Questions

Q1.

A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leqslant t \leqslant 10$$

(a) Explain the restriction, $0 \leq t \leq 10$

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

(Total for question = 9 marks)

Q2.

Unless otherwise indicated, wherever a numerical value of g is required, take g = 9.8 m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

A particle, P, moves along the x-axis. At time t seconds, $t \geq 0$, the displacement,

x metres, of P from the origin O, is given by $x = \frac{1}{2}t^2(t^2 - 2t + 1)$

(a) Find the times when P is instantaneously at rest.

(5)

(b) Find the total distance travelled by P in the time interval $0 \leq t \leq 2$

(3)

(c) Show that P will never move along the negative x-axis.

(2)

(Total for question = 10 marks)

Q3.

A particle, P, moves along a straight line such that at time t seconds, $t \geq 0$, the velocity of P, v m s^{-1}, is modelled as

$$v = 12 + 4t - t^2$$

Find

(a) the magnitude of the acceleration of P when P is at instantaneous rest,

(5)

(b) the distance travelled by P in the interval $0 \leq t \leq 3$

(3)

(Total for question = 8 marks)

Q4.

A particle P moves along a straight line such that at time t seconds, $t \geq 0$, after leaving the point O on the line,
the velocity, v m s^{-1}, of P is modelled as

$$v = (7 - 2t)(t + 2)$$

(a) Find the value of t at the instant when P stops accelerating.

(4)

(b) Find the distance of P from O at the instant when P changes its direction of motion.

(5)

In this question, solutions relying on calculator technology are not acceptable.

(Total for question = 9 marks)

Q5.

At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

(Total for question = 6 marks)

Q6.

Unless otherwise stated, whenever a numerical value of g is required, take g = 9.8 m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

At time t seconds, where $t \geq 0$, a particle P moves in the x-y plane in such a way that its velocity **v** m s^{-1} is given by

$$\mathbf{v} = t^{-\frac{1}{2}}\mathbf{i} - 4t\mathbf{j}$$

When $t = 1$, P is at the point A and when $t = 4$, P is at the point B.

Find the exact distance AB.

(6)

(Total for question = 6 marks)

Q7.

[*In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geq 0$, a particle, P, moves so that its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})$ m .

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

(Total for question = 6 marks)

Q8.

A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})$ m s^{-2}

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})$ m s^{-1}

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T.

(4)

At time $t = 4$ seconds, P is at the point B.

(b) Find the distance AB.

(4)

(Total for question = 8 marks)

Q9.

A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})$ m s^{-2}

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-1}

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

(Total for question = 8 marks)

Q10.

(i) At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration **a** m s^{-2} is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is 36**i** m s^{-1}

(a) Find the velocity of P when $t = 4$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to **i**

(3)

(ii) At time t seconds, where $t \geq 0$, a particle Q moves so that its position vector

r metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of t at the instant when the speed of Q is 5 m s^{-1}

(6)

(Total for question = 12 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Substitution of both $t = 0$ and $t = 10$	M1	2.1
	$s = 0$ for both $t = 0$ and $t = 10$	A1	1.1b
	Explanation ($s > 0$ for $0 < t < 10$) since $s = \frac{1}{10}t^2(t-10)^2$	A1	2.4
		(3)	
(b)	Differentiate displacement s w.r.t. t to give velocity, v	M1	1.1a
	$v = \frac{1}{10}(4t^3 - 60t^2 + 200t)$	A1	1.1b
	Interpretation of 'rest' to give $v = \frac{1}{10}(4t^3 - 60t^2 + 200t) = \frac{2}{5}t(t-5)(t-10) = 0$	M1	1.1b
	$t = 0, 5, 10$	A1	1.1b
	Select $t = 5$ and substitute their $t = 5$ into s	M1	1.1a
	Distance $= 62.5$ m	A1 ft	1.1b
		(6)	
			(9 marks)

Notes

(a) M1 for substituting $t = 0$ and $t = 10$ into s expression
A1 for noting that $s = 0$ at both times
A1 Since s is a perfect square, $s > 0$ for all other t- values.

(b) 1st M1 for differentiating s w.r.t. t to give v (powers of t reducing by 1)
1st A1 for a correct v expression in any form
2nd M1 for equating v to 0 and factorising
2nd A1 for correct t values
3rd M1 for substituting their intermediate t value into s
3rd A1 ft following an incorrect t-value.

Q2.

Question	Scheme	Marks	AOs
(a)	Multiply out and differentiate *wrt* to time (or use of product rule i.e. must have two terms with correct structure)	M1	1.1a
	$v = 2t^3 - 3t^2 + t$	A1	1.1b
	$2t^3 - 3t^2 + t = 0$ and solve: $t(2t-1)(t-1) = 0$	DM1	1.1b
	$t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two	A1	1.1b
	All three	A1	1.1b
		(5)	
(b)	Find x when $t = 0, \frac{1}{2}$, 1 and 2 : $(0, \frac{1}{32}, 0, 2)$	M1	2.1
	Distance $= \frac{1}{32} + \frac{1}{32} + 2$	M1	2.1
	$2\frac{1}{16}$ (m) oe or 2.06 or better	A1	1.1b
		(3)	
(c)	$x = \frac{1}{2}t^2(t-1)^2$	M1	3.1a
	$\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative	A1 cso	2.4
		(2)	
			(10 marks)

Notes:
(a) **M1:** Must have 3 terms and at least two powers going down by 1 **A1:** A correct expression **DM1:** Dependent on first M, for equating to zero and attempting to solve a <u>cubic</u> **A1:** Any two of the three values (Two correct answers can imply a correct method) **A1:** The third value
(b) **M1:** For attempting to find the values of x (at least two) at their t values found in (a) or at $t=2$ or equivalent e.g. they may integrate their v and sub in at least two of their t values **M1:** Using a correct strategy to combine their distances (must have at least 3 distances) **A1:** $2\frac{1}{16}$ (m) oe or 2.06 or better
(c) **M1:** Identify strategy to solve the problem such as: (i) writing x as $\frac{1}{2} \times$ perfect square (ii) or using x values identified in (b). (iii) or using calculus i.e. identifying min points on x–t graph. (iv) or using x-t graph. **A1 cso :** Fully correct explanation to show that $x \geq 0$ i.e. never negative

Q3.

Question	Scheme	Marks	AOs	Notes
(a)	$v = 12 + 4t - t^2 = 0$ and solving	M1	3.1a	Equating v to 0 and solving the quadratic If no evidence of solving, and at least one answer wrong, M0
	$t = 6$ (or -2)	A1	1.1b	6 but allow -2 as well at this stage
	Differentiate v wrt t	M1	1.1a	For differentiation (both powers decreasing by 1)
	$(a = \frac{dv}{dt} =)\ 4 - 2t$	A1	1.1b	Cao; only need RHS
	When $t = 6$, $a = -8$; Magnitude is 8 (m s^{-2})	A1	1.1b	Substitute in $t = 6$ and get 8 (m s^{-2}) as the answer . Must be **positive.** (A0 if two answers given)
		(5)		
(b)	Integrate v wrt t $(s =)12t + 2t^2 - \frac{1}{3}t^3 (+C)$	M1 A1	3.1a 1.1b	For integration (at least two powers increasing by 1) Correct expression (ignore C) only need RHS Must be used in part (b)
	$t = 3$ => distance $= 45$ (m)	A1	1.1b	Correct distance. Ignore units
		(3)		
			(8 marks)	

Q4.

Question	Scheme	Marks	AOs
(a)	$v = 3t - 2t^2 + 14$ and differentiate	M1	3.1a
	$a = \frac{dv}{dt} = 3 - 4t$ or $(7-2t) - 2(t+2)$ using product rule	A1	1.1b
	$3 - 4t = 0$ and solve for t	M1	1.1b
	$t = \frac{3}{4}$ oe	A1	1.1b
		(4)	
(b)	Solve problem using $v = 0$ to find a value of t $\left(t = \frac{7}{2}\right)$	M1	3.1a
	$v = 3t - 2t^2 + 14$ and integrate	M1	1.1b
	$s = \frac{3t^2}{2} - \frac{2t^3}{3} + 14t$	A1	1.1b
	Substitute $t = \frac{7}{2}$ into their s expression (M0 if using *suvat*)	M1	1.1b
	$s = \frac{931}{24} = 38\frac{19}{24} = 38.79166..(\text{m})$ Accept 39 or better	A1	1.1b
		(5)	
		(9 marks)	

Notes:		
(a)	M1	Multiply out and attempt to differentiate, with at least one power decreasing
	A1	Correct expression
	M1	Equate their a to 0 and solve for t
	A1	cao
(b)	M1	Uses $v = 0$ to obtain a value of t
	M1	Attempt to integrate, with at least one power increasing
	A1	Correct expression
	M1	Substitute in their value of t ,which must have come from using $v = 0$, into their s (must have integrated)
	A1	39 or better

Q5.

Question	Scheme	Marks	AOs
	Integrate **a** w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of **C**)	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1ft	1.1b
			(6 marks)

Notes:

1st **M1:** for integrating **a** w.r.t. time (powers of t increasing by 1)
1st **A1:** for a correct **v** expression without **C**
2nd **A1:** for a correct **v** expression including **C**
2nd **M1:** for putting $t = 4$ into their **v** expression
3rd **M1:** for finding magnitude of their **v**
3rd **A1:** **ft** for 100 m s^{-1}, follow through on an incorrect **v**

Q6.

Question	Scheme	Marks	AOs
	Integrate **v** w.r.t. time	M1	1.1a
	$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j}\ (+\ \mathbf{C})$	A1	1.1b
	Substitute $t = 4$ and $t = 1$ into their **r**	M1	1.1b
	$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C});\ t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4, -32); (2, -2)$	A1	1.1b
	$\sqrt{2^2 + (-30)^2}$	M1	1.1b
	$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
		(6)	
		(6 marks)	

Notes: Allow column vectors throughout

M1: At least one power increasing by 1.

A1: Any correct (unsimplified) expression

M1: Must have attempted to integrate **v**. Substitute $t = 4$ and $t = 1$ into their **r** to produce 2 vectors (or 2 points if just working with coordinates).

A1: $4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C})$ and $2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4, -32)$ and $(2, -2)$. These can be seen or implied.

M1: Attempt at distance of form $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for their points. Must have 2 non zero terms.

A1: $\sqrt{904} = 2\sqrt{226}$ or any equivalent surd (exact answer needed)

Q7.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\mathbf{a} = 6\mathbf{i} - \frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	M1	This mark is given for a method to differentiate the expression for **v**
		A1	This mark is given for correctly differentiating the expression for **v**
	$= 6\mathbf{i} - 15\mathbf{j}$ m s^{-1}	A1	This mark is given for substituting $t = 4$ to find a correct vector expression for the acceleration of P
(b)	$\mathbf{r} = (\mathbf{r}_0) + 3t^2\mathbf{i} - 2t^{\frac{5}{2}}\mathbf{j}$	M1	This mark is given for a method to integrate the expression for **v**
		A1	This mark is given for correctly integrating the expression for **v**
	$(-20\mathbf{i} + 20\mathbf{j}) + (48\mathbf{i} - 64\mathbf{j})$ $= 28\mathbf{i} - 44\mathbf{j}$ m	A1	This mark is given for substituting $t = 4$ to find a correct position vector of P
			(Total 6 marks)

Q8.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$ $\mathbf{v} = (-\mathbf{i} + 4\mathbf{j}) + (2\mathbf{i} - 3\mathbf{j})t$	M1	This mark is given for a method to find a vector expression for **v**
	$= (-1 + 2t)\mathbf{i} + (4 - 3t)\mathbf{j}$	A1	This mark is given for finding a correct vector expression for **v**
	$\frac{4-3T}{1+2T} = \frac{-4}{3}$	M1	This mark is given for a correct use of ratios as a method to find the value of T
	$12 - 9T = 4 - 8T$ $T = 12 - 4 = 8$	A1	This mark is given for finding the correct value of T
(b)	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ $\mathbf{s} = (-\mathbf{i} + 4\mathbf{j})t + \frac{1}{2}(2\mathbf{i} - 3\mathbf{j})t^2$	M1	This mark is given for a method to find a vector expression for the distance AB
	$= (-t + t^2)\mathbf{i} + \left(4t - \frac{3}{2}t^2\right)\mathbf{j}$	A1	This mark is given for finding a correct vector expression for the distance AB
	$AB = \sqrt{12^2 + 8^2}$	M1	This mark is given for a method to find the distance AB using Pythagoras and substituting $t = 4$
	$= 14.4$ m	A1	This mark is given for find a correct value for the distance AB
			(Total 8 marks)

Q9.

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or integrate to give: $\mathbf{v} = (-2\mathbf{i} + 2\mathbf{j}) + 2(4\mathbf{i} - 5\mathbf{j})$	M1	3.1a
	$(6\mathbf{i} - 8\mathbf{j})$ (m s^{-1})	A1	1.1b
		(2)	
(b)	Solve problem through use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or integration (M0 if $\mathbf{u} = 0$) **Or** any other complete method e.g use $\mathbf{v} = \mathbf{u} + \mathbf{a}T$ and $\mathbf{r} = \frac{(\mathbf{u}+\mathbf{v})T}{2}$:	M1	3.1a
	$-4.5\mathbf{j} = 2t\mathbf{j} - \frac{1}{2}t^2 5\mathbf{j}$ (**j** terms only)	A1	1.1b
	The first two marks could be implied if they go straight to an algebraic equation.		
	Attempt to equate **j** components to give equation in T only $(-4.5 = 2T - \frac{5}{2}T^2)$	M1	2.1
	$T = 1.8$	A1	1.1b
		(4)	
(c)	Solve problem by substituting <u>their</u> T value (M0 if $T < 0$) into the **i** component equation to give an equation in λ only: $\lambda = -2T + \frac{1}{2}T^2 \times 4$	M1	3.1a
	$\lambda = 2.9$ or 2.88 or $\frac{72}{25}$ oe	A1	1.1b
		(2)	
Notes: Accept column vectors throughout		**(8 marks)**	

Notes: Accept column vectors throughout		(8 marks)
2a	M1	For any complete method to give a v expression with correct no. of terms with $t = 2$ used, so if integrating, must see the initial velocity as the constant. Allow sign errors.
	A1	Cao isw if they go on to find the speed.
2b	M1	For any complete method to give a vector expression for **j** component of displacement in t (or T) only, using **a** = (4**i** – 5**j**), so if integrating, RHS of equation must have the correct structure. Allow sign errors.
	A1	Correct **j** vector equation in t or T. Ignore **i** terms.
	M1	Must have earned 1st M mark. Equate **j** components to give equation in T (allow t) only (no **j**'s) which has come from a displacement. Equation must be a 3 term quadratic in T.
	A1	cao
2c	M1	Must have earned 1st M mark in (b) Complete method - must have an equation in λ only (no **i**'s) which has come from an appropriate displacement.. (e.g M0 if **a** = 0 has been used) Expression for λ must be a quadratic in T
	A1	cao

Q10.

Question	Scheme	Marks	AOs
(i)(a)	Integrate **a** wrt t to obtain velocity	M1	3.4
	$\mathbf{v} = (t-2t^2)\mathbf{i} + \left(3t - \frac{1}{3}t^3\right)\mathbf{j}$ (+C)	A1	1.1b
	$8\mathbf{i} - \frac{28}{3}\mathbf{j}$ (m s^{-1})	A1	1.1b
		(3)	
(i)(b)	Equate **i** component of **v** to zero	M1	3.1a
	$t - 2t^2 + 36 = 0$	A1ft	1.1b
	$t = 4.5$ (ignore an incorrect second solution)	A1	1.1b
		(3)	
(ii)	Differentiate **r** wrt to t to obtain velocity	M1	3.4
	$\mathbf{v} = (2t-1)\mathbf{i} + 3\mathbf{j}$	A1	1.1b
	Use magnitude to give an equation in t only	M1	2.1
	$(2t-1)^2 + 3^2 = 5^2$	A1	1.1b
	Solve problem by solving this equation for t	M1	3.1a
	$t = 2.5$	A1	1.1b
		(6)	
			(12 marks)

Notes: Accept column vectors throughout		
(i)(a)	M1	At least 3 terms with powers increasing by 1 (but M0 if clearly just multiplying by t)
	A1	Correct expression
	A1	Accept $8\mathbf{i} - 9.3\mathbf{j}$ or better. Isw if speed found.
(i)(b)	M1	Must have an equation in t only (Must have integrated to find a velocity vector)
	A1 ft	Correct equation follow through on their **v** but must be a 3 term quadratic
	A1	cao
(ii)	M1	At least 2 terms with powers decreasing by 1 (but M0 if clearly just dividing by t)
	A1	Correct expression
	M1	Use magnitude to give an equation in t only, must have differentiated to find a velocity (M0 if they use $\sqrt{x^2 - y^2}$)
	A1	Correct equation $\sqrt{(2t-1)^2 + 3^2} = 5$
	M1	Solve a 3 term quadratic for t which has come from differentiating and using a magnitude. This M mark can be implied by a correct answer with no working.
	A1	2.5