1. A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j})\text{ms}^{-1}$. At time $t = 6$ s P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j})$m. Find the distance of P from the origin at time $t = 2$ s.

(Total 5 marks)

2. *[In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]*

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is s km. When $t = 0$, s = $9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, s = $21\mathbf{i} + 10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that s = $(3t + 9)\,\mathbf{i} + (4t - 6)\,\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

(6)

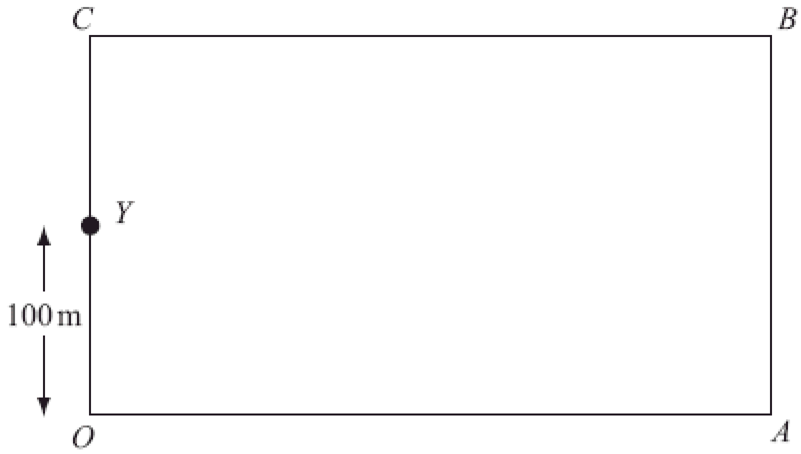
(Total 14 marks)

3. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker H is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

(a) Find the speed of H.

(2)



A horizontal field $OABC$ is rectangular with OA due east and OC due north, as shown in the diagram above. At twelve noon hiker H is at the point Y with position vector 100 **j** m, relative to the fixed origin O.

(b) Write down the position vector of H at time t seconds after noon.

(2)

At noon, another hiker K is at the point with position vector (9**i** + 46**j**) m. Hiker K is moving with constant velocity (0.75**i** + 1.8**j**) m s^{-1}.

(c) Show that, at time t seconds after noon,

$$\overrightarrow{HK} = [(9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}] \text{ meters.}$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

(Total 13 marks)

4. A particle P moves with constant acceleration (2**i** – 5**j**) m s^{-2}. At time t = 0, P has speed u m s^{-1}. At time t = 3 s, P has velocity (–6**i** + **j**) m s^{-1}.

Find the value of u.

(Total 5 marks)

5. A boat B is moving with constant velocity. At noon, B is at the point with position vector $(3\mathbf{i} - 4\mathbf{j})$ km with respect to a fixed origin O. At 1430 on the same day, B is at the point with position vector $(8\mathbf{i} + 11\mathbf{j})$ km.

(a) Find the velocity of B, giving your answer in the form $p\mathbf{i} + q\mathbf{j}$.

(3)

At time t hours after noon, the position vector of B is $\mathbf{b}$ km.

(b) Find, in terms of t, an expression for $\mathbf{b}$.

(3)

Another boat C is also moving with constant velocity. The position vector of C, $\mathbf{c}$ km, at time t hours after noon, is given by

$$\mathbf{c} = (-9\mathbf{i} + 20\mathbf{j}) + t(6\mathbf{i} + \lambda\mathbf{j}),$$

where λ is a constant. Given that C intercepts B,

(c) find the value of λ,

(5)

(d) show that, before C intercepts B, the boats are moving with the same speed.

(3)
(Total 14 marks)

6. A particle P of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ newtons. When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$ s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find

(a) the acceleration of P in terms of $\mathbf{i}$ and $\mathbf{j}$,

(2)

(b) the magnitude of $\mathbf{F}$,

(4)

(c) the velocity of P at time t =6s.

(3)
(Total 9 marks)

7. [*In this question the unit vectors* **i** *and* **j** *are due east and north respectively.*]

A ship *S* is moving with constant velocity (–2.5**i** + 6**j**) km h^{-1}. At time 1200, the position vector of *S* relative to a fixed origin *O* is (16**i** + 5**j**) km. Find

(a) the speed of *S*,

(2)

(b) the bearing on which *S* is moving.

(2)

The ship is heading directly towards a submerged rock *R*. A radar tracking station calculates that, if *S* continues on the same course with the same speed, it will hit *R* at the time 1500.

(c) Find the position vector of *R*.

(2)

The tracking station warns the ship's captain of the situation. The captain maintains *S* on its course with the same speed until the time is 1400. He then changes course so that *S* moves due north at a constant speed of 5 km h^{-1}. Assuming that *S* continues to move with this new constant velocity, find

(d) an expression for the position vector of the ship *t* hours after 1400,

(4)

(e) the time when *S* will be due east of *R*,

(2)

(f) the distance of *S* from *R* at the time 1600.

(3)
(Total 15 marks)

8. *[In this question the horizontal unit vectors* **i** *and* **j** *are due east and due north respectively.]*

A model boat *A* moves on a lake with constant velocity (–**i** + 6**j**) m s^{-1}. At time $t = 0$, *A* is at the point with position vector (2**i** – 10**j**) m. Find

(a) the speed of *A*,

(2)

(b) the direction in which *A* is moving, giving your answer as a bearing.

(3)

At time $t = 0$, a second boat B is at the point with position vector $(-26\mathbf{i} + 4\mathbf{j})$ m.

Given that the velocity of B is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1},

(c) show that A and B will collide at a point P and find the position vector of P.

(5)

Given instead that B has speed 8 m s^{-1} and moves in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$,

(d) find the distance of B from P when $t = 7$ s.

(6)
(Total 16 marks)

9. [*In this question, the unit vectors* **i** *and* **j** *are horizontal vectors due east and north respectively.*]

At time $t = 0$, a football player kicks a ball from the point A with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of the ball,

(2)

(b) the position vector of the ball after t seconds.

(2)

The point B on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m.

(c) Find the time when the ball is due north of B.

(2)

At time $t = 0$, another player starts running due north from B and moves with constant speed v m s^{-1}. Given that he intercepts the ball,

(d) find the value of v.

(6)

(e) State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic.

(1)
(Total 13 marks)

10. Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector $(20\mathbf{i} + 10\mathbf{j})$ km relative to a fixed origin O. At the same time, Q is at the point with position vector $(14\mathbf{i} - 6\mathbf{j})$ km. Three hours later, P is at the point with position vector $(29\mathbf{i} + 34\mathbf{j})$ km. The ship Q travels with velocity $12\mathbf{j}$ km h^{-1}. At time t hours after midnight, the position vectors of P and Q are $\mathbf{p}$ km and $\mathbf{q}$ km respectively. Find

(a) the velocity of P, in terms of $\mathbf{i}$ and $\mathbf{j}$,

(2)

(b) expressions for $\mathbf{p}$ and $\mathbf{q}$, in terms of t, $\mathbf{i}$ and $\mathbf{j}$.

(4)

At time t hours after midnight, the distance between P and Q is d km.

(c) By finding an expression for $\overrightarrow{PQ}$, show that

$$d^2 = 25t^2 - 92t + 292.$$

(5)

Weather conditions are such that an observer on P can only see the lights on Q when the distance between P and Q is 15 km or less. Given that when $t = 1$, the lights on Q move into sight of the observer,

(d) find the time, to the nearest minute, at which the lights on Q move out of sight of the observer.

(5)
(Total 16 marks)

11. A particle P moves in a horizontal plane. The acceleration of P is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-2}. At time $t = 0$, the velocity of P is $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

(a) Find, to the nearest degree, the angle between the vector $\mathbf{j}$ and the direction of motion of P when $t = 0$.

(3)

At time t seconds, the velocity of P is $\mathbf{v}$ m s^{-1}. Find

(b) an expression for $\mathbf{v}$ in terms of t, in the form $a\mathbf{i} + b\mathbf{j}$,

(2)

(c) the speed of P when $t = 3$,

(3)

(d) the time when P is moving parallel to $\mathbf{i}$.

(2)
(Total 10 marks)

12. A small boat S, drifting in the sea, is modelled as a particle moving in a straight line at constant speed. When first sighted at 0900, S is at a point with position vector $(4\mathbf{i} - 6\mathbf{j})$ km relative to a fixed origin O, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north respectively. At 0945, S is at the point with position vector $(7\mathbf{i} - 7.5\mathbf{j})$ km. At time t hours after 0900, S is at the point with position vector $\mathbf{s}$ km.

(a) Calculate the bearing on which S is drifting.

(4)

(b) Find an expression for $\mathbf{s}$ in terms of t.

(3)

At 1000 a motor boat M leaves O and travels with constant velocity $(p\mathbf{i} + q\mathbf{j})$ km h^{-1}. Given that M intercepts S at 1015,

(c) calculate the value of p and the value of q.

(6)
(Total 13 marks)

13. [*In this question the vectors* **i** *and* **j** *are horizontal unit vectors in the directions due east and due north respectively*.]

Two boats *A* and *B* are moving with constant velocities. Boat *A* moves with velocity 9**j** km h^{-1}. Boat *B* moves with velocity (3**i** + 5**j**) km h^{-1}.

(a) Find the bearing on which *B* is moving.
(2)

At noon, *A* is at point *O*, and *B* is 10 km due west of *O*. At time *t* hours after noon, the position vectors of *A* and *B* relative to *O* are **a** km and **b** km respectively.

(b) Find expressions for **a** and **b** in terms of *t*, giving your answer in the form *p***i** + *q***j**.
(3)

(c) Find the time when *B* is due south of *A*.
(2)

At time *t* hours after noon, the distance between *A* and *B* is *d* km. By finding an expression for $\overrightarrow{AB}$,

(d) show that $d^2 = 25t^2 - 60t + 100$.
(4)

At noon, the boats are 10 km apart.

(e) Find the time after noon at which the boats are again 10 km apart.
(3)
(Total 14 marks)

14. A particle *P* of mass 3 kg is moving under the action of a constant force **F** newtons. At $t = 0$, *P* has velocity (3**i** – 5**j**) m s^{-1}. At $t = 4$ s, the velocity of *P* is (–5**i** + 11**j**) m s^{-1}. Find

(a) the acceleration of *P*, in terms of **i** and **j**.
(2)

(b) the magnitude of **F**.

(4)

At t = 6 s, P is at the point A with position vector (6**i** – 29**j**) m relative to a fixed origin O. At this instant the force **F** newtons is removed and P then moves with constant velocity. Three seconds after the force has been removed, P is at the point B.

(c) Calculate the distance of B from O.

(6)
(Total 12 marks)

15. A particle P moves with constant acceleration (2**i** – 3**j**) m s^{-2}. At time t seconds, its velocity is **v** m s^{-1}. When t = 0, **v** = –2**i** + 7**j**.

(a) Find the value of t when P is moving parallel to the vector **i**.

(4)

(b) Find the speed of P when t = 3.

(3)

(c) Find the angle between the vector **j** and the direction of motion of P when t = 3.

(3)
(Total 10 marks)

16. Two ships P and Q are moving along straight lines with constant velocities. Initially P is at a point O and the position vector of Q relative to O is (6**i** + 12**j**) km, where **i** and **j** are unit vectors directed due east and due north respectively. The ship P is moving with velocity 10**j** km h^{-1} and Q is moving with velocity (–8**i** + 6**j**) km h^{-1}. At time t hours the position vectors of P and Q relative to O are **p** km and **q** km respectively.

(a) Find **p** and **q** in terms of t.

(3)

(b) Calculate the distance of Q from P when t = 3.

(3)

(c) Calculate the value of t when Q is due north of P.

(2)
(Total 8 marks)

17. [In this question, the horizontal unit vectors **i** and **j** are directed due East and North respectively.]

A coastguard station O monitors the movements of ships in a channel. At noon, the station's radar records two ships moving with constant speed. Ship A is at the point with position vector $(-5\mathbf{i} + 10\mathbf{j})$ km relative to O and has velocity $(2\mathbf{i} + 2\mathbf{j})$ km h^{-1}. Ship B is at the point with position vector $(3\mathbf{i} + 4\mathbf{j})$ km and has velocity $(-2\mathbf{i} + 5\mathbf{j})$ km h^{-1}.

(a) Given that the two ships maintain these velocities, show that they collide.

(6)

The coast guard radios ship A and orders it to reduce its speed to move with velocity $(\mathbf{i} + \mathbf{j})$ km h^{-1}.

Given that A obeys this order and maintains this new constant velocity,

(b) find an expression for the vector $\overrightarrow{AB}$ at time t hours after noon.

(2)

(c) find, to 3 significant figures, the distance between A and B at 1400 hours,

(3)

(d) find the time at which B will be due north of A.

(2)

(Total 13 marks)

1. $(-4\mathbf{i} - 7\mathbf{j}) = \mathbf{r} + 4(-3\mathbf{i} + 2\mathbf{j})$ A1

$\mathbf{r} = (8\mathbf{i} - 15\mathbf{j})$ A1

$|\mathbf{r}| = \sqrt{8^2 + (-15)^2} = 17$ m A1 ft

[5]

2. (a) $v = \dfrac{21\mathbf{i} + 10\mathbf{j} - (9\mathbf{i} - 6\mathbf{j})}{4} = 3\mathbf{i} + 4\mathbf{j}$ A1

speed is $\sqrt{(3^2 + 4^2)} = 5(\text{km h}^{-1})$ A1 4

(b) $\tan\theta = \dfrac{3}{4} (\Rightarrow \theta \approx 36.9°)$

bearing is 37, 36.9, 36.87, … A1 2

(c) $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j} + t(3\mathbf{i} + 4\mathbf{j})$

$= (3t + 9)\mathbf{i} + (4t - 6)\mathbf{j}$ * cso A1 2

(d) Position vector of S relative to L is

$(3T + 9)\mathbf{i} + (4T - 6)\mathbf{j} - (18\mathbf{i} + 6\mathbf{j}) = (3T - 9)\mathbf{i} + (4T - 12)\mathbf{j}$ A1

$(3T - 9)^2 + (4T - 12)^2 = 100$

$25T^2 - 150T + 125 = 0$ or equivalent DM1 A1

$(T^2 - 6T + 5 = 0)$

$T = 1, 5$ A1 6

[14]

3. (a) $|v| = \sqrt{1.2^2 + (-0.9)^2} = 1.5 \text{m s}^{-1}$ A1 2

(b) $(\mathbf{r}_H =) 100\mathbf{j} + t(1.2\mathbf{i} - 0.9\mathbf{j})$ m A1 2

(c) $(\mathbf{r}_K =) 9\mathbf{i} + 46\mathbf{j} + t(0.75\mathbf{i} + 1.8\mathbf{j})$ m A1

$\overrightarrow{HK} = \mathbf{r}_K - \mathbf{r}_H = (9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}$

m **Printed Answer** A1 4

(d) Meet when $\overrightarrow{HK} = 0$

$(9 - 0.45t) = 0$ and $(2.7t - 54) = 0$ A1

$t = 20$ from both equations A1

$\mathbf{r}_K = \mathbf{r}_H = (24\mathbf{i} + 82\mathbf{j})$ m DM1 A1 **cso** 5

[13]

4.		$-6\mathbf{i} + \mathbf{j} = \mathbf{u} + 3(2\mathbf{i} - 5\mathbf{j})$	A1	
		$\Rightarrow\ \mathbf{u} = -12\mathbf{i} + 16\mathbf{j}$	A1 cso	
		$\Rightarrow\ u = \sqrt{(-12)^2 + 16^2} = 20$	A1	

[5]

5.	(a)	$\mathbf{v} = \dfrac{8\mathbf{i} + 11\mathbf{j} - (3\mathbf{i} - 4\mathbf{j})}{2.5}$ or any equivalent	M1A1	
		$\mathbf{v} = 2\mathbf{i} + 6\mathbf{j}$	A1	3
	(b)	$\mathbf{b} = 3\mathbf{i} - 4\mathbf{j} + \mathrm{v}t$ ft their $\mathbf{v}$	M1A1ft	
		$= 3\mathbf{i} - 4\mathbf{j} + (2\mathbf{i} + 6\mathbf{j})t$	A1cao	3
	(c)	$\mathbf{i}$ component: $-9 + 6t = 3 + 2t$		
		$t = 3$	M1A1	
		$\mathbf{j}$ component: $20 + 3\lambda = -4 + 18$		
		$\lambda = -2$	A1	5
	(d)	$v_B = \sqrt{(2^2 + 6^2)}$ or $v_C = \sqrt{(6^2 + (-2)^2)}$		
		Both correct	A1	
		The speeds of B and C are the same — cso	A1	3

[14]

6.	(a)	$\mathbf{a} = \dfrac{(15\mathbf{i} - 4\mathbf{j}) - (3\mathbf{i} + 2\mathbf{j})}{4} = 3\mathbf{i} - 15\mathbf{j}$	M1A1	2
	(b)	N2L $\mathbf{F} = m\mathbf{a} = 6\mathbf{i} - 3\mathbf{j}$ — ft their $\mathbf{a}$	M1A1	
		$\lvert\mathbf{F}\rvert = \sqrt{(6^2 + 3^2)} \approx 6.71$ (N) — accept $\sqrt{45}$, awrt 6.7	M1A1	4
	(c)	$\mathbf{v_6} = (3\mathbf{i} + 2\mathbf{j}) + (3\mathbf{i} - 1.5\mathbf{j})6$ — ft their $\mathbf{a}$	M1A1ft	
		$= 21\mathbf{i} - 7\mathbf{j}$ (m s^{-1})	A1	1

[9]

7.	(a)	Speed $= \sqrt{(2.5^2 + 6^2)} = \underline{6.5 \text{ km h}^{-1}}$	A1	2
		needs square, add and $\sqrt{}$correct components		
	(b)	Bearing $= 360 - \arctan(2.5 / 6) \approx \underline{337}$	A1	2
		for finding acute angle = arctan (2.5 / 6)		

or arctan (6 / 2.5) (i.e. 67° / 23°).
Accept answer as AWRT 337.

(c) $\mathbf{R} = (16 - 3 \times 2.5)\mathbf{i} + (5 + 3 \times 6)\mathbf{j}$

$= 8.5\mathbf{i} + 23\mathbf{j}$ A1 2

needs non-zero initial p.v. used + 'their 3' × velocity vector

(d) At 1400 s $= 11\mathbf{i} + 17\mathbf{j}$ A1

At time t, $\mathbf{s} = 11\mathbf{i} + (17 + 5t)\mathbf{j}$ ↓ A1 4

Allow 1st even if non-zero initial p.v. not used here

(e) East of $R \Rightarrow 17 + 5t = 23$

$\Rightarrow t = 6 / 5 \Rightarrow$ <u>1512 hours</u> A1 2

A1 is for answer as a time of the day

(f) At 1600 $\mathbf{s} = 11\mathbf{i} + 27\mathbf{j}$

$\mathbf{s} - \mathbf{r} = 2.5\mathbf{i} + 4\mathbf{j}$

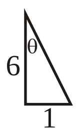
Distance $= \sqrt{(2.5^2 + 4^2)} \approx 4.72$ km A1 3

1st for using t = 2 or 4 (but not 200, 400, 6, 16 etc)
and forming $\mathbf{s} - \mathbf{r}$ *or* $\mathbf{r} - \mathbf{s}$

[15]

8. (a) Speed of $A = \sqrt{(1^2 + 6^2)} \approx$ <u>6.08 m s^{-1}</u> A1 2

(b)



$\tan\theta = 1/6 \Rightarrow \theta \approx 9.46°$ A1

Bearing ≈ <u>351</u> A1 3

(c) p.v. *of A* at time $t = (2 - t)\mathbf{i} + (-10 + 6t)\mathbf{j}$

p.v. of B at time $t = (-26 + 3t)\mathbf{i} + (4 + 4t)\mathbf{j}$ B1 (either)

(E.g.) **i** components equal $\Rightarrow 2 - t = -26 + 3t \Rightarrow t = 7$ A1

j components at $t = 7$: A: $-10 + 6t = 32$
B: $4 + 4t = 32$

Same, so collide at $t = 7$ s at point with p.v. $(-5\mathbf{i} + 32\mathbf{j})$ m A1 cso 5

	(d)	New velocity of $B = 8\,(3\mathbf{i} + 4\mathbf{j})$ m s^{-1}	B1	
		P.v. of B at 7 s $= -26\mathbf{i} + 4\mathbf{j} + 1.6(3\mathbf{i} + 4\mathbf{j}) \times 7 = 7.6\mathbf{i} + 48.8\mathbf{j}$	A1	
		$\underline{PB} = \mathbf{b} - \mathbf{p} = 12.6\mathbf{i} + 16.8\mathbf{j}$ (in numbers)		
		Distance $= \sqrt{(12.6^2 + 16.8^2)} = \underline{21\text{ m}}$	A1	6
				[16]
9.	(a)	Speed of ball $= \sqrt{(5^2 + 8^2)} \approx \underline{9.43\text{ m s}^{-1}}$	A1	2
		Valid attempt at speed (square, add and squ. root cpts)		
	(b)	p.v. of ball $= (2\mathbf{i} + \mathbf{j}) + (5\mathbf{i} + 8\mathbf{j})t$	A1	2
		needs non-zero p.v. + (attempt at veloc vector) x t. Must be vector		
	(c)	North of B when $\mathbf{i}$ components same, i.e. $2 + 5t = 10$		
		$t = \underline{1.6\text{ s}}$	A1	2
	(d)	When $t = 1.6$, p.v. of ball $= 10\mathbf{i} + 13.8\mathbf{j}$ (or $\mathbf{j}$ component $= 13.8$)	M1 A1	
			↓	
		Distance travelled by 2[nd] player $= 13.8 - 6 = 6.8$	M1 A1	
			↓	
		Speed $= 6.8 \div 1.6 = \underline{4.25\text{ m s}^{-1}}$	M1 A1	6
		or $[(2 + 5t)\mathbf{i} +]\ (1 + 8t)\mathbf{j} = [10\mathbf{i} +]\ (7 + vt)\mathbf{j}$	M1 A1	
		(pv's or $\mathbf{j}$ *components same)*		
			↓	
		Using $t = 1.6$: $1 + 12.8 = 7 + 1.6v$ (equn in v only)	M1 A1	
			↓	
		$v = \underline{4.25\text{ m s}^{-1}}$	M1 A1	
		2[nd] – allow if finding displacement vector (e.g. if using wrong time)		
		3[rd] for getting speed as a scalar (and final answer must be as a scalar). But if they get e.g. '4.25j', allow	*A0*	
	(e)	Allow for friction on field (i.e. velocity of ball not constant)	B1	1
		or allow for vertical component of motion of ball		
		Allow 'wind', 'spin', 'time for player to accelerate', size of ball		
		Do not allow on their own 'swerve', 'weight of ball'.		
				[13]

10.	(a)	$\mathbf{v}_P = \{(29\mathbf{i} + 34\mathbf{j}) - (20\mathbf{i} + 10\mathbf{j})\}/3 = \underline{(3\mathbf{i} + 8\mathbf{j})\text{ km h}^{-1}}$	A1	2
	(b)	$\mathbf{p} = (20\mathbf{i} + 10\mathbf{j}) + (3\mathbf{i} + 8\mathbf{j})t$	A1ft	
		$\mathbf{q} = (14\mathbf{i} - 6\mathbf{j}) + 12t\mathbf{j}$	A1	4
	(c)	$\mathbf{q} - \mathbf{p} = (-6 - 3t)\mathbf{i} + (-16 + 4t)\mathbf{j}$	A1 ↓	
		$d^2 = (-6 - 3t)^2 + (-16 + 4t)^2$	↓	
		$= 36 + 36t + 9t^2 + 16t^2 - 128t + 256$		
		$= 25t^2 - 92t + 292$ (*)	A1 (cso)	5
	(d)	$25t^2 - 92t + 292 = 225$		
		$25t^2 - 92t + 67 = 0$	A1 ↓	
		$(t - 1)(25t - 67) = 0$		
		$t = 67/25$ or 2.68	A1	
		time ≈ 161 mins, or 2 hrs 41 mins, or 2.41 am, or 0241	A1	5

[16]

11.	(a)	$\tan\theta = \frac{3}{2}$ ($\theta = 56.3°$)		
		angle between $\mathbf{v}$ and $\mathbf{j} = 90 + 56.3 \approx 146°$	A1	3
	(b)	$\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + (-\mathbf{i} + 2\mathbf{j})t$ $= (2 - t)\mathbf{i} + (-3 + 2t)\mathbf{j}$	A1	2
	(c)	$t = 3$, $\mathbf{v} = -\mathbf{i} + 3\mathbf{j}$ speed $= \sqrt{(1^2 + 3^2)} = \underline{\sqrt{10} \text{ or } 3.16\text{ m s}^{-1}}$	A1	3
	(d)	$\mathbf{v}$ parallel to $\mathbf{i} \Rightarrow -3 + 2t = 0$ $\Rightarrow t = \underline{1.5\text{ s}}$	A1	2

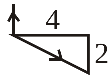
[10]

12.	(a)	Direction of $\mathbf{v} = (7\mathbf{i} - 7.5\mathbf{j}) - (4\mathbf{i} - 6\mathbf{j}) = 3\mathbf{i} - 1.5\mathbf{j}$		
		$\tan q = \frac{1.5}{3} = 0.5 \Rightarrow q = 26.565\ldots$	A1	
		Bearing = $\underline{117}$ (accept awrt)	A1	4

	(b)	$\mathbf{v} = (3\mathbf{i} - 1.5\mathbf{j}) \div \frac{3}{4} = 4\mathbf{i} - 2\mathbf{j}$	B1	
		$\mathbf{s} = \underline{(4\mathbf{i} - 6\mathbf{j}) + t(4\mathbf{i} - 2\mathbf{j})}$	A1f.t.	3
	(c)	At 1015 $\mathbf{s} = (4\mathbf{i} - 6\mathbf{j}) + \frac{5}{4}(4\mathbf{i} - 2\mathbf{j})$ ($= 9\mathbf{i} - 8.5\mathbf{j}$)	A1	
		$\mathbf{m} = 0.25\,(p\mathbf{i} + q\mathbf{j})$	B1 ↓	
		$\mathbf{s} = \mathbf{m} \Rightarrow \underline{p = 36,\ q = -34}$	A1, A1	6
				[13]
13.	(a)	$\tan\theta = \frac{3}{5} \Rightarrow \theta = 031°$	A1	2
	(b)	$\mathbf{a} = 9t\,\mathbf{j}$	B1	
		$\mathbf{b} = (-10 + 3t)\mathbf{I} + 5t\,\mathbf{j}$	A1	3
	(c)	B south of A $\Rightarrow -10 + 3t = 0$		
		$t = 3\frac{1}{3} \Rightarrow$ <u>1520 hours</u>	A1	2
	(d)	AB $= \mathbf{b} - \mathbf{a} = (3t - 10)\mathbf{I} + 5t\,\mathbf{i}$	A1 ↓	
		$d^2 = \lvert\mathbf{b} - \mathbf{a}\rvert^2 = (3t - 10)^2 + 16t^2$		
		$= 25t^2 - 60t + 100$ (*)	A1	4
	(e)	$d = 10 \Rightarrow d^2 = 100 \Rightarrow 25t^2 - 60t = 0$		
		$\Rightarrow t =$ (0 or) 2.4	A1	
		$\Rightarrow$ time <u>1424 hours</u>	A1	3
				[15]
14.	(a)	$a = \frac{1}{4}[(5\mathbf{i} + 11\mathbf{j}) - (3\mathbf{i} - 5\mathbf{j})] = -2\mathbf{i} + 4\mathbf{j}$	A1	2
	(b)	$\mathbf{F} = m\mathbf{a} = -6\mathbf{i} + 12\mathbf{j}$	A1	
		$\lvert\mathbf{F}\rvert = \sqrt{180} \cong 13.4$ N (AWRT)	A1	4
		[**OR** $\lvert\mathbf{a}\rvert = \sqrt{20} \cong 4.47 \Rightarrow \lvert\mathbf{F}\rvert = 3 \times 4.47 \cong 13.4$ N]		

	(c)	$t = 6$ $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j} + 6(-2\mathbf{i} + 4\mathbf{j})$ $[= -9\mathbf{i} + 19\mathbf{j}]$	A1	
		At B: $\mathbf{r} = (6\mathbf{i} - 29\mathbf{j}) + 3(-9\mathbf{i} + 19\mathbf{j})$ $[= -21\mathbf{i} + 28\mathbf{j}]$	A1ft	
		OB = $\sqrt{(21^2 + 28^2)}$ = <u>35 m</u>	A1	6

[12]

15.	(a)	"$v = u + at$" : $\mathrm{v} = (-2 + 2t)\mathrm{i} + (7 - 3t)\mathrm{j}$	A1	
		v parallel to i $\Rightarrow 7 - 3t = 0 \Rightarrow t = 2\frac{1}{3}$ s	A1	4
	(b)	$t = 3$, $\mathrm{v} = 4\mathrm{i} - 2\mathrm{j}$		
		$\lvert\mathrm{v}\rvert = \sqrt{20} \approx \underline{4.47}$ m s^{-1}	A1	3
	(c)			
		Angle = (arctan $\frac{2}{4}$), + 90° = 116.6° (accept 117°)	A1	3
		[or 180° – (arctan $\frac{4}{2}$)]	[A1]	

[10]

16.	(a)	$\mathbf{p} = 10t\mathbf{j}$	B1	
		$\mathbf{q} = (6\mathbf{i} + 12\mathbf{j}) + (-8\mathbf{i} + 6\mathbf{j})t$	A1	3
	(b)	$t = 3$: $\mathbf{p} = 30\mathbf{j}$, $\mathbf{q} = -18\mathbf{i} + 30\mathbf{j}$	A1	
		$\Rightarrow$ dist. apart = 18 km	A1	3

Alt. (b)

$\mathbf{PQ} = \mathbf{q} - \mathbf{p} = (6 - 8t)\mathbf{i} + (12 - 4t)\mathbf{j}$

$t = 3$: $\mathbf{PQ} = -18\mathbf{i} + 0\mathbf{j}$	or $\lvert\mathbf{PQ}\rvert^2 = (6 - 8t)^2 + (12 - 4t)^2$	A1
Dist. = 18 km	$t = 3 \rightarrow \lvert\mathbf{PQ}\rvert = 18$	A1

	(c)	Q north of $P \Rightarrow 6 - 8t = 0$		
		$t = \frac{3}{4}$	A1	2

[8]

17.	(a)	At time t	$\mathbf{r}_A = (-5 + 2t)\mathbf{i} + (10 + 2t)\mathbf{j}$	B1	
			$\mathbf{r}_B = (3 - 2t)\mathbf{i} + (4 + 5t)\mathbf{j}$	B1	
			$\mathbf{i}$ components equal when $-5 + 2t = 3 - 2t \Rightarrow t = 2$ h	A1	
			$t = 2$: $\mathbf{r}_A = -\mathbf{i} + 14\mathbf{j}$; $\mathbf{r}_B = -\mathbf{i} + 14\mathbf{j} \Rightarrow$ collide	A1	6
	(b)	New $\mathbf{r}_A = (-5 + t)\mathbf{i} + (10 + t)\mathbf{j}$			
	(c)	$\Rightarrow AB = \mathbf{r}_B - \mathbf{r}_A = (8 - 3t)\mathbf{i} + (-6 + 4t)\mathbf{j}$		A1	2
		$t = 2$: $\overrightarrow{AB} = 2\mathbf{i} + 2\mathbf{j}$, $\Rightarrow$ dist. $= \sqrt{(2^2 + 2^2)} \approx 2.83$ km		A1	3
	(d)	B north of $A \Rightarrow 8 - 3t = 0 \Rightarrow t = 8/3 \Rightarrow$ time 1440 hours			

[11]

1. This proved to be a tricky opening question for many of the candidates. The most popular approach was to find the starting position and then use it to find the position vector at $t = 2$. Errors in sign were fairly common at some stage of the working. A significant minority did not use a valid method at all, some just multiplying the given velocity vector by 2 or using a time of 6 only, and others becoming confused with constant acceleration formulae. A number of candidates failed to find the magnitude of their position vector to obtain the distance as required; there were follow through marks available for this even if the vector had been determined incorrectly. A few found the distance from the starting point rather than from the origin. Nevertheless, there were a fair number of entirely correct solutions.

2. There was some confusion in parts (a), (b) and (c) over which vectors were velocities and which were displacements, with some even using acceleration. In the first part, many did not appreciate the distinction between velocity and speed and in part (b) many were unable to convert an appropriate angle into a bearing. The third part tended to be well-answered but a few used ‘verification’ at $t = 0$ and $t = 4$ and scored nothing. Part (d) was a good discriminator and the less able were often unable to make much progress. The majority of candidates who used Pythagoras to find the magnitude of the relative position vector and equated it to 10 scored at least 3/6 but many often lost the accuracy marks due to poor algebra. There were a number of other methods seen which used the fact that the lighthouse was on the path of the ship and that the speed of the ship was 5 km/h and these received full credit.

3. Most candidates were able to gain the first six marks and most seemed to know that, in part (c), they needed to perform a subtraction on $\mathbf{r}_H$ and $\mathbf{r}_K$ although some were unsure which way round to do it. Another common error was to equate the position vectors and then fudge the answer. This received no credit.

In part (d) many candidates assumed that the hikers would meet and equated just one pair of components to produce $t = 20$. If they then used just one hiker to find $24\mathbf{i} + 82\mathbf{j}$ they scored only 2 out of 5, if they used **both** hikers, they scored full marks. There were a number of other ways of obtaining $t = 20$, some spurious, but provided that the candidate verified that both hikers were at the point with position vector $24\mathbf{i} + 82\mathbf{j}$ at $t = 20$, they could score all of the marks.

4. Most candidates realised that they needed to apply $\mathbf{v} = \mathbf{u} + \mathbf{a}\,t$ and many arrived at $12\mathbf{i} - 16\mathbf{j}$ but then failed to go on and find the speed, losing the final two marks. This showed a lack of understanding of the relationship between speed and velocity. A small minority found magnitudes at the start and then tried to use $v = u + at$, gaining no marks. Some candidates lost the third mark because of errors in the manipulation of negative numbers.

5. There was some evidence that a number of weaker candidates were unable to complete this question but it wasn’t clear whether they ran out of time or simply couldn’t do it.
In parts (a) and (b) some candidates confused the use of position vectors and velocity vectors.

(a) This was well answered by most candidates. Where errors did occur they often involved adding the position vectors, not dividing by the time or miscalculating the time or else doing the subtraction incorrectly or the wrong way round.
Particular examples:
errors in dividing by 2.5, particularly the **j**-component of the vector.
errors in time, using 2.3 or 4.5 hours.

some candidates changed the time into minutes, others into seconds.
not enough care was taken in looking at the compatibility of length and time units.
use of inappropriate formulae to solve the problem.

A few candidates clearly did not know how to deal with it at all.

(b) This was often correct. Errors that did occur were usually in the position vector, either using 8**i** + 11**j** or else leaving it out completely. Also some candidates used a position vector for **v**. A few candidates found the speed or velocity. However for those who had an answer to part (a) most were successful in carrying it correctly forward into this part.

(c) Most knew they had to equate the position vectors but a number did not then go on to equate coefficients of **i** and **j**. Those that did were largely successful in getting the right values out.
Others tried to solve the equation for □ by crossing out all the '*t*'s or all the '**i**'s and '**j**'s. Some tried to divide vectors whilst others just substituted in random values for *t*.

(d) Relatively few got full marks here. Most, who got part (a) correct ,were able to get the first mark. Common errors seen were finding the position at $t = 3$ and then using Pythagoras, or else using $\mathbf{v} \times t$. Some candidates just stated that the vectors were the same. Many of those who did carry out the correct calculations either left it at that, without making a statement, or else declared that the velocities rather than the speeds were equal. There were a few instances where 6**i** + 2**j** was taken as the second speed, with no obvious connection to their previous work, using the fact that the speeds must be equal! A few also guessed λ in part (d) and then placed this value at the end of a page of incomprehensible working in part (c).

6. In part (a) most candidates knew the method and it was often fully correct but a number failed to find the magnitude of the force in the second part, with some, subtracting the squares of the components instead of adding them. Part (c) was well answered.

7. **A**s always the vector question proved to be challenging for many weaker candidates. A number did not attempt it at all. For those who did, most found the speed correctly in part (a), but answers to part (b) were very variable: many chose the wrong vector to use (the position vector, not the velocity); others could not find the correct bearing from the acute angle obtained from their diagrams. Most with any understanding of the topic could complete parts (c) and (d), though there was quite a lot of confusion about the 24 hour clock (with e.g. values for t of 1600 or 200 being used [instead of 2 etc]). In part (e) some equated the j component to zero, rather than to the value obtained from part (c); others obtained t as 1.2 correctly but could not put it back into the context of the question as a time of day. Those who got as far as part (f) could usually make a good attempt (though a number used t = 4 rather than 2), and a number of correct final answers were seen.

8. Part (a) was generally well answered, most knowing how to calculate a 'speed' from a 'velocity'. In part (b), most calculated an angle, but as often as not failed to give any indication which angle it was in relation to the data; they then often failed to deal with the angle correctly to find the correct bearing. Candidates should be encouraged to show clearly their working and, if they are calculating an angle, which angle in a figure it is. In part (c), most correctly equated the two general position vectors to find a value of t, but some failed to make the full check to ensure that the value of t obtained produced equality for both coordinates (hence implying a collision). Part (d) proved to be more discriminating. Several could not find the new velocity of B correctly (given its magnitude and direction though with the direction not given in the form of a unit vector); however, a number of candidates did manage to pick up the method marks here by proceeding correctly with what they thought the velocity was.

9. The question proved again to be a good discriminator. The calculations involved were relatively simple, though a correct solution did require a proper understanding of the physical situation. Part (a) was generally well done, though not universally: some evidently did not know the meaning of the word ‘speed’. Part (b) was mostly correct. In part (c) a significant minority equated the **j** components, rather than the **i** components. In part (d), many got to the end result, apparently correctly, though the working presented often proved to be very challenging to decipher. Others used the wrong vectors or distances involved. In part (e) presentation was again somewhat inadequate, with some effectively stating one of the assumptions (e.g. the field being smooth), rather than saying that the opposite would be a factor needing to be taken into account (i.e. friction). Again some relevant responses were given, but also a number of irrelevant (or unclear) ones.

10. This was probably the most discriminating question on the paper with only the top grade of candidates tending to complete the whole question successfully. Most could make a good attempt at part (a), and the writing down of the two position vectors in part (b) was generally well done. Parts (c) and (d) were however more taxing. Several could not start part (c) at all by subtracting the two position vectors; others could not progress because they failed to collect expressions for the components before finding the modulus of this vector. In part (d), several successfully restarted even though they had not reached the given answer in part (c). Many offered well presented answers to the solution of the quadratic equation. Some however failed to equate the d of the given expression to 15 (some using 15.1 or 16).

11. Part (a) was found to be the hardest part of this question: most could find a relevant acute angle but very few could convert this to the angle required in the question. In part (b), several failed to give their answers in the required form ‘ai + bj’, collecting together the i and j components. In part (c), most could find the velocity (as a vector) but many then stopped, not appreciating the difference between ‘velocity’ and ‘speed’. In part (d), the majority found the correct method, but a significant minority equated the i component of the velocity to zero.

12. The vector question was found to be quite difficult by a number of candidates. In part (a), many failed to realise that they had to find the direction of motion, and simply tried to find a direction associated with one or other of the position vectors given. Part (b) was generally better done: even those who did not realise how to tackle part (a) could still find the velocity in part (b). In part (c), there was often considerable confusion about the times involved: many used a value for t in minutes rather than hours, and several effectively assumed that S and M had been moving for the same time. The interpretation of the actual times (‘0900’, ‘1015’ etc) caused a number of problems for many. Hence although many could adopt a correct general approach, fully accurate answers to this part were seen only by the better candidates. The standard of presentation here was also not good: many very scrappy answers were presented with little or no explanation given for what was being attempted and working strewn all over the paper.

13. Once again vector work proved to be challenging for many at this level. Quite a few failed to find the correct angle for the bearing required in part (a). In part (b), most could make a reasonable attempt to write down expressions of the two position vectors at a general time *t*. In part (c) however, clear methods were not seen very often and many failed to realise that they had to equate the i components of the position vectors. In part (d), many appeared to be fudging their working to produce the given answer, and clear methods finding an expression for the vector **AB** or the vector **BA**, and then finding its magnitude, were not that common.

However, a number managed to produce a good solution in part (e) even where they had struggled with the vector work earlier in the question. A number who successfully completed all the vector work lost a mark or two by failing to give their answers to parts (c) and (e) as a specific time, rather than just the value of a variable *t*: interpreting the results obtained from the algebra in terms of the original context of the question was expected here.

14. As always vector work proved to be challenging to many of the weaker candidates. Most could cope reasonably well with parts (a) and (b), and it was pleasing to see that only a minority this time failed to understand the significance of the demand to find the 'magnitude' of the force in part (b). Part (c) however was beyond many and several could not make a start at all here. There was some confusion between displacements and velocities, and some tried to use constant acceleration equations in vector form, often with little success. However, the question was by no means impossible and a number of the more able candidates scored full marks here.

15. Vectors once again proved the most challenging part of the paper for many candidates. Further, many failed to explain their work clearly so that it was not always easy to give candidates credit for correct methods even when the final answers were wrong. In part (a), very few seemed able to write down an expression for the velocity at a general time *t*, or then use this to find when the particle was moving parallel to i. Some partially recovered themselves in part (b), perhaps feeling more comfortable with a numerical value of *t* to use; however, a very large proportion of candidates simply found the velocity of *P*, failing to appreciate the distinction been 'velocity' and 'speed'. In part (c) several attempted to find an angle, but as often as not, simply found an acute angle for the direction of their velocity vector, failing to realise that with a vector such as 4i – 2j, an obtuse angle would be involved.

16. Most were successful in doing part (a) correctly. In part (b) there was a similar problem to that of understanding the meaning of 'magnitude' in qu. 3 in that many assumed that the 'distance' of *P* from *Q* was 18i instead of 18 km: again the problem of interpreting vectors properly caused some problems. Answers to part (c) were rather variable: several were completely correct, but a fair proportion of candidates tried to equate the **j** components.

17. No Report available for this question.