Questions

Q1.

A car is moving along a straight horizontal road with constant acceleration.

There are three points A, B and C, in that order, on the road, where $AB = 22\ m$ and $BC = 104\ m$.

The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(7)

(Total for question = 7 marks)

Q2.

Unless otherwise indicated, wherever a numerical value of g is required, take g = 9.8 m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

A man throws a tennis ball into the air so that, at the instant when the ball leaves his hand, the ball is 2 m above the ground and is moving vertically upwards with speed 9 m s^{-1}

The motion of the ball is modelled as that of a particle moving freely under gravity and the acceleration due to gravity is modelled as being of constant magnitude 10 m s^{-2}

The ball hits the ground T seconds after leaving the man's hand.

Using the model, find the value of T.

(4)

(Total for question = 4 marks)

Q3.

At time $t = 0$, a parachutist falls vertically from rest from a helicopter which is hovering at a height of 550 m above horizontal ground.

The parachutist, who is modelled as a particle, falls for 3 seconds before her parachute opens.

While she is falling, and before her parachute opens, she is modelled as falling freely under gravity.

The acceleration due to gravity is modelled as being 10 m s^{-2}.

(a) Using this model, find the speed of the parachutist at the instant her parachute opens.

(1)

When her parachute is open, the parachutist continues to fall vertically.

Immediately after her parachute opens, she decelerates at 12 m s^{-2} for 2 seconds before reaching a constant speed and she reaches the ground with this speed.

The total time taken by the parachutist to fall the 550 m from the helicopter to the ground is T seconds.

(b) Sketch a speed–time graph for the motion of the parachutist for $0 \le t \le T$.

(2)

(c) Find, to the nearest whole number, the value of T.

(5)

In a refinement of the model of the motion of the parachutist, the effect of air resistance is included before her parachute opens and this refined model is now used to find a new value of T.

(d) How would this new value of T compare with the value found, using the initial model, in part (c)?

(1)

(e) Suggest one further refinement to the model, apart from air resistance, to make the model more realistic.

(1)

(Total for question = 10 marks)

Q4.

At time $t = 0$, a small ball is projected vertically upwards with speed U m s^{-1} from a point A that is 16.8 m
above horizontal ground.

The speed of the ball at the instant immediately before it hits the ground for the first time is 19 m s^{-1}

The ball hits the ground for the first time at time $t = T$ seconds.

The motion of the ball, from the instant it is projected until the instant just before it hits the ground for the first time, is modelled as that of a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude 10 m s^{-2}

Using the model,

(a) show that $U = 5$

(2)

(b) find the value of T,

(2)

(c) find the time from the instant the ball is projected until the instant when the ball is 1.2 m below A.

(4)

(d) Sketch a velocity-time graph for the motion of the ball for $0 \leq t \leq T$, stating the coordinates of the start point and the end point of your graph.

(2)

In a refinement of the model of the motion of the ball, the effect of air resistance on the ball is included and this refined model is now used to find the value of U.

(e) State, with a reason, how this new value of U would compare with the value found in part (a),
using the initial unrefined model.

(1)

(f) Suggest one further refinement that could be made to the model, apart from including air resistance, that would make the model more realistic.

(1)

(Total for question = 12 marks)

Q5.

At time $t = 0$, a small stone is thrown vertically upwards with speed 14.7 m s^{-1} from a point A.

At time $t = T$ seconds, the stone passes through A, moving downwards.

The stone is modelled as a particle moving freely under gravity throughout its motion.

Using the model,

(a) find the value of T,

(2)

(b) find the total distance travelled by the stone in the first 4 seconds of its motion.

(4)

(c) State one refinement that could be made to the model, apart from air resistance, that would make the model more realistic.

(1)

(Total for question = 7 marks)

Q6.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is $\mathbf{r}$ metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(b) Find $\mathbf{r}$ in terms of t.

(2)

(c) Find the value of t when the boat is north-east of O.

(3)

(d) Find the value of t when the boat is moving in a north-east direction.

(3)

(Total for question = 10 marks)

Q7.

Unless otherwise stated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O.*]

A particle P moves with constant acceleration.
At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}
At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m.

(a) Show that the magnitude of the acceleration of P is 2.5 m s^{-2}

(4)

At the instant when P leaves the point A, the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})$ m s^{-2}

At the instant when P reaches the point B, the direction of motion of P is north east.

(b) Find the time it takes for P to travel from A to B.

(4)

(Total for question = 8 marks)

Q8.

A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})$ m s^{-2}

At time $t = 0$, P is moving with velocity $4\mathbf{i}$ m s^{-1}

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})$ m.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

(Total for question = 4 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(i)(ii)	Using a correct strategy for solving the problem by setting up two equations in a and u only and solving for either	M1	3.1b
	Equation in a and u only	M1	3.1b
	$22 = 2u + \frac{1}{2} a\, 2^2$	A1	1.1b
	Another equation in a and u only	M1	3.1b
	$126 = 6u + \frac{1}{2} a\, 6^2$	A1	1.1b
	5 m s^{-2}	A1	1.1b
	6 m s^{-1}	A1 ft	1.1b
			(7 marks)

Notes

1st M1 for solving the problem by setting up two equations in a and u only and solving for either
2nd M1 use of (one or more) *suvat* formulae to produce equation in u and a only
1st A1 for a correct equation
3rd M1 use of (one or more) *suvat* formulae to produce another equation in u and a only
2nd A1 for a correct equation
3rd A1 for correct accln 5 m s^{-2}
4th A1 for correct speed 6 m s^{-1} (The second of these A marks is an **ft** mark, following an incorrect value for u or a, depending on which has been found first)
N.B. Do not award the **ft** mark for absurd answers e.g. $a > 15$, $u > 50$

See alternative on next page

ALTERNATIVE

Question	Scheme	Marks	AOs
(i)(ii)	Using a correct strategy for solving the problem by obtaining actual speeds at two times and using $a =$ change in speed / time taken.	M1	3.1b
	Actual speed at $t = 1 =$ Average speed over interval	M1	3.1b
	$22/2 = 11$	A1	1.1b
	Actual speed at $t = 4 =$ Average speed over interval	M1	3.1b
	$104/4 = 26$	A1	1.1b
	5 m s^{-2}	A1	1.1b
	6 m s^{-1}	**A1 ft**	1.1b
			(7 marks)

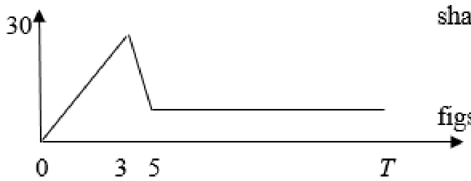
Notes

1st M1 for solving the problem by obtaining two actual speeds and use of $a = (v - u)/ t$
2nd M1 use of speed at half-time = av speed over interval to produce a speed at $t = 1$
1st A1 for a correct speed
3rd M1 use of speed at half-time = av speed over interval to produce a speed at $t = 4$
2nd A1 for a correct speed
3rd A1 for correct accln 5 m s^{-2}
4th A1 **ft** for correct speed 6 m s^{-1} (This is an ft mark, following an incorrect value of a)
N.B. Do not award the **ft** mark for absurd answers e.g. $a > 15$, $u > 50$

Q2.

Question	Scheme	Marks	AOs
	Equation in t only	M1	2.1
	$-2 = 9t - \frac{1}{2} \leftrightarrow 10t^2$	A1	1.1b
	$5t^2 - 9t - 2 = 0 = (5t+1)(t-2)$	DM1	1.1b
	$T = 2$ (only)	A1	1.1b
		(4)	
			(4 marks)
Notes:			
M1: Complete method to give equation in t only. This mark is for a complete method for the TOTAL time i.e. for finding sufficient equations, with usual rules, correct no. of terms in each equation but condone sign errors and g does not need to be substituted **A1:** A correct equation **or** correct equations (e.g. if they find the speed, 11 ms^{-1}, when the ball strikes the ground and then use that to find the total time **or** if they split the time (e.g. 0.9s up and 1.1s down or 0.9s + 0.9s + 0.2s)) N.B. $g = 10$ must be substituted in all equations used. **DM1**: Dependent on first M1, for solving a 3 term quadratic to find T **or** for solving their equations to find T **or** for solving their equations and adding their split times to find T **A1**: $T = 2$ only (i.e. A0 if they give two times) *N.B.* If solving a correct quadratic, the DM1 can be implied by a correct answer i.e. the method does not need to be shown, but if there is no method shown and the answer is wrong then award DM0 A0.			

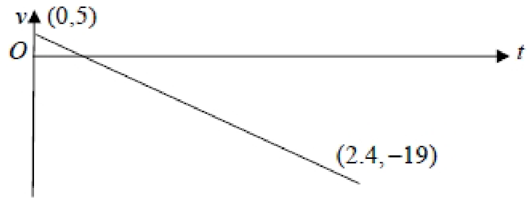
Q3.

Question	Scheme	Marks	AOs	Notes
(a)	$V = 30$ (m s^{-1})	B1	3.4	cao
		(1)		
(b)	 shape	B1	1.1b	Overall shape of the graph, starting at the origin. Dotted vertical line at end is OK but solid vertical line is B0
	figs	B1ft	1.1b	3, 5 and T marked on the t-axis, and **ft** on their 30 marked on the speed axis. 3 must be where graph reaches a peak. Allow delineators: 3, 2 and $T-5$ or a mixture
		(2)		
(c)	Using total area = 550 to set up an equation in **one unknown,** Or they may use *suvat* on one or more of the sections (but must still be considering all sections) M0 if they use one *suvat* equation for the whole motion	M1	2.1	Need all sections to be included, with correct structure for each section. e.g. triangle + trapezium + rectangle oe = 550 to give an equation in **one unknown (may not be *T*)**
	$\frac{1}{2}\times3\times30+\frac{(30+6)}{2}\times2+6(T-5)=550$ **OR**: $\frac{1}{2}\times3\times30+\frac{1}{2}\times2\times24+6(T-3)=550$ **OR:** $\frac{1}{2}\times3\times30+\frac{1}{2}\times2\times24+(2\times6)+6(T-5)=550$	A2 ft	1.1b	**ft** on their answer to (a). -1 each error. **N.B.** If '6' is incorrect, treat as one error, unless it is correct ft from their 30.
	Solve for T	M1	1.1b	Attempt to solve for T provided they have tried to find the area using at least 3 sections. (M0 if they only solve for their unknown and never try to find T)
	$T = 83$ (nearest whole number)	A1	1.1b	83 is the only answer
		(5)		

(d)	New value of T would be bigger (ignore their reasons whether correct or not)	B1	3.5 a	Clear statement about the value of T Allow '*it* would increase, get larger etc' B0 for 'Takes longer' or 'the value of T would be longer'
		(1)		
(e)	e.g. effect of wind; allow for dimensions of parachutist; use a more accurate value for g; parachutist does not fall vertically after chute opens; smooth changes in v; time for parachute to open; deceleration not constant (but B0 if they say *acceleration* not constant); smooth changes in a; B0 for: moves horizontally; mass/weight of parachutist; upthrust; air pressure;air resistance; terminal velocity	B1	3.5 c	Any appropriate refinement of the model. B0 if incorrect (or irrelevant) extras
		(1)		
		(10 marks)		

Q4.

Question	Scheme	Marks	AOs
(a)	$19^2 = (-U)^2 + 2\times10\times16.8$ (Allow use of $g = 9.8$ for this M mark)	M1	2.1
	$U = 5$ *	A1*	1.1b
		(2)	
	For consistent use of $g = 9.8$ in parts (b), (c) and (d), treat as a MR. i.e. max (b) M1A0 (c)M1A0M(A)0A1ft (d)B1B1ft		
(b)	$19 = -5 + 10T$ **OR** $16.8 = \dfrac{(-5+19)}{2}T$ **OR** $16.8 = -5T + \dfrac{1}{2}\times10T^2$ **OR** $16.8 = 19T - \dfrac{1}{2}\times10T^2$	M1	2.1
	$T = 2.4$	A1	1.1b
		(2)	
(c)	$1.2 = -5t + \dfrac{1}{2}\times10\times t^2$	M1	2.1
	$5t^2 - 5t - 1.2 = 0$	A1	1.1b
		M(A)1	1.1b
	$t = 1.2$ (s)	A1	1.1b
		(4)	

Question	Scheme	Marks	AOs
(d)		B1 shape	1.1b
	$(0,5)$ and $(2.4,-19)$ Allow these to be marked on the axes.	B1ft	1.1b
		(2)	
(e)	Greater since air resistance would slow the ball down.	B1	3.5a
		(1)	
(f)	Take into account: spin, wind effects, use a more accurate value of g, not model the ball as a particle	B1	3.5c
		(1)	
			(12 marks)

Notes:		
(a)	M1	Complete method to find U, condone sign errors and use of $g = 9.8$
	A1*	$U = 5$ cao correctly obtained – allow U^2 instead of $(-U)^2$. Allow verification.
(b)	M1	Complete method to find T, condone sign errors
	A1	$T = 2.4$
(c)	M1	Complete method to find t, condone sign errors
	A1	Correct equation with at most one error
	(A)1	Correct equation
	A1	$t = 1.2$ (s)
(d)	B1	Graph could be reflected in the t-axis.
	B1ft	Follow through on their T value. If graph is reflected, $(0, -5)$ and $(2.4, 19)$
(e)	B1	Any similar appropriate comment
(f)	B1	B0 if any incorrect extras e.g. weight/mass included

Q5.

Question	Scheme	Marks	AOs
(a)	$14.7 = -14.7 + 9.8T$ or $0 = 14.7T - \frac{1}{2} \times 9.8T^2$ or $0 = 14.7 - 9.8 \times \left(\frac{1}{2}T\right)$ oe	M1	3.4
	$T = 3$	A1	1.1b
		(2)	
(b)	$s_1 = \frac{(14.7+0)}{2} \times 1.5$ $(11.025$ or $\frac{441}{40})$	M1	1.1b
	$s_2 = \frac{1}{2} \times 9.8 \times 2.5^2$ $(30.625$ or $\frac{245}{8})$ **OR** $s_3 = 14.7 \times 1 + \frac{1}{2} \times 9.8 \times 1^2$ $(19.6$ or $\frac{98}{5})$ **OR** $-s_3 = 14.7 \times 4 - \frac{1}{2} \times 9.8 \times 4^2$ (- 19.6) (allow omission of – on LHS)	M1	1.1b
	Total distance = $s_1 + s_2$ **OR** $2s_1 + s_3$	M1	2.1
	= 41.7 m or 42 m	A1	1.1b
		(4)	
(c)	e.g. Take account of the dimensions of the stone (e.g. allow for spin), do not model the stone as a particle, use a more accurate value for g	B1	3.5c
		(1)	
			(7 marks)

Notes:		**If they use g = 9.81 or 10, penalise once for whole question.**
a	M1	Complete method to find T, condone sign errors (M0 if they only find time to top)
	A1	$T = 3$ correctly obtained.
b	M1	Complete method to find one key distance
	M1	Correct method to find another key distance
	M1	Complete method to find the total distance
	A1	41.7 or 42 (after use of $g = 9.8$)
c	B1	B0 if there are incorrect extra refinements but ignore extra incorrect statements.

Q6.

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2} Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}\,t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})\,t^2$	A1	1.1b
		(2)	
(c)	Equating the $\mathbf{i}$ and $\mathbf{j}$ components of $\mathbf{r}$	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7\,t^2 = 0.6\,t - \frac{1}{2} \leftarrow 0.1\,t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})\,t$	M1	3.1b
	Equating the $\mathbf{i}$ and $\mathbf{j}$ components of $\mathbf{v}$	M1	3.1b
	$t = 0.75$	A1 ft	1.1b
		(3)	
			(10 marks)

Notes:

(a)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

A1: for given answer correctly obtained

(b)

M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

A1: for a correct expression for $\mathbf{r}$ in terms of t

(c)

M1: for equating the $\mathbf{i}$ and $\mathbf{j}$ components of their $\mathbf{r}$

A1ft: for a correct equation following their $\mathbf{r}$

A1: for $t = 1.5$

(d)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for a general t

M1: for equating the $\mathbf{i}$ and $\mathbf{j}$ components of their $\mathbf{v}$

A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation

Q7.

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $(7\mathbf{i} - 10\mathbf{j}) = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}\mathbf{a}2^2$	M1	3.1b
	$\mathbf{a} = (1.5\mathbf{i} - 2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{a}\rvert = \sqrt{1.5^2 + (-2)^2}$	M1	1.1b
	$= 2.5 \text{ m s}^{-2}$ * GIVEN ANSWER	A1*	2.1
		(4)	
(b)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 3\mathbf{j}) + 2(1.5\mathbf{i} - 2\mathbf{j})$	M1	3.1b
	$= (5\mathbf{i} - 7\mathbf{j})$	A1	1.1b
	$\mathbf{v} = (5\mathbf{i} - 7\mathbf{j}) + t(4\mathbf{i} + 8.8\mathbf{j}) = (5 + 4t)\mathbf{i} + (8.8t - 7)\mathbf{j}$ and $(5 + 4t) = (8.8t - 7)$	M1	3.1b
	$t = 2.5$ (s)	A1	1.1b
		(4)	
			(8 marks)

Notes: Allow column vectors throughout
(a) **No credit for individual component calculations** **M1:** Using a complete method to obtain the acceleration. N.B. Equation, in **a** only, could be obtained by two integrations **ALTERNATIVE** **M1:** Use velocity at half-time (t = 1) = Average velocity over time period So at $t = 1$, $\mathbf{v} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j})$ so $\mathbf{a} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j}) - (2\mathbf{i} - 3\mathbf{j})$ N.B. could see $(7\mathbf{i} - 10\mathbf{j}) = (4\mathbf{i} - 6\mathbf{j}) + 2\mathbf{a}$ as first line of working **A1:** Correct **a** vector **M1:** Attempt to find magnitude of their **a** using form $\sqrt{a^2 + b^2}$ **A1*:** Correct GIVEN ANSWER obtained correctly
(b) **M1:** Using a complete method to obtain the velocity at A e.g.by use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with t = 2 and $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$ and their **a** OR: by use of $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ OR: by integrating their **a**, with addition of **C** = 2**i** – 3**j**, and putting t = 2 **A1:** correct vector **M1:** Complete method to find equation in t only
e.g. by using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, with their **u** and equating **i** and **j** components **OR:** by integrating (4**i** + 8.8**j**), with addition of a constant, and equating **i** and **j** components. N.B. Must be equating **i** and **j** components of <u>a velocity vector</u> and must be their velocity at A, to give an equation in t only for this M mark **A1:** 2.5 (s)

Q8.

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 2$: $\mathbf{v} = 4\mathbf{i} + 2(2\mathbf{i} - 3\mathbf{j})$ **OR integration**: $\mathbf{v} = (2\mathbf{i} - 3\mathbf{j})t + 4\mathbf{i}$, with $t = 2$	M1	3.1a
	$\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ at $t = 3$: $(\mathbf{i} + \mathbf{j}) + \left[3 \times 4\mathbf{i} + \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR**: find $\mathbf{v}$ at $t = 3$: $4\mathbf{i} + 3(2\mathbf{i} - 3\mathbf{j}) = (10\mathbf{i} - 9\mathbf{j})$ then use $\mathbf{r} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ $(\mathbf{i} + \mathbf{j}) + \left[\frac{1}{2}[4\mathbf{i} + (10\mathbf{i} - 9\mathbf{j})] \times 3\right]$ or $\mathbf{r} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $(\mathbf{i} + \mathbf{j}) + \left[3 \times (10\mathbf{i} - 9\mathbf{j}) - \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR integration**: $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \left[(2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2 + 4t\mathbf{i}\right]$, with $t = 3$	M1	3.1a
	$\mathbf{r} = 22\mathbf{i} - 12.5\mathbf{j}$	A1	2.2a
		(2)	
		(4 marks)	

Notes:	**Accept column vectors throughout**	
a	M1	Complete method to find $\mathbf{v}$, using ruva$t$ or integration (M0 if $\mathbf{i}$ and/or $\mathbf{j}$ is missing)
	A1	Apply isw if they also find the speed
b	M1	Complete method to find the p.v. but this mark can be scored if they omit $(\mathbf{i} + \mathbf{j})$ i.e. the M1 is for the expression in the square bracket If they integrate, the M1 is earned once the expression in the square bracket is seen with $t = 3$ (M0 if $\mathbf{i}$ and/or $\mathbf{j}$ is missing)
	A1	cao