1. A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j})\text{ms}^{-1}$. At time $t = 6$ s P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j})$m. Find the distance of P from the origin at time $t = 2$ s.

(Total 5 marks)

2. Two cars P and Q are moving in the same direction along the same straight horizontal road. Car P is moving with constant speed 25 m s^{-1}. At time $t = 0$, P overtakes Q which is moving with constant speed 20 ms^{-1}. From $t = T$ seconds, P decelerates uniformly, coming to rest at a point X which is 800 m from the point where P overtook Q. From $t = 25$ s, Q decelerates uniformly, coming to rest at the same point X at the same instant as P.

(a) Sketch, on the same axes, the speed-time graphs of the two cars for the period from $t = 0$ to the time when they both come to rest at the point X.

(4)

(b) Find the value of T.

(8)
(Total 12 marks)

3. An athlete runs along a straight road. She starts from rest and moves with constant acceleration for 5 seconds, reaching a speed of 8 ms^{-1}. This speed is then maintained for T seconds. She then decelerates at a constant rate until she stops. She has run a total of 500 m in 75 s.

(a) Sketch a speed-time graph to illustrate the motion of the athlete.

(3)

(b) Calculate the value of T.

(5)
(Total 8 marks)

4. *[In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]*

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is s km. When $t = 0$, s = 9**i** – 6**j**. When $t = 4$, s = 21**i** + 10**j**. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that s = $(3t + 9)$ **i** + $(4t – 6)$ **j**.

(2)

A lighthouse L is located at the point with position vector (18**i** + 6**j**) km. When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

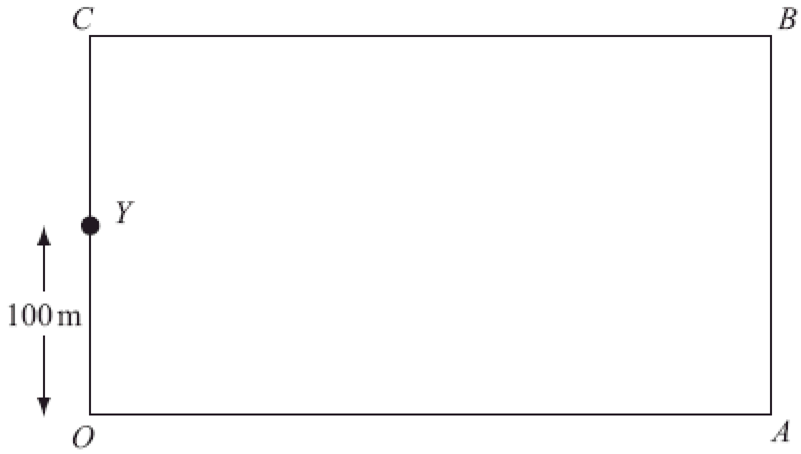
(6)
(Total 14 marks)

5. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

A hiker H is walking with constant velocity (1.2**i** – 0.9**j**) m s^{-1}.

(a) Find the speed of H.

(2)



A horizontal field *OABC* is rectangular with *OA* due east and *OC* due north, as shown in the diagram above. At twelve noon hiker *H* is at the point *Y* with position vector 100 **j** m, relative to the fixed origin *O*.

(b) Write down the position vector of *H* at time *t* seconds after noon.

(2)

At noon, another hiker *K* is at the point with position vector (9**i** + 46**j**) m. Hiker *K* is moving with constant velocity (0.75**i** + 1.8**j**) m s^{-1}.

(c) Show that, at time *t* seconds after noon,

$$\overrightarrow{HK} = [(9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}] \text{ meters.}$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

(Total 13 marks)

6. At time $t = 0$, a particle is projected vertically upwards with speed *u* m s_{-1} from a point 10 m above the ground. At time *T* seconds, the particle hits the ground with speed 17.5 m s^{-1}. Find

(a) the value of *u*,

(3)

(b) the value of T.

(4)
(Total 7 marks)

7. A firework rocket starts from rest at ground level and moves vertically. In the first 3 s of its motion, the rocket rises 27 m. The rocket is modelled as a particle moving with constant acceleration a m s^{-2}. Find

(a) the value of a,

(2)

(b) the speed of the rocket 3 s after it has left the ground.

(2)

After 3 s, the rocket burns out. The motion of the rocket is now modelled as that of a particle moving freely under gravity.

(c) Find the height of the rocket above the ground 5 s after it has left the ground.

(4)
(Total 8 marks)

8. A car moves along a horizontal straight road, passing two points A and B. At A the speed of the car is 15 m s^{-1}. When the driver passes A, he sees a warning sign W ahead of him, 120 m away. He immediately applies the brakes and the car decelerates with uniform deceleration, reaching W with speed 5 m s^{-1}. At W, the driver sees that the road is clear. He then immediately accelerates the car with uniform acceleration for 16 s to reach a speed of V m s^{-1} ($V > 15$). He then maintains the car at a constant speed of V m s^{-1}. Moving at this constant speed, the car passes B after a further 22 s.

(a) Sketch, in the space below, a speed-time graph to illustrate the motion of the car as it moves from A to B.

(3)

(b) Find the time taken for the car to move from A to B.

(3)

The distance from A to B is 1 km.

(c) Find the value of V.

(5)
(Total 11 marks)

9. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A particle P is moving with constant velocity $(-5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of P,

(2)

(b) the direction of motion of P, giving your answer as a bearing.

(3)

At time $t = 0$, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m relative to a fixed origin O. When $t = 3$ s, the velocity of P changes and it moves with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}, where u and v are constants. After a further 4 s, it passes through O and continues to move with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}.

(c) Find the values of u and v.

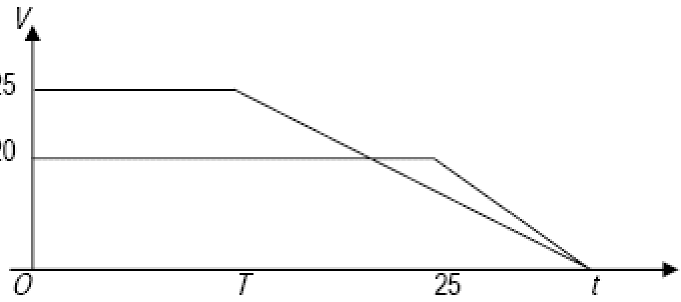
(5)

(d) Find the total time taken for P to move from A to a position which is due south of A.

(3)
(Total 13 marks)

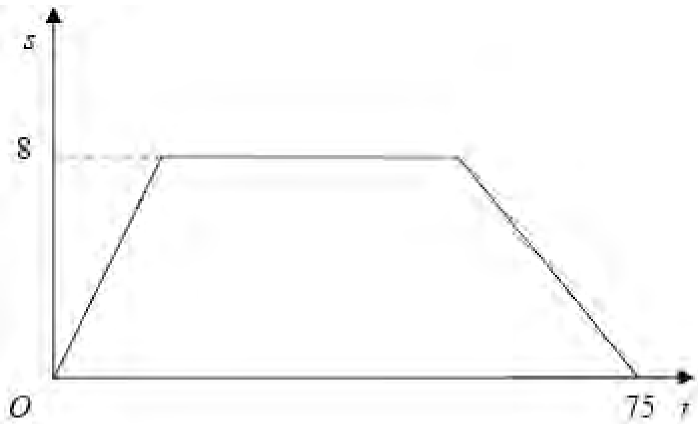
1.	$(-4\mathbf{i} - 7\mathbf{j}) = \mathbf{r} + 4(-3\mathbf{i} + 2\mathbf{j})$	A1		
	$\mathbf{r} = (8\mathbf{i} - 15\mathbf{j})$	A1		
	$\lvert\mathbf{r}\rvert = \sqrt{8^2 + (-15)^2} = 17$ m	A1 ft		
				[5]

2.	(a)	Shape (both)	B1		
		Cross	B1		
		Meet on t-axis	B1		
		Figures 25, 20, T, 25	B1	4	



(b)	For Q: $20\left(\dfrac{t+25}{2}\right) = 800$	A1		
	$t = 55$	DM1 A1		
	For P: $25\left(\dfrac{T+55}{2}\right) = 800$	A1		
	solving for T: $T = 9$	DM1 A1	8	
				[12]

3. (a)



First two line segments	B1
Third line segment	B1

		8, 75	B1	3
	(b)	$\frac{1}{2}\times 8\times (T+75)=500$	A2 (1,0)	
		Solving to T = 50	DM1 A1	5

[8]

4.	(a)	v = $\frac{21\mathbf{i}+10\mathbf{j}-(\mathbf{9i}-\mathbf{6j})}{4} = 3\mathbf{i}+4\mathbf{j}$	A1	
		speed is $\sqrt{(3^2+4^2)} = 5(\text{km h}^{-1})$	A1	4
	(b)	$\tan\theta = \frac{3}{4}(\Rightarrow \theta \approx 36.9°)$		
		bearing is 37, 36.9, 36.87, …	A1	2
	(c)	$\mathbf{s} = 9\mathbf{i} - 6\mathbf{j} + t(3\mathbf{i} + 4\mathbf{j})$		
		$= (3t+9)\mathbf{i} + (4t-6)\mathbf{j}$ * cso	A1	2
	(d)	Position vector of S relative to L is		
		$(3T+9)\mathbf{i} + (4T-6)\mathbf{j} - (18\mathbf{i} + 6\mathbf{j}) = (3T-9)\mathbf{i} + (4T-12)\mathbf{j}$	A1	
		$(3T-9)^2 + (4T-12)^2 = 100$		
		$25T^2 - 150T + 125 = 0$ or equivalent	DM1 A1	
		$(T^2 - 6T + 5 = 0)$		
		$T = 1, 5$	A1	6

[14]

5.	(a)	$\lvert v\rvert = \sqrt{1.2^2 + (-0.9)^2} = 1.5\text{m s}^{-1}$	A1	2
	(b)	$(\mathbf{r}_H =)100\mathbf{j} + t(1.2\mathbf{i} - 0.9\mathbf{j})$ m	A1	2
	(c)	$(\mathbf{r}_K =)9\mathbf{i} + 46\mathbf{j} + t(0.75\mathbf{i} + 1.8\mathbf{j})$ m	A1	
		$\overrightarrow{HK} = \mathbf{r}_K - \mathbf{r}_H = (9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}$ m **Printed Answer**	A1	4

	(d)	Meet when $\overrightarrow{HK} = 0$		
		$(9 - 0.45t) = 0$ and $(2.7t - 54) = 0$	A1	
		$t = 20$ from both equations	A1	
		$\mathbf{r}_K = \mathbf{r}_H = (24\mathbf{i} + 82\mathbf{j})$ m	DM1 A1 **cso**	5

[13]

6.	(a)	$v^2 = u^2 + 2as \Rightarrow 17.5^2 = u^2 + 2 \times 9.8 \times 10$	A1	
		Leading to $u = 10.5$	A1	3
	(b)	$v = u + at \Rightarrow 17.5 = -10.5 + 9.8T$	M1A1ft	
		$T = 2\frac{6}{7}$ (s)	DM1A1	4
		Alternatives		
		$s = \left(\frac{u+v}{2}\right)T \Rightarrow 10 = \left(\frac{17.5 + -10.5}{2}\right)T$	M1A1ft	
		$\frac{20}{7} = T$	DM1A1	4
	OR	$s = ut + \frac{1}{2}at^2 \Rightarrow -10 = 10.5t - 4.9t^2$	M1A1ft	
		Leading to $T = 2\frac{6}{7}, \left(-\frac{5}{7}\right)$ Rejecting negative	DM1A1	4
		(b) can be done independently of (a)		
		$s = vt - \frac{1}{2}at^2 \Rightarrow -10 = -17.5t + 4.9t^2$	M1A1	
		Leading to $T = 2\frac{6}{7}, \frac{5}{7}$	DM1	
		For final A1, second solution has to be rejected. $\frac{5}{7}$ leads to a negative u.	A1	4

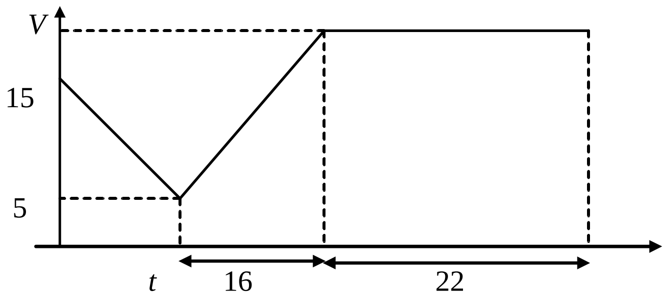
[7]

7.	(a)	$27 = 0 + ½.a.3^2 \Rightarrow a = \underline{6}$	M1A1	2
	(b)	$v = 6 \times 3 = \underline{18 \text{ m s}^{-1}}$	M1A1ft	2

(c) From $t = 3$ to $t = 5$, $s = 18 \times 2 - \frac{1}{2} \times 9.8 \times 2^2$
Total ht. = $s + 27$ = 43.4 m, 43 m — M1A1 — 4

[8]

8. (a)



Shape 'V' — B1
Shape for last 22s (with $V > 15$) — B1
Figures — B1 — 3

(b) $\frac{1}{2}(15 + 5) \times t = 120$
$\Rightarrow t = 12 \rightarrow T = 12 + 16 + 22 = \underline{50\text{ s}}$ — M1A1 — 3

(c) $120 + \underline{\frac{1}{2}(V + 5).16} + 22V = 1000$ — M1B1A1
Solve: $30V = 840 \Rightarrow V = \underline{28}$ — DM1A1 — 5

[11]

9. (a) Speed = $\sqrt{(5^2 + 8^2)} \approx \underline{9.43\text{ m s}^{-1}}$ — M1A1 — 2

(b) Forming arctan 8/5 or arctan 5/8 oe
Bearing = 360 – arctan 5/8 or 270 + arctan 8/5 = 328 — DM1A1 — 3

(c) At $t = 3$, p.v. of $P = (7 - 15)\mathbf{i} + (-10 + 24)\mathbf{j} = -8\mathbf{i} + 14\mathbf{j}$ — M1A1
Hence $-8\mathbf{i} + 14\mathbf{j} + 4(u\mathbf{i} + v\,\mathbf{j}) = \mathbf{0}$
$\Rightarrow \underline{u = 2, v = -3.5}$ — DM1A1 — 5

(d) p.v. of P t secs after changing course = $(-8\mathbf{i} + 14\mathbf{j}) + t(2\boldsymbol{i} - 3.5\mathbf{j})$
$= 7\mathbf{i} + \ldots..$ — DM1
Hence total time = 10.5 s — A1 — 3

[13]

1. This proved to be a tricky opening question for many of the candidates. The most popular approach was to find the starting position and then use it to find the position vector at $t = 2$. Errors in sign were fairly common at some stage of the working. A significant minority did not use a valid method at all, some just multiplying the given velocity vector by 2 or using a time of 6 only, and others becoming confused with constant acceleration formulae. A number of candidates failed to find the magnitude of their position vector to obtain the distance as required; there were follow through marks available for this even if the vector had been determined incorrectly. A few found the distance from the starting point rather than from the origin. Nevertheless, there were a fair number of entirely correct solutions.

2. A large number of entirely (or almost) correct solutions were seen to this question. Most candidates drew their velocity-time graphs correctly and included appropriate annotations, with the most common error being that the lines drawn did not cross. This did not deny candidates access to full marks in the rest of the question though and many went on to solve the problem correctly. Most realised that they needed to equate the expressions for area under the graph to 800 for both P and Q. Attempts to use constant acceleration formulae over the whole distance were occasionally seen and scored no marks although a few used this approach in a valid way for the separate parts of the motion. Most commonly, a combination of rectangles and triangles were used to represent area rather than the area of a trapezium which made the subsequent algebra more difficult, and there were occasional errors seen in simplification. A relatively common error was to calculate a correct time for Q ($t = 30$) but to misinterpret this as the time when they both came to rest leading to errors in the motion of P.

3. In part (a) the speed-time graph was almost universally correct. Most candidates realised, in the second part, that the area under the graph was equal to the distance travelled and were able to calculate the correct area of 20 for the first part of the motion. Errors in the interpretation of T caused most of the problems in the calculations of the other areas. Comparatively few used an area of a trapezium which provided the neatest solution.

4. There was some confusion in parts (a), (b) and (c) over which vectors were velocities and which were displacements, with some even using acceleration. In the first part, many did not appreciate the distinction between velocity and speed and in part (b) many were unable to convert an appropriate angle into a bearing. The third part tended to be well-answered but a few used 'verification' at $t = 0$ and $t = 4$ and scored nothing. Part (d) was a good discriminator and the less able were often unable to make much progress. The majority of candidates who used Pythagoras to find the magnitude of the relative position vector and equated it to 10 scored at least 3/6 but many often lost the accuracy marks due to poor algebra. There were a number of other methods seen which used the fact that the lighthouse was on the path of the ship and that the speed of the ship was 5 km/h and these received full credit.

5. Most candidates were able to gain the first six marks and most seemed to know that, in part (c), they needed to perform a subtraction on $\mathbf{r}_H$ and $\mathbf{r}_K$ although some were unsure which way round to do it. Another common error was to equate the position vectors and then fudge the answer. This received no credit.

In part (d) many candidates assumed that the hikers would meet and equated just one pair of components to produce $t = 20$. If they then used just one hiker to find $24\mathbf{i} + 82\mathbf{j}$ they scored only 2 out of 5, if they used **both** hikers, they scored full marks. There were a number of other ways of obtaining $t = 20$, some spurious, but provided that the candidate verified that both hikers were at the point with position vector $24\mathbf{i} + 82\mathbf{j}$ at $t = 20$, they could score all of the marks.

6. There were various approaches that could be applied successfully to answer this question. Those who fully understood the implications of projecting from above ground level could achieve full marks by the most direct method although sign errors were not uncommon. Another popular approach was to split the motion into two stages (to and from the highest point) in both part (a) to find the initial velocity, and in part (b) to find the whole time. Although this required more working, there tended to be fewer sign errors. Premature approximation occasionally led to inaccuracy in the final answer. The weakest candidates sometimes only considered motion to or from the highest point. It should be noted that the rubric requires g = 9.8 to be used and not 9.81, which was penalised.

7. Parts (a) and (b) were usually done correctly although there were a number of candidates who thought that velocity = distance/time and used this as a basis for the solutions to the first two parts of the question and there are still some who are unable to quote the '*suvat*' formulae correctly. The use of a "follow through" in the mark scheme was of great benefit to some students.

Part (c) caused more problems with many candidates using incorrect information in their equations. A few found the time to the top and then used the remaining time to work out the distance fallen. The continuity of the motion was the major stumbling block for students who produced incorrect solutions. Students wrongly used $u = 0$, $t = 5$ or the wrong acceleration here and there were many sign errors. A considerable number of students failed to use the '27' appropriately with students subtracting their answers from '27' or not even attempting this section. There are still students who forget that they need to explain work to the examiner rather than just quote an answer. The instructions on the front of the exam paper are clear.

8. In the first part the majority were able to produce a reasonable speed-time graph – errors usually occurring due to the omission of time details; also a few had $V < 15$ and some candidates included a section for the time before $t = 0$.

In the final part, the majority considered the three separate parts of the journey with most errors occurring in the calculation of the distance covered in the second part, with many splitting the area into a triangle, whose height they incorrectly took as V, and a rectangle.
There were several variations along this theme.

9. In parts (a) and (b) most were able to find the speed of the particle and were also able to obtain an appropriate angle associated with it. Many were then unable to use this angle correctly to obtain the correct bearing.

There were a great many correct solutions for (c), but also many incorrect attempts. The majority of errors tended to come from those candidates who had not read the question carefully enough and did not incorporate the velocity vector (–5**i** + 12**j**) into their working or from those candidates making errors with directions. Many candidates were able to visualise the situation well, realising that 7**i** was involved, even though they may have made earlier errors in interpretation.