **A1:** Correct equation in tan α **only** **M1:** Solve equation for α **A1:** $\alpha = 14°$ or better (No restriction on accuracy since g's cancel)

N.B. If they quote the equation of the trajectory $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$ oe **AND** put in values for x and y, could score first 5 marks, M1A1M1A1M1 (nothing for the equation only); wrong x value loses first A mark and wrong y value loses second A mark
(c) **B1:** Give one limitation of the model e.g. the ball will have dimensions, or there will be air resistance or wind effects or spin N.B. B0 if any incorrect extra(s) but ignore extra consequences.
(d) **M1:** Using their α to find a value for U **A1: Treat as M1:** Using their U to find a value for t **B1: Treat as A1 :** $t = 1.1$ or 1.10 (since depends on $g = 9.8$) **(d) ALTERNATIVE** **M1:** Using their α to find a value for U **A1: Treat as M1:** Using their U to find a value for t **B1: Treat as A1 :** $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	Horizontal speed = 20 cos 30° = $10\sqrt{3}$ m s^{-1}	B1	This mark is given for a correct expression for the horizontal speed of P
	$v = u + at$ Vertical speed = 20 sin 30° – 19.6 = –9.6 m s^{-1}	M1	This mark is given for a method to find the vertical speed of P
		A1	This mark is given for a correct value for the vertical speed of P
	$\theta = \tan^{-1} \pm \dfrac{9.6}{10\sqrt{3}}$	M1	This mark is given finding an expression for the value of θ
	Speed = $\sqrt{(100 \times 3) + 9.6^2}$	M1	This mark is given for using Pythagoras to find the magnitude of the speed of P
	9.8 m s^{-1} downwards at 29.0° to the horizontal	A1	This mark is given for finding the correct velocity of P (showing both magnitude and direction)

(b)	Sum of horizontal distances = 50 m	M1	This mark is given for stating the sum of the horizontal distances
	$(u \cos \theta) \times 2 = 50 - (20 \cos 30°) \times 2$ $u \cos \theta = 25 - 20 \cos 30°$	A1	This mark is given for a correct expression for the horizontal distance
	Vertical distances equal $(20 \sin 30°) \times 2 - \dfrac{g}{2} \times 4 = (u \sin \theta) \times 2 - \dfrac{g}{2} \times 4$	M1	This mark is given for equating the vertical distances
	$u \sin \theta = 20 \sin 30°$	A1	This mark is given for a correct expression for the vertical distance
	$\theta = 52.5°$, $u = 12.6$ m s^{-1}	M1	This mark is given for a correct method to find θ and u
		A1	This mark is given for finding correct values of θ and u
(c)	For example: The effect of the wind The effect of the spinning of the balls The size of the balls	B1	This mark is given for one correct limitation of the model stated

Q4.

Question	Scheme	Marks	AOs
(a)	Using horizontal motion	M1	3.3
	$U\cos 45°t = 100$	A1	1.1b
	Using vertical motion	M1	3.4
	$U\sin 45°t - \frac{1}{2}gt^2 = -25$	A1	1.1b
	Solve problem by eliminating t and solving for U	M1	3.1b
	$U = 28$*	A1*	1.1b
		(6)	
(b)	Using vertical motion	M1	3.4
	$0^2 = (28\sin 45°)^2 - 2gh$	A1	1.1b
	Greatest height = 45 m	A1	1.1b
		(3)	
(c)	New value > 28	B1	3.5a
		(1)	
(d)	e.g. wind effects, more accurate value of g, spin of ball, include size of the ball, not model as a particle, shape of ball	B1	3.5c
		(1)	
			(11 marks)

Notes:		
a	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors
	A1	Correct equation (g does not need to be substituted)
	M1	Must have earned the previous two M marks. Eliminate t and solve for U. N.B. They may solve for t first ($100-\frac{1}{2}gt^2=-25$) and then use it to find U.
	A1*	Exact given answer correctly obtained with no wrong working (e.g. $g = 9.81$ used) or approximation seen.
b	M1	Complete method to give equation in h only (allow if U not substituted), condone sin/cos confusion and sign errors
	A1	Correct equation (g does not need to be substituted) (A0 if U is used instead of 28)
	A1	cao
c	B1	Clear statement
d	B1	Penalise incorrect extras i.e. B0 if there are incorrect extras. The ground being horizontal, the cliff being vertical, .. are not part of the model so B0 Include weight/mass of the ball B0

Q5.

Question	Scheme	Marks	AOs
	Note that $g = 10$; penalise once for whole question if $g = 9.8$		
(a)	Use $s = ut + \frac{1}{2}at^2$ vertically or any complete method to give an equation in t only	M1	3.4
	$-70 = 65\sin\alpha \times t - \frac{1}{2} \times g \times t^2$	A1	1.1b
		M (A)1	1.1b
	$t = 7$ (s)	A1	1.1b
		(4)	
(b)	Horizontal velocity component at $A = 65\cos\alpha$ (60)	B1	3.4
	Complete method to find vertical velocity component at A	M1	3.4
	$65\sin\alpha - g \times 7$ **OR** $\sqrt{(-25)^2 + 2g \times 70}$ (45)	A1ft	1.1b
	Sub for trig and square, add and square root : $\sqrt{60^2 + (-45)^2}$	M1	3.1b
	75 Accept 80 ($\mathrm{m\,s^{-1}}$)	A1	1.1b
		(5)	
(c)	e.g. an approximate value of g has been used, the dimensions of the stone could affect its motion, spin of the stone, $g = 10$ instead of 9.8 has been used, g has been assumed to be constant, wind effect, shape of the stone	B1	3.5b
		(1)	
			(10 marks)

Notes:		
a	M1	Complete method, correct no. of terms, condone sign errors and sin/cos confusion
	A1	Correct equation in t only with at most one error
	M(A)1	Correct equation in t only
		N.B. For 'up and down' methods etc, the two A marks are for all the equations that they use, lose a mark for each error.
	A1	Cao ($g = 9.8$, 7.1 or 7.11) ($g = 9.81$, 7.1 or 7.12)
b	B1	Seen, including on a diagram.
	M1	Condone sign errors and sin/cos confusion
	A1ft	Correct expression; accept negative of this, follow their t
	M1	Sub for trig and use Pythagoras
	A1	Cao ($g = 9.8$ or 9.81, 75 or 74.8)
c	B1	B0 if incorrect extras