1. A particle P moves along a straight line.

At time t seconds, the velocity $v\,\text{m s}^{-1}$ of P is modelled as

$$v = 10t - t^2 - k \qquad t \geqslant 0$$

where k is a constant.

(a) Find the acceleration of P at time t seconds.

(2)

The particle P is instantaneously at rest when $t = 6$

(b) Find the other value of t when P is instantaneously at rest.

(4)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 6$

(4)

a) $a = \frac{dv}{dt}$ $\quad v = 10t - t^2 - k$

$\frac{dv}{dt} = 10 - 2t$ (2)

b) instantaneously at rest when $v = 0$

$\therefore v = 0$ when $t = 6$

$0 = 10(6) - (6)^2 - k$

$0 = 60 - 36 - k$

$k = 24$ (1)

$v = 10t - t^2 - 24$ set $v = 0$

$0 = 10t - t^2 - 24$

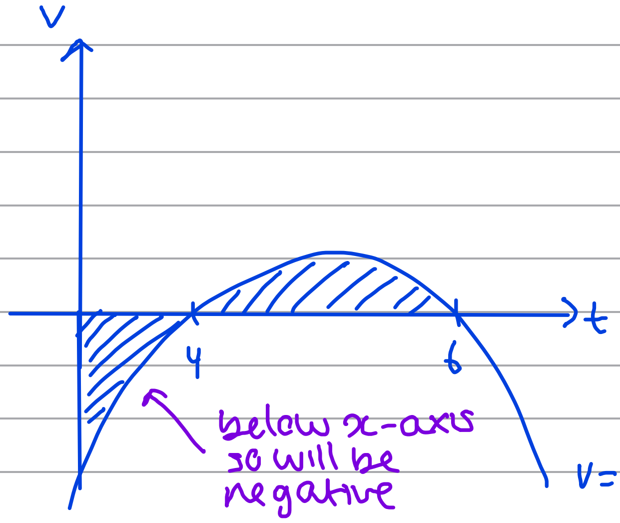
$0 = t^2 - 10t + 24$ (1)

$= (t-6)(t-4)$ (1)

$\therefore t = 6$ already found or $\underline{\underline{t = 4}}$ (1)

Question 1 continued

c)



Distance travelled $0 \le t \le 6$ = area under curve between $t=0$ and $t=6$

$$= -\int_0^4 (10t - t^2 - 24)\,dt + \int_4^6 (10t - t^2 - 24)\,dt \quad (1)$$

$$= -\left[5t^2 - \frac{1}{3}t^3 - 24t\right]_0^4 + \left[5t^2 - \frac{1}{3}t^3 - 24t\right]_4^6 \quad (1)\ (1)$$

$$= -\left(-\frac{112}{3} - 0\right) + \left(-36 + \frac{112}{3}\right)$$

$$= \frac{116}{3}\ \text{m} \quad (1)$$

2. A fixed point O lies on a straight line.

A particle P moves along the straight line.

At time t seconds, $t \geqslant 0$, the distance, s metres, of P from O is given by

$$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$$

(a) Find the acceleration of P at each of the times when P is at instantaneous rest.

(6)

(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(3)

a) find v and a:

$$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$$

$$v = \frac{ds}{dt} = t^2 - 5t + 6 \quad (2)$$

$$a = \frac{dv}{dt} = 2t - 5 \quad (1)$$

instantaneously at rest when $v = 0$:

$$t^2 - 5t + 6 = 0 \quad (1)$$

$$(t-2)(t-3) = 0$$

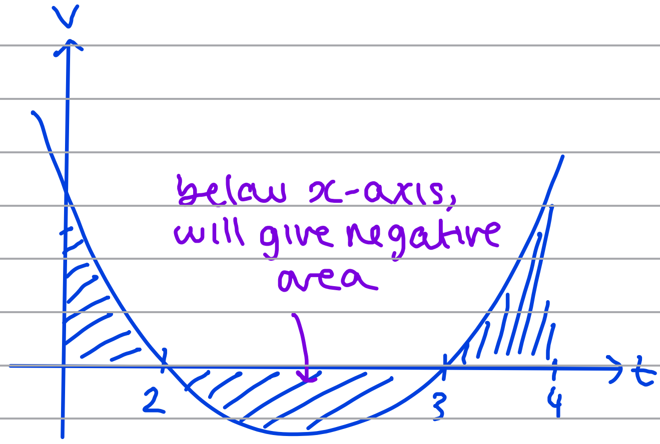
$$t = 2 \text{ or } t = 3 \quad (1)$$

if $t = 2$, $a = 2(2) - 5 = -1$

if $t = 3$, $a = 2(3) - 5 = 1 \quad (1)$

Question 2 continued

b)



Distance travelled from $0 \leq t \leq 4$ = area under curve between $t=0$ and $t=4$

$$\text{Distance} = \int_0^2 t^2 - 5t + 6 \, dt - \int_2^3 t^2 - 5t + 6 \, dt + \int_3^4 t^2 - 5t + 6 \, dt$$

$$= \left[S \right]_0^2 - \left[S \right]_2^3 + \left[S \right]_3^4 \quad (1)$$

$$= \frac{14}{3} - \left(\frac{9}{2} - \frac{14}{3} \right) + \left(\frac{16}{3} - \frac{9}{2} \right) \quad (1)$$

$$= \frac{17}{3} \quad (1)$$

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A fixed point O lies on a straight line.

A particle P moves along the straight line such that at time t seconds, $t \geqslant 0$, after passing through O, the velocity of P, $v \text{ m s}^{-1}$, is modelled as

$$v = 15 - t^2 - 2t$$

(a) Verify that P comes to instantaneous rest when $t = 3$

(1)

(b) Find the magnitude of the acceleration of P when $t = 3$

(3)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(4)

a) sub $t=3$ into v: $v = 15 - (3)^2 - 2(3)$
$= 15 - 9 - 6$
$= 0$ (1)

b) $v = 15 - t^2 - 2t$

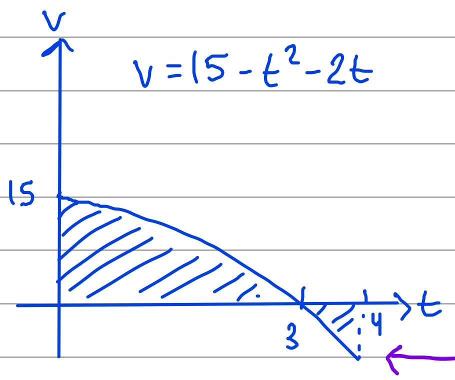
$a = \frac{dv}{dt}$

(1) $\frac{dv}{dt} = -2t - 2$ (1)

$\left.\frac{dv}{dt}\right|_{t=3} = -2(3) - 2 = -8$

magnitude $= 8 \text{ ms}^{-2}$ (1)

c)



Distance travelled = area under graph

$= \int_0^3 15 - t^2 - 2t \, dt - \int_3^4 15 - t^2 - 2t \, dt$ (1)

area under x-axis is negative so we must subtract this portion

$= \left[15t - \frac{1}{3}t^3 - t^2\right]_0^3$ (1) $-$ $\left[15t - \frac{1}{3}t^3 - t^2\right]_3^4$ (1)

$= 27 - 0 - \left(\frac{68}{3} - 27\right) = \frac{94}{3}$ (1)

4. At time t seconds, a particle P has velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed 10 m s^{-1}

(6)

a) $\underline{a} = \frac{d\underline{v}}{dt}$ $\qquad \underline{v} = 3t^{1/2}\underline{i} - 2t\underline{j}$

$\underline{a} = \frac{3}{2}t^{-1/2}\underline{i} - 2\underline{j}$ (1) (1)

b) $\underline{v} = k(\underline{i} - \underline{j})$

$\underline{v} = \begin{pmatrix} k \\ -k \end{pmatrix} = \begin{pmatrix} 3\sqrt{t} \\ -2t \end{pmatrix}$

comparing elements: $k = 3\sqrt{t}$

$-k = -2t$

$\Rightarrow 3\sqrt{t} = 2t$ (1)

$0 = \sqrt{t}(2\sqrt{t} - 3)$ (1)

$\therefore t = 0$ or $\sqrt{t} = \frac{3}{2} \Rightarrow t = \frac{9}{4}$ (1)

ignore

c) $\underline{r} = \int \underline{v}\,dt$ $\qquad \underline{v} = 3t^{1/2}\underline{i} - 2t\underline{j}$

$\underline{r} = 2t^{3/2}\underline{i} - t^2\underline{j} + \underline{c}$ (1) (1)

Question 4 continued

$\underline{r} = 2t^{3/2}\underline{i} - t^2\underline{j} + \underline{c}$

when $t=1$, $\underline{r} = -\underline{j}$

$-\underline{j} = 2\underline{i} - \underline{j} + \underline{c}$

$0 = 2\underline{i} + \underline{c}$

$\underline{c} = -2\underline{i}$

$\underline{r} = (2t^{3/2} - 2)\underline{i} - t^2\underline{j}$ (1)

d) speed $= 10$

$\sqrt{(3\sqrt{t})^2 + (-2t)^2} = 10$ (1)

$9t + 4t^2 = 100$ (1)

$4t^2 + 9t - 100 = 0$

$(t-4)(4t+25) = 0$

$t = 4$ or $t = -6.25$

$t > 0 \therefore t = 4$ (1)

$\underline{r} = (2(4)^{3/2} - 2)\underline{i} - (4)^2\underline{j}$

$\underline{r} = 14\underline{i} - 16\underline{j}$ (1)

Distance $OP = \sqrt{14^2 + (-16)^2}$ (1)

$= 2\sqrt{113}$ m (1)

5. [*In this question, position vectors are given relative to a fixed origin.*]

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j})$ m.

(c) Find the position vector of P at time $t = 1$ second.

(4)

a) sub $t=2$ into $\underline{v}$: $\underline{v} = 3(2)^2\underline{i} - 6(2)^{1/2}\underline{j}$

$\underline{v} = 12\underline{i} - 6\sqrt{2}\underline{j}$ (1)

speed $= \sqrt{12^2 + (-6\sqrt{2})^2} = 6\sqrt{6} = 15\ \mathrm{ms^{-1}}$ (2sf) (1)

b) $\underline{a} = \frac{d\underline{v}}{dt} = 6t\underline{i} - 3t^{-1/2}\underline{j}$ (1) (1)

c) $\underline{r} = \int \underline{v}\,dt = t^3\underline{i} - 4t^{3/2}\underline{j} + \underline{c}$ (1) (1)

sub in $t=4$, $\underline{r} = \underline{i} - 4\underline{j}$

$\underline{i} - 4\underline{j} = (4)^3\underline{i} - 4(4)^{3/2}\underline{j} + \underline{c}$ (1)

$\underline{i} - 4\underline{j} = 64\underline{i} - 32\underline{j} + \underline{c}$

$\underline{c} = -63\underline{i} + 28\underline{j}$

$\therefore\ \underline{r} = (t^3 - 63)\underline{i} + (-4t^{3/2} + 28)\underline{j}$

sub in $t=1$

$\underline{r} = -62\underline{i} + 24\underline{j}$ (1)

6. At time t seconds, where $t \geqslant 0$, a particle P has velocity $\mathbf{v}$ m s^{-1} where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$ **(3)**

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$ **(2)**

(c) the acceleration of P at time t seconds **(2)**

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$ **(2)**

(a) $v = (0^2 - 3(0) + 7)i + (2(0)^2 - 3)j$

$v = 7i - 3j$ (1)

speed $= |v|$

$= \sqrt{7^2 + (-3)^2}$ (1)

$= \sqrt{58}$

$= 7.6 \text{ ms}^{-1}$ (1)

(b) $t^2 - 3t + 7 = 2t^2 - 3$ (1)

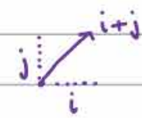
$t^2 + 3t - 10 = 0$

$(t + 5)(t - 2) = 0$

$\therefore t = -5$ or $t = 2$

$t = 2$ (1) because time can't be less than 0.

parallel to $(i + j)$ means coefficients of i and j are equal:



so $xi = xj$

(c) $\frac{dv}{dt}$ (1) $= (2t - 3)i + (4t)j$ ← acceleration is rate of change of speed over time, so find $\frac{dv}{dt}$

$\therefore a = (2t - 3)i + (4t)j$ (1)

(d) $2t - 3 = 0$ (1) ← 'perpendicular to i' means i-coefficient is 0.

$t = \frac{3}{2}$ seconds (1)

so $0i + xj$