1. A particle P moves on a straight line that contains the point O. At time t seconds the displacement of P from O is s metres, where $s = t^3 - 3t^2 + 3$.

(a) Determine the times when the particle has zero **velocity**. [3]

(b) Find the distances of P from O at the times when it has zero velocity. [2]

2. **In this question you must show detailed reasoning.**

A boy plays on a path that runs north-south through an origin O. His displacement x metres north of O at time t seconds is given by

$$x = -0.7t^2 + 4t \text{ for } 0 \le t \le 10.$$

(a) Determine the direction in which he is moving when $t = 7$. [3]

(b) Find the furthest distance from O reached by the boy for $0 \le t \le 10$. [5]

3. A car travels along a straight track for 5 seconds. Its displacement s metres after t seconds is given by

$$s = 3t + 0.1t^3 .$$

Show that the car does not have constant acceleration. [3]

4. **In this question you must show detailed reasoning.**

Fig. 6 shows the velocity-time graph for a car as it travels along a straight road. The car sets off from some traffic lights and stops momentarily at a road junction. The velocity v ms^{-1} of the car at time t s after leaving the traffic lights is modelled by

$$v = 0.025t^3 - 0.8t^2 + 6.4t \text{ for } 0 \le t \le 20.$$

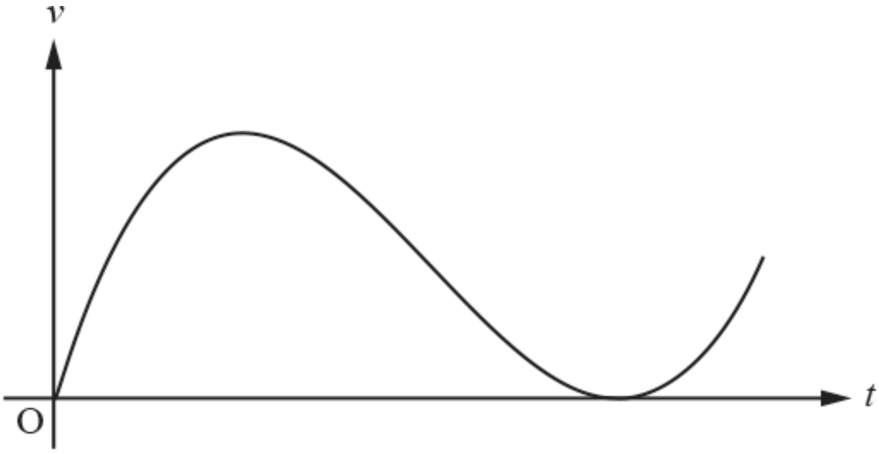


Fig. 6

Calculate the distance from the traffic lights to the road junction. [6]

5. The velocity of a car, v ms^{-1} at time *t* seconds, is being modelled. Initially the car has velocity 5 ms^{-1} and it accelerates to 11.4 ms^{-1} in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(a) Find an expression for the velocity of the car at time *t* using this model. [3]

(b) Explain why this model is not appropriate in the long term. [1]

Model A is refined so that the velocity remains constant once the car reaches 17.8 ms^{-1}.

(c) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. [3]

(d) Calculate the displacement of the car in the first 20 seconds according to this refined model. [3]

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

(e) Show that this model gives an appropriate value for *v* when $t = 4$. [1]

(f) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. [3]

(g) Show that model B gives the same value as model A for the displacement at time 20 s. [3]

6. In a laboratory experiment, the motion of a small object moving in a straight line is being studied. A model for its velocity v m s^{-1} at time t s is given by

$$v = at^4 + bt^3,$$

where a and b are constants and $t > 0$.

The velocity has maximum value of 0.1 m s^{-1} when $t = 1$.

(a) Determine the values of a and b. [5]

(b) Find the time at which the particle changes direction. [2]

(c) Explain why the model would not be suitable for very large values of t. [1]

7. Fig. 8 shows the velocity-time graph of a car that is travelling in a straight line as it manoeuvres then drives away. Its velocity $v$ms^{-1} at time t s is given by $v = 0.1t^3 + 0.9t^2 - 1$.

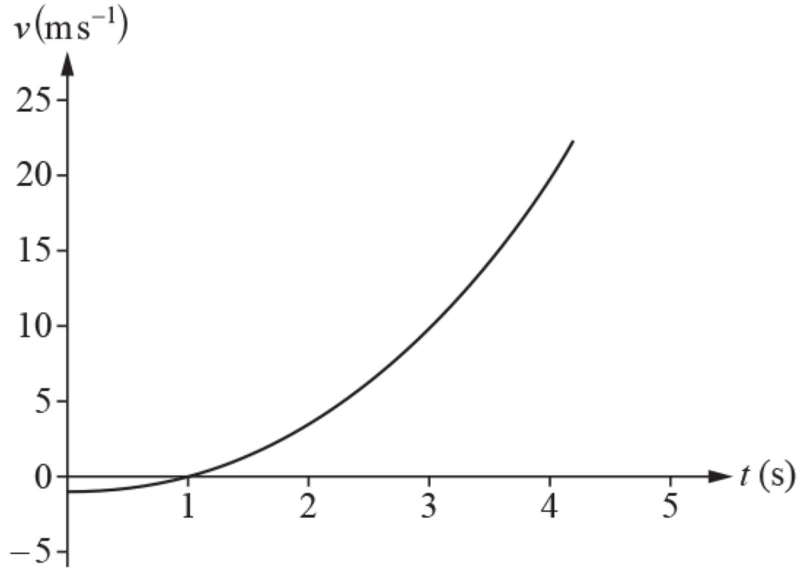


Fig. 8

(a) Describe two features of the motion of the car in the first 4 seconds. [2]

(b) **In this question you must show detailed reasoning.**
Calculate the total distance travelled in the first 4 seconds. [5]

(c) Find an expression for the acceleration of the car in terms of t. [2]

END OF QUESTION paper

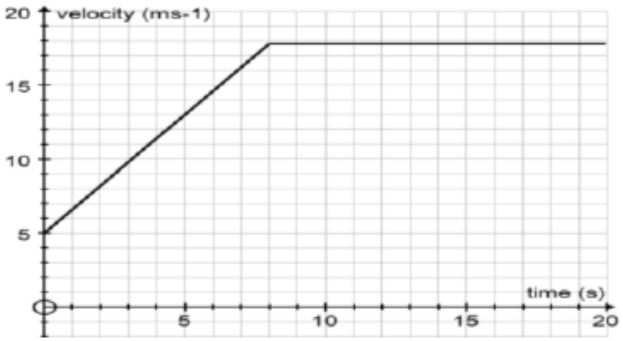
Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		a	Velocity v is $\frac{ds}{dt} = 3t^2 - 6t$ $= 0$ $t = 0$ or 2	M1(AO1.1a) M1(AO1.1) A1(AO1.1) [3]	Attempt to find $\frac{ds}{dt}$ $\frac{ds}{dt} = 0$ must be stated Both roots found
		b	$s(0) = 3$ so distance 3 m $s(2) = 8 - 12 + 3 = -1$ so distance is 1 m	A1(AO1.1) A1(AO3.4) [2]	Accept seeing 3 without comment –1 for s must be seen as well as 1 m for distance
			Total	5	
2		a	DR $v = -0.7 \times 2t + 4$ When $t = 7$, $v = -5.8$ Boy is moving due south since v is negative	M1(AO 3.1b) M1(AO 3.4) E1(AO 2.2a)	For attempt to differentiate For substitution in their v

				[3]	Dependent on correct −5.8
		b	DR Max/min s when $v = 0$, so $-1.4t + 4 = 0$ $t = \frac{20}{7}$ When $t = \frac{20}{7}$, $s = \frac{40}{7} = 5.71$ to 3sf When $t = 10$, $s = -30$ and when $t = 0$, $s = 0$ Greatest distance is 30 m	M1(AO 3.1b) A1(AO 1.1b) A1FT(AO 1.1b) B1(AO 1.1b) B1FT(AO 3.2a) [5]	For attempt to solve their $v = 0$ Follow through their t For checking both end-points Must select (their) correct distance, must be positive and must have units
			Total	8	
3			$v = \frac{ds}{dt} = 3 + 0.3t^2$ and $a = \frac{dv}{dt} = 0.3 \times 2t$ $a = 0.6t$ Acceleration is a function of t so is not constant Alternative method	M1(AO 1.1a) A1(AO 2.1) E1(AO 2.2a) M1	Attempt to differentiate twice Must be clearly argued

			For constant acceleration, $s = ut + \frac{1}{2}at^2\ (+c)$ i.e. s is a quadratic function of t So this cubic function is not constant acceleration	A1 E1 [3]	Comparing with standard formula Explicit identification of quadratic, oe Deduction clearly stated
			Total	3	
4			DR $0.025t^3 - 0.8t^2 + 6.4t = 0$ $0.025t(t^2 - 32t + 256) = 0 \Rightarrow 0.025t(t - 16)^2 = 0$ $t = 0$ or 16 Distance is $\int_0^{16} (0.025t^3 - 0.8t^2 + 6.4t)\,\mathrm{d}t$ $= \left[0.025\frac{t^4}{4} - 0.8\frac{t^3}{3} + 6.4\frac{t^2}{2}\right]_0^{16}$ $= \left(0.025 \times \frac{16^4}{4} - 0.8 \times \frac{16^3}{3} + 6.4 \times \frac{16^2}{2}\right) - (0)$	M1(AO3.4) A1(AO2.1) M1(AO3.4) A1(AO1.1b) M1(AO2.1) B1(AO1.1b)	Equating v to 0 for time at junction Factorising seen; method must be clear Limits not required for this mark Correct integration, and limits soi

			Distance = 137 m (3 sf)	[6]	Use of limits seen; substitution of limits into integral must be seen — $\frac{2048}{5}-\frac{16\,384}{15}+\frac{4096}{5}$ Allow for any method www — $\frac{2048}{15}=136.53\ldots$
			Total	6	
5		a	$u = 5$, $v = 11.4$, $t = 4$ $a=\frac{v-u}{t}=\frac{11.4-5}{4}=1.6$ $v = 5 + 1.6t$	M1 (AO 3.1b) A1 (AO 1.1b) A1 (AO 3.3) [3]	Using suvat equation(s) leading to value for a Any form FT their a Examiner's Comments The key to this question was to calculate the acceleration of the car. The required expression is then found by substituting the values for u and a into the equation $v = u + a\,t$. Many fully correct answers were seen.
		b	The car would not be able to accelerate indefinitely – the velocity would become too large	E1 (AO 3.5b) [1]	Examiner's Comments

					Candidates were required to recognise the limitations of this model. Most successful answers stated that the velocity would eventually get much too big or that the car would have to slow down or stop at some point.
		c	When $v = 17.8$ $$t = \frac{17.8 - 5}{1.6} = 8$$ 	B1 (AO 1.1a) G1 (AO1.1a) G1 (AO 3.5c) [3]	Calculation or point on graph labelled at $t = 8$ Two line segments with one horizontal Axes labelled. (0, 5) and constant speed 17.8 clear on vertical scale Mark intent for 17.8 – allow for a linear scale beyond 17.8 **Examiner's Comments** Most candidates correctly had a graph consisting of two line segments but a common error was to begin the graph at the origin when the initial velocity was 5 ms^{-1}. Some did not fully label their graph so lost a mark. AfL Make sure the axes are labelled and that all key points are clearly indicated on the graph.
		d	Dividing area into sections Area under trapezium $= \frac{1}{2}(5+17.8)\times 8 = 91.2$	M1 (AO 3.1b) A1 (AO 1.1a)	FT their graph if linear for M1 A0 for a triangle or trapezium area

			Area rectangle 12 × 17.8 = 213.6 Total displacement = 304.8 m	A1 (AO 1.1b) [3]	May be found as sum of areas. May be implied by correct total FT their distance found for first 8s — 213.6 must be added to another distance **Examiner's Comments** Many candidates were successful in finding the area under their graph with only a few arithmetical errors.
		e	When $t = 4$ $v = 5 + 0.3 \times 4^2 - 0.05 \times 4^3 = 11.4$ ms^{-1} Which matches the given value	B1 (AO 3.4) [1]	Allow without comment **Examiner's Comments** This mark was credited for seeing the substitution of $t = 4$ into the equation. It would have been good to see this followed by a comment that the value was close to the given value.
		f	$\frac{dv}{dt} = 0.6 \times 2t - 0.05 \times 3t^2 \ \left[= 1.2t - 0.15t^2 \right]$ When $t = 8$ $v = 1.2 \times 8 - 0.15 \times 64 = 0$ Acceleration is zero at $t = 8$ which means that the car reaches its maximum speed without the sudden change in acceleration in model A.	M1 (AO 1.1a) A1 (AO 3.2a) E1 (AO 3.2a) [3]	Need not be simplified Must mention acceleration Must compare with model A Final mark can be awarded independently for a statement about change in acceleration as long as supported by some numerical evidence

<table>
<tr><td></td><td></td><td></td><td></td><td></td><td><u>Examiner's Comments</u>

Some candidates were very vague in their answer here and were not credited many marks. There is a very clear instruction that it is the value of acceleration that is needed, so 2 of the 3 marks were given for finding this. Many candidates were able to comment that model B gives a gradual change in acceleration as it approaches the maximum speed avoiding the very sudden change seen in model A.</td></tr>
<tr><td></td><td></td><td>g</td><td>EITHER
$$\int_0^8 \left(5+0.6t^2-0.05t^3\right) \mathrm{d}t = \left[5t+0.2t^3-0.0125t^4\right]_0^8$$ $=91.2$ m

which is same as model A for the first 8 s Distance is the same for the remainder of the time So this is the same as model A at $t = 20$

OR
$$\int_0^8 \left(5+0.6t^2-0.05t^3\right) \mathrm{d}t = \left[5t+0.2t^3-0.0125t^4\right]_0^8$$
= 91.2 m

Distance at 17.8 ms^{-1} 213.6

Total distance 304.8m

[which is the same as model A]</td><td>M1 (AO 2.1)
A1 (AO 1.1b)

E1 (AO 2.1)
[3]

M1
A1

A1</td><td><table><tr><td>BC

Must consider to $t = 20$

BC</td><td>Allow for correct definite integral stated and calculator used. Also allow M1A1 for $5\times8 - 0.2 \times8^3 - 0.0125 \times 8^4$ seen

Allow for correct definite integral stated and calculator used.</td></tr></table></td></tr>
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					Must consider to $t = 20$ **Examiner's Comments** Many candidates realised that the distance was the definite integral that gave the distance travelled in the first 8s. It would have been sufficient to clearly write the integral with its limits and use a calculator to evaluate it. Most candidates decided to give a full solution with the substitution of limits made clear. Only a few candidates omitted the part of the journey beyond 8s.
			Total	17	
6		a	$v = 0.1$ when $t = 1$ gives $a + b = 0.1$ $\frac{dv}{dt} = 4at^3 + 3bt^2$ Maximum v when $t = 1$ gives $4a + 3b = 0$ Solving simultaneous equation for a and b $a = -0.3$ and $b = 0.4$	M1 (AO 3.3) M1 (AO 3.1b) M1 (AO 3.3) M1 (AO 1.1a) A1 (AO 1.1) [5]	Using given information to find an equation linking a and b Equating the derivative to zero to find an equation linking a and b Method may be implied, e.g. if **BC** cao

		b	Changes direction when $v = 0$, so $-0.3t^4 + 0.4t^3 = 0 \Rightarrow 0.3t^4 \Rightarrow 0.4t^3 \Rightarrow t = \frac{4}{3}$ (as $t > 0$) Particle changes direction when $t = \frac{4}{3}$	M1 (AO 3.4) A1 (AO 1.1) [2]	For equating v to 0 and solving for t (may be **BC**); ignore any inclusion of $t = 0$ at this point cao
		c	Model is not suitable for large values of t as the object's velocity would increase without limit	B1 (AO 3.5b) [1]	oe, e.g. 'velocity gets very large'
			Total	8	
7		a	Two valid comments, e.g.: The car's direction of motion changes from negative to positive [at time $t = 1$] The initial speed of the car is 1 m s^{-1} [in the negative direction] After 1 s the car is momentarily stationary The car accelerates [in the positive direction] reaching a speed of 19.8 m s^{-1} [after 4 seconds]	B1 (AO 2.2a) B1 (AO 2.2a) [2]	For a comment involving the change of direction For any essentially different sensible comment about the motion

		b	DR Consideration of two separate phases of the motion $s = \int (0.1t^3 + 0.9t^2 - 1)\,dt = 0.025t^4 + 0.3t^3 - t\,(+c)$ For 1st second: $s = (0.025\times1^4 + 0.3\times1^3 - 1) - 0 = -0.675$ For the next three seconds: $s = (0.025\times4^4 + 0.3\times4^3 - 4) - (-0.675)$ $= 22.275$ Total distance = 22.275 + 0.675 = 22.95m	M1 (AO 3.1b) M1 (AO 1.1a) A1 (AO 1.1b) M1 (AO 1.1a) B1 (AO 1.1b) [5]	May be implied Attempt to integrate the terms is needed Correct indefinite integration (may be seen as working for a definite integral) — + c not required here For substitution of limits, oe — Allow this mark for limits 0 and 4 Allow for correct answer seen, www
		c	$a = \dfrac{dv}{dt} = 0.3t^2 + 1.8t$	M1 (AO 1.1a) A1 (AO 1.1b) [2]	Attempt to differentiate cao
			Total	9	