1. At time $t = 0$, a small stone is thrown vertically upwards with speed $14.7\,\text{m s}^{-1}$ from a point A.

At time $t = T$ seconds, the stone passes through A, moving downwards.

The stone is modelled as a particle moving freely under gravity throughout its motion.

Using the model,

(a) find the value of T,

(2)

(b) find the total distance travelled by the stone in the first 4 seconds of its motion.

(4)

(c) State one refinement that could be made to the model, apart from air resistance, that would make the model more realistic.

(1)

a) $t=0$ $\quad u=14.7 \uparrow$ A $\downarrow$ $\quad t=T$

$s=0$
$u=14.7$
$v=$
$a=-g$
$t=T$

using "$s=ut+\frac{1}{2}at^2$"

$0 = 14.7T - \frac{g}{2}T^2$ (1)

$0 = T(14.7-4.9T)$

$T=0$ or $14.7-4.9T=0$

$T=\frac{14.7}{4.9}=3$

$\therefore T=3$ (1)

b) separate distances going up and going down:

up:
$s=s_1$
$u=14.7$
$v=0$
$a=-g$
$t=1.5$

"$s=\frac{u+v}{2}t$"

$s_1=\frac{14.7+0}{2}\times 1.5$ (1)

$s_1=11.025$ (1)

down:
$s=s_2$
$u=0$
$v=$
$a=g$
$t=2.5$

"$s=ut+\frac{1}{2}at^2$"

$s_2=0+\frac{1}{2}g(2.5)^2$ (1)

$s_2=30.625$

Total distance $=s_1+s_2=11.025+30.625=41.65\text{m}=42\text{m (2sf)}$ (1)

c) account for dimensions of the stone (1)

2. The point A is 1.8 m vertically above horizontal ground.

At time $t = 0$, a small stone is projected vertically upwards with speed U m s^{-1} from the point A.

At time $t = T$ seconds, the stone hits the ground.

The speed of the stone as it hits the ground is 10 m s^{-1}

In an initial model of the motion of the stone as it moves from A to where it hits the ground

- the stone is modelled as a particle moving freely under gravity
- **the acceleration due to gravity is modelled as having magnitude 10 m s^{-2}**

Using the model,

(a) find the value of U,

(3)

(b) find the value of T.

(2)

(c) Suggest one refinement, apart from including air resistance, that would make the model more realistic.

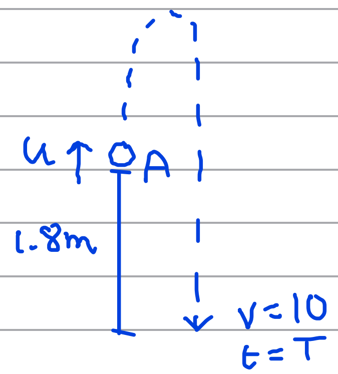
(1)

In reality the stone will not move freely under gravity and will be subject to air resistance.

(d) Explain how this would affect your answer to part (a).

(1)

a)



for movement A to ground: (↑+)

$s = -1.8$
$u = U$
$v = -10$
$a = -10$
$t =$

"$v^2 = u^2 + 2as$"

$(-10)^2 = U^2 + 36$ 2

$100 = U^2 + 36$

$U^2 = 64$

$U = 8$ 1

v is negative here because we chose the upwards direction as positive.

b) using "$v = u + at$"

$-10 = 8 - 10T$ 1

$10T = 18$

$T = 1.8\text{s}$ 1

c) use a more accurate value for g, e.g. $g = 9.8$ (1)

d) U would be greater. (1)

3. A small stone is projected vertically upwards with speed $39.2\,\text{m s}^{-1}$ from a point O.

The stone is modelled as a particle moving freely under gravity from when it is projected until it hits the ground $10\,\text{s}$ later.

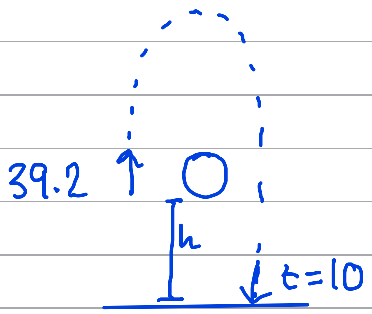
Using the model, find

(a) the height of O above the ground, **(3)**

(b) the total length of time for which the speed of the stone is less than or equal to $24.5\,\text{m s}^{-1}$ **(3)**

(c) State one refinement that could be made to the model that would make your answer to part (a) more accurate. **(1)**

a)



motion from O to ground (↑+) (1)

$s = -h$

$u = 39.2$

$v =$

$a = -g$

$t = 10$

since we took upwards as positive, we take displacement downwards negative. So we use $-h$.

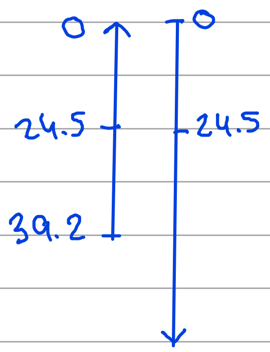
using "$s = ut + \frac{1}{2}at^2$"

$-h = 39.2(10) - \frac{1}{2}g(10)^2$ (1)

$-h = 392 - 490$

$h = 98\text{ m}$ (1)

b)



Motion from when speed upwards is 24.5 until speed downwards is 24.5 (1)

$s =$

$u = 24.5$

$v = -24.5$

$a = -g$

$t = t$

"$v = u + at$"

$-24.5 = 24.5 - gt$ (1)

$t = \dfrac{24.5 + 24.5}{g} = 5$ (1)

c) could include air resistance (1)

4. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\text{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

a) $\underline{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ when $t=0$, $\underline{v} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\underline{v} = \underline{u} + \underline{a}t$

$= \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ 1

$= \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

$\underline{v} = 8\underline{i} - 6\underline{j}$ 1

b) when $t=0$, $\underline{r}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underline{s} = \underline{r} - \underline{r}_0$

$\therefore \ \underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 4 \\ 0 \end{pmatrix} + \frac{3^2}{2}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ 1

$= \begin{pmatrix} 22 \\ -25/2 \end{pmatrix}$

$\underline{r} = 22\underline{i} - 12.5\underline{j}$ 1

5. A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of 3.2 m s^{-2}

Find

(a) the speed of the car after 5 s,

(1)

(b) the distance travelled by the car in the first 5 s.

(2)

(a) $v = u + at$

$v = 0 + 3.2 \times 5$

$v = 16 \text{ ms}^{-1}$ (1)

(b) $s = \frac{1}{2}(u+v)t$ ← other suvat equations would also work

$s = \frac{1}{2} \times (0 + 16) \times 5$ (1)

$s = 40\text{m}$ (1)

6. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P passes through the point A.

At time $t = 5\,\text{s}$, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$

(a) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})\,\text{m}$.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})\,\text{m}$.

(b) Find the value of T.

(3)

(c) Find the value of c.

(3)

(a) $v = u + at$

$V_B = (-16i - 3j) + (2.4i + j) \times 5$ (1)

$V_B = -4i + 2j$ (1)

speed $= |v|$

$= \sqrt{(-4)^2 + 2^2}$ (1)

$= \sqrt{20}$

$= 2\sqrt{5}$

$= 4.5\ \text{ms}^{-1}$ (1)

start : $A = (44i - 10j)$

end : $C = (4i + cj)$

(b) $s = ut + \frac{1}{2}at^2$

$\underset{\text{end}}{(4i + cj)} = (-16i - 3j)T + \frac{1}{2}(2.4i + j)T^2 + \underset{\text{start}}{(44i - 10j)}$ (1)

i-components : $4 = -16T + 1.2T^2 + 44$ (1)

$1.2T^2 - 16T + 40 = 0$

$T = 10$ or $T = \frac{10}{3}$

$T > 5$ so $T = 10$ seconds (1)

(c) j-components: $C = -3T + \frac{1}{2}T^2 - 10$ (1)

$C = -3(10) + \frac{1}{2}(10^2) - 10$ (1)

from part (b)

$C = 10$ (1)

7.

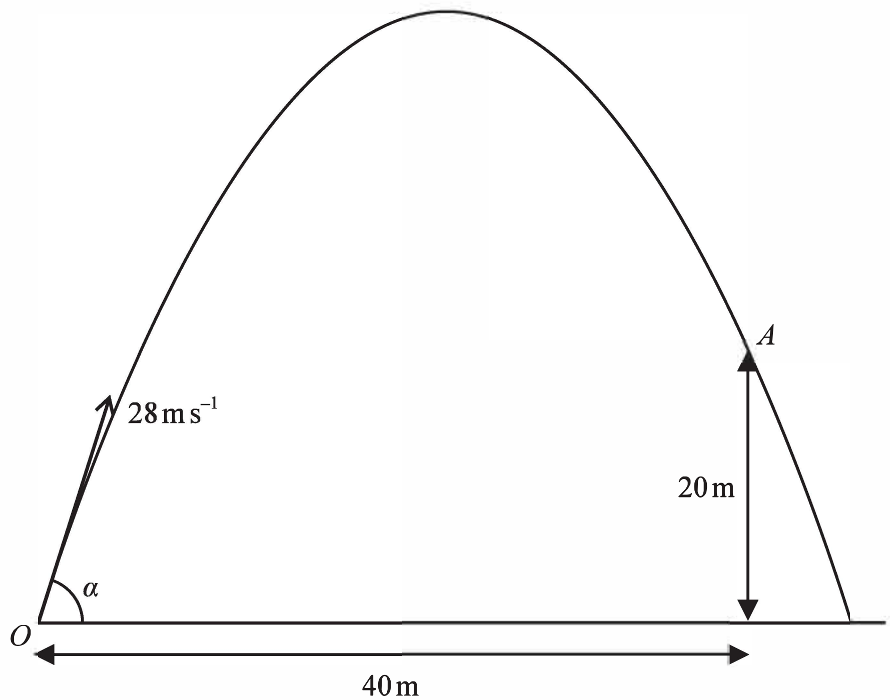


Figure 2

A small ball is projected with speed $28\,\text{m s}^{-1}$ from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

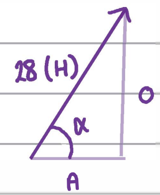
(3)

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

(a)



Initial speed (28) has horizontal and vertical components.

Horizontal (A) $= 28\cos\alpha$
Vertical (O) $= 28\sin\alpha$] using SOHCAHTOA

Horizontally:

$$28\cos\alpha = \frac{40}{T} \quad (1) \quad \leftarrow \text{speed} = \frac{\text{distance}}{\text{time}}$$

$$28\cos\alpha \times T = 40$$

$$T = \frac{40}{28\cos\alpha}$$

$$T = \frac{10}{7\cos\alpha} \quad (1)$$

(b) Vertically:

$$20 = (28\sin\alpha \times T) + \left(\tfrac{1}{2} \times -g \times T^2\right) \quad (1) \quad \leftarrow s = ut + \tfrac{1}{2}at^2$$

$$20 = (28\sin\alpha \times T) - \tfrac{1}{2}gT^2 \quad (1)$$

$$20 = \left[28\sin\alpha \times \frac{10}{7\cos\alpha}\right] - \left[\frac{1}{2}g\left(\frac{10}{7\cos\alpha}\right)^2\right] \quad (1)$$

$$\frac{\sin x}{\cos x} = \tan x$$

$$20 = 40\frac{\sin\alpha}{\cos\alpha} - \frac{g}{2} \times \frac{100}{49\cos^2\alpha}$$

$$20 = 40\tan\alpha - \frac{100g}{98} \times \frac{1}{\cos^2\alpha}$$

$$\frac{1}{\cos^2 x} = \sec^2 x$$

$$\sec^2 x = 1 + \tan^2 x$$

$$20 = 40\tan\alpha - \frac{100 \times 9.8}{98} \times (1 + \tan^2\alpha) \quad (1)$$

$$20 = 40\tan\alpha - 10 - 10\tan^2\alpha$$

$$10\tan^2\alpha - 40\tan\alpha + 30 = 0$$

$$\tan^2\alpha - 4\tan\alpha + 3 = 0 \quad (1)$$

(c) $\tan^2\alpha - 4\tan\alpha + 3 = 0$

$(\tan\alpha - 3)(\tan\alpha - 1) = 0$

$\therefore \tan\alpha = 3 \qquad \tan\alpha = 1$ ← select larger value of α to

$\alpha = 71.6° \qquad \alpha = 45°$ (1) obtain "greatest possible height"

$v^2 = u^2 + 2as$ ← at highest point, vertical velocity is 0.

$0 = (28\sin\alpha)^2 + (2\times -g \times H)$ (1)

$0 = (28 \times \sin(71.6°))^2 - 2\times 9.8 \times H$

$0 = 26.56^2 - 19.6H$

$19.6H = 705.43$

$H = 35.99$

$H = 36.0m$ to 3.s.f (1)

(d) Ball is modelled as a particle. (1)