1. A and B are two points on a line of greatest slope of a plane inclined at 45° to the horizontal and $AB = 2$ m. A particle P of mass 0.4 kg is projected from A towards B with speed 5 m s^{-1}. The coefficient of friction between the plane and P is 0.2.

   i. Given that the level of A is above the level of B, calculate the speed of P when it passes through the point B, and the time taken to travel from A to B.

   [7]

   ii. Given instead that the level of A is below the level of B,

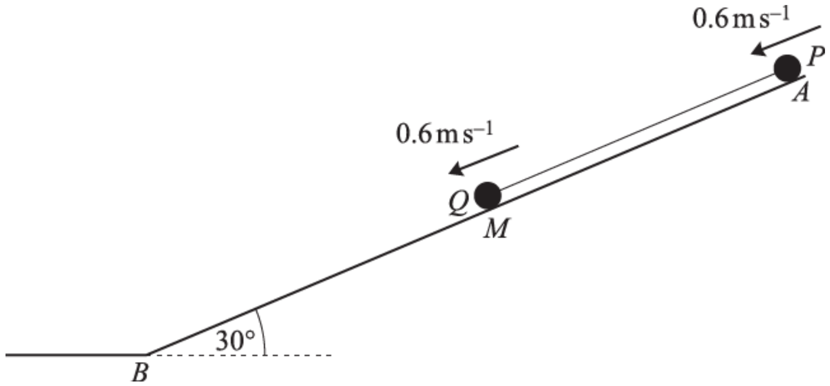
      a. show that P does not reach B,

      [3]

      b. calculate the difference in the momentum of P for the two occasions when it is at A.

      [4]

2.



A and *B* are points at the upper and lower ends, respectively, of a line of greatest slope on a plane inclined at 30° to the horizontal. *M* is the mid-point of *AB*. Two particles *P* and *Q*, joined by a taut light inextensible string, are placed on the plane at *A* and *M* respectively. The particles are simultaneously projected with speed 0.6 ms^{-1} down the line of greatest slope (see diagram). The particles move down the plane with acceleration 0.9 ms^{-2}. At the instant 2 s after projection, *P* is at *M* and *Q* is at *B*. The particle *Q* subsequently remains at rest at *B*.

i. Find the distance *AB*.

[3]

The plane is rough between *A* and *M*, but smooth between *M* and *B*.

ii. Calculate the speed of *P* when it reaches *B*.

[4]

P has mass 0.4 kg and *Q* has mass 0.3 kg.

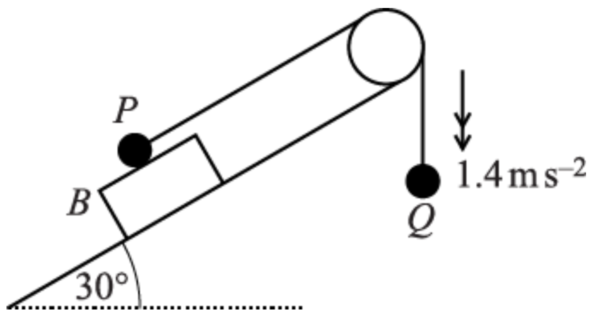
iii. By considering the motion of *Q*, calculate the tension in the string while both particles are moving down the plane.

[3]

iv. Calculate the coefficient of friction between *P* and the plane between *A* and *M*.

[6]

3.



A block B is placed on a plane inclined at 30° to the horizontal. A particle P of mass 0.6 kg is placed on the upper surface of B. The particle P is attached to one end of a light inextensible string which passes over a smooth pulley fixed to the top of the plane. A particle Q of mass 0.5 kg is attached to the other end of the string. The portion of the string attached to P is parallel to a line of greatest slope of the plane, the portion of the string attached to Q is vertical and the string is taut. The particles are released from rest and start to move with acceleration 1.4 m s^{-2} (see diagram). It is given that B is in equilibrium while P moves on its upper surface.

i. Find the tension in the string while P and B are in contact.

[3]

ii. Calculate the coefficient of friction between P and B.

[5]

iii. Given that the weight of B is 7 N, calculate the set of possible values of the coefficient of friction between B and the plane.

[7]

4. A particle P of weight 8 N rests on a horizontal surface. A horizontal force of magnitude 3 N acts on P, and P is in limiting equilibrium.

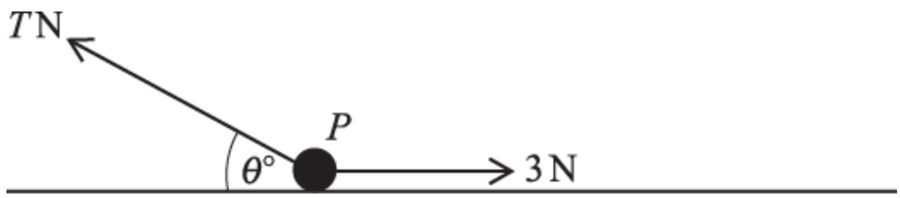
i. Calculate the coefficient of friction between P and the surface.

[2]

ii. Find the magnitude and direction of the contact force exerted by the surface on P.

[4]

iii.



The initial 3 N force continues to act on P in its original direction. An additional force of magnitude T N, acting in the same vertical plane as the 3 N force, is now applied to P at an angle of $\theta°$ above the horizontal (see diagram). P is again in limiting equilibrium.

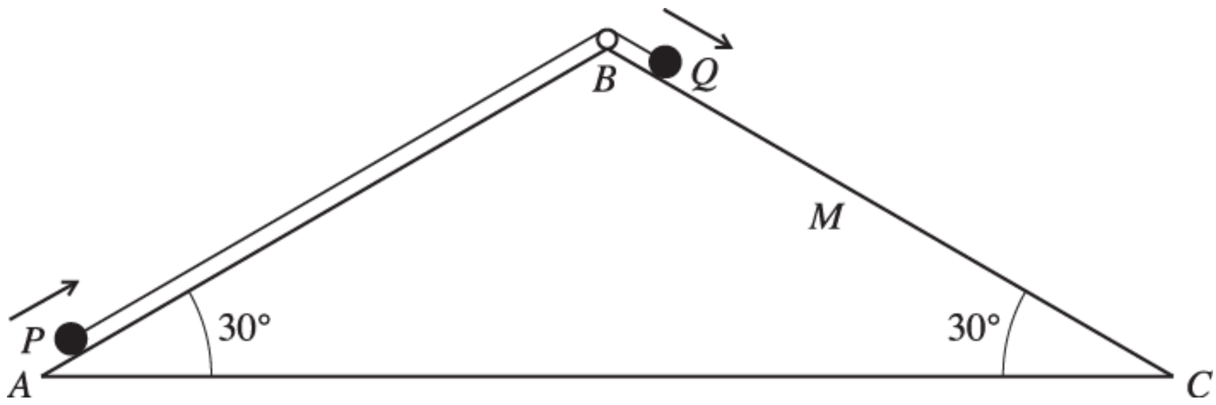
a. Given that $\theta = 0$, find T.

[2]

b. Given instead that $\theta = 30$, calculate T.

[6]

5.



AB and *BC* are lines of greatest slope on a fixed triangular prism, and *M* is the mid-point of *BC*. *AB* and *BC* are inclined at 30° to the horizontal. The surface of the prism is smooth between *A* and *B*, and between *B* and *M*. Between *M* and *C* the surface of the prism is rough. A small smooth pulley is fixed to the prism at *B*. A light inextensible string passes over the pulley. Particle *P* of mass 0.3 kg is fixed to one end of the string, and is placed at *A*. Particle *Q* of mass 0.4 kg is fixed to the other end of the string and is placed next to the pulley on *BC*. The particles are released from rest with the string taut. *P* begins to move towards the pulley, and *Q* begins to move towards *M* (see diagram).

i. Show that the initial acceleration of the particles is 0.7 m s^{-2}, and find the tension in the string.

[5]

The particle *Q* reaches *M* 1.8 s after being released from rest.

ii. Find the speed of the particles when *Q* reaches *M*.

[2]

After *Q* passes through *M*, the string remains taut and the particles decelerate uniformly. *Q* comes to rest between *M* and *C* 1.4 s after passing through *M*.

iii. Find the deceleration of the particles while *Q* is moving from *M* towards *C*.

[2]

iv.

a. By considering the motion of *P*, find the tension in the string while *Q* is moving from *M* towards *C*.

[3]

b. Calculate the magnitude of the frictional force which acts on *Q* while it is moving from *M* towards *C*.

[3]

6. A particle P of mass 0.4 kg is at rest on a horizontal surface. The coefficient of friction between P and the surface is 0.2. A force of magnitude 1.2 N acting at an angle of $\theta°$ above the horizontal is then applied to P. Find the acceleration of P in each of the following cases:

i. $\theta = 0$;

[3]

ii. $\theta = 20$;

[3]

iii. $\theta = 70$;

[3]

iv. $\theta = 90$.

[2]

7. Three forces act on a particle. The first force has magnitude P N and acts horizontally due east. The second force has magnitude 5 N and acts horizontally due west. The third force has magnitude $2P$ N and acts vertically upwards. The resultant of these three forces has magnitude 25 N.

i. Calculate P and the angle between the resultant and the vertical.

[7]

The particle has mass 3 kg and rests on a rough horizontal table. The coefficient of friction between the particle and the table is 0.15.

ii. Find the acceleration of the particle, and state the direction in which it moves.

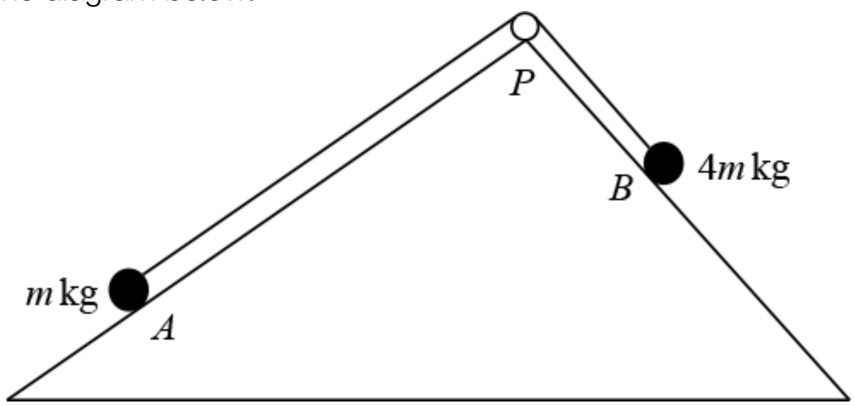
[5]

8. A body of mass 20 kg is on a rough plane inclined at angle α to the horizontal. The body is held at rest on the plane by the action of a force of magnitude P N acting up the plane in a direction parallel to a line of greatest slope of the plane. The coefficient of friction between the body and the plane is μ.

(a) When $P = 100$, the body is on the point of sliding down the plane. Show that $g\sin\alpha = g\mu\cos\alpha + 5$. [4]

(b) When P is increased to 150, the body is on the point of sliding up the plane. Using this and your answer to part (a), find an expression for α in terms of g. [3]

9. Particle A, of mass m kg, lies on the plane $\Pi 1$ inclined at an angle of $\tan^{-1}\frac{3}{4}$ to the horizontal. Particle B, of $4m$ kg, lies on the plane $\Pi 2$ inclined at an angle of $\tan^{-1}\frac{4}{3}$ to the horizontal. The particles are attached to the ends of a light inextensible string which passes over a smooth pulley at P. The coefficient of friction between particle A and $\Pi 1$ is image and plane $\Pi 2$ is smooth. Particle A is initially held at rest such that the string is taut and lies in a line of greatest slope of each plane.
This is shown on the diagram below.



(a) Show that when A is released it accelerates towards the pulley at $\frac{7g}{15}\,\mathrm{m\,s^{-2}}$. [6]

(b) Assuming that A does not reach the pulley, show that it has moved a distance of $\frac{1}{4}$m when its speed is $\sqrt{\frac{7g}{30}}\,\mathrm{m\,s^{-1}}$. [2]

10. A particle P of weight W lies on the surface of a rough plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$. The coefficient of friction between the particle and the plane is $\frac{1}{2}$. A horizontal force of magnitude H is applied to P. This force acts in the vertical plane

through a line of greatest slope. It is given that H is the greatest value for which P remains in equilibrium.

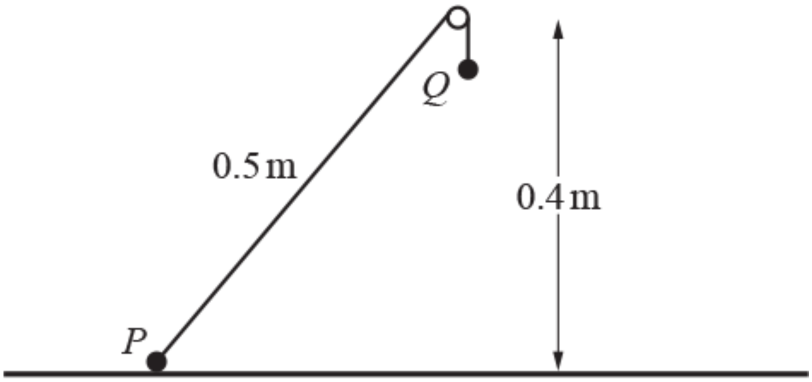
(a) Indicate on a diagram the forces acting on P. [1]

(b) Show that $H = \frac{11}{2}W$. [5]

The horizontal force acting on P is now removed.

(c) Find the acceleration of P in terms of g. [4]

11.



A particle P of mass 0.4 kg is attached to one end of a light inextensible string. The string passes over a small smooth fixed pulley, and a particle Q of mass 0.1 kg is attached to the other end of the string. P rests in limiting equilibrium on a horizontal surface which is 0.4 m below the pulley, with the string taut and in the same vertical plane as P, Q and the pulley. P is 0.5 m from the pulley (see diagram).

(i) Calculate the coefficient of friction and the magnitude of the contact force exerted on P by the surface. [7]

Q is now moved to the position on the surface below the pulley such that the portion of the string attached to Q is vertical. P hangs freely below the pulley and the portion of the string attached to P is vertical. Both particles are at rest when Q is released.

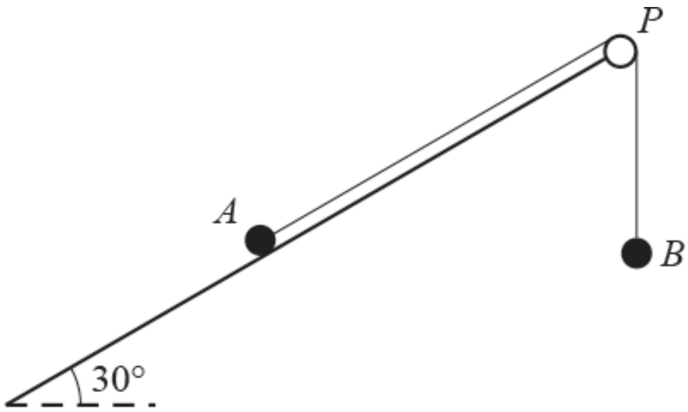
(ii) Find the acceleration of the particles and the tension in the string while P is descending. [5]

P strikes the surface and remains at rest. Q comes to instantaneous rest immediately before reaching the pulley.

(iii) Find the length of the string. [5]

12. One end of a light inextensible string is attached to a particle A of mass m kg. The other end of the string is attached to a second particle B of mass λm kg, where λ is a constant. Particle A is in contact with a rough plane inclined at 30° to the horizontal. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is

parallel to a line of greatest slope of the plane. The particle *B* hangs freely below *P* (see diagram).



The coefficient of friction between *A* and the plane is μ.

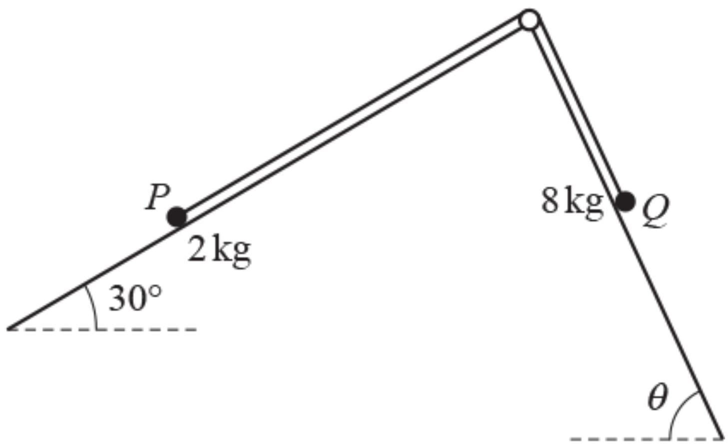
(a) It is given that *A* is on the point of moving down the plane.

(i) Find the exact value of μ when $\lambda = \frac{1}{4}$. [7]

(ii) Show that the value of λ must be less than $\frac{1}{2}$. [2]

(b) Given instead that λ = 2 and that the acceleration of *A* is $\frac{1}{4}g \text{ m s}^{-2}$, find the exact value of μ. [5]

13.



Two particles P and Q, with masses 2 kg and 8 kg respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at a point on the intersection of two fixed inclined planes. The string lies in a vertical plane that contains a line of greatest slope of each of the two inclined planes. Plane Π_1 is inclined at an angle of 30° to the horizontal and plane Π_2 is inclined at an angle of θ to the horizontal. Particle P is on Π_1 and Q is on Π_2 with the string taut (see diagram).

Π_1 is rough and the coefficient of friction between P and Π_1 is $\frac{\sqrt{3}}{3}$.

Π_2 is smooth.

The particles are released from rest and P begins to move towards the pulley with an acceleration of
$g \cos \theta$ m s^{-2}.

(a) Show that θ satisfies the equation

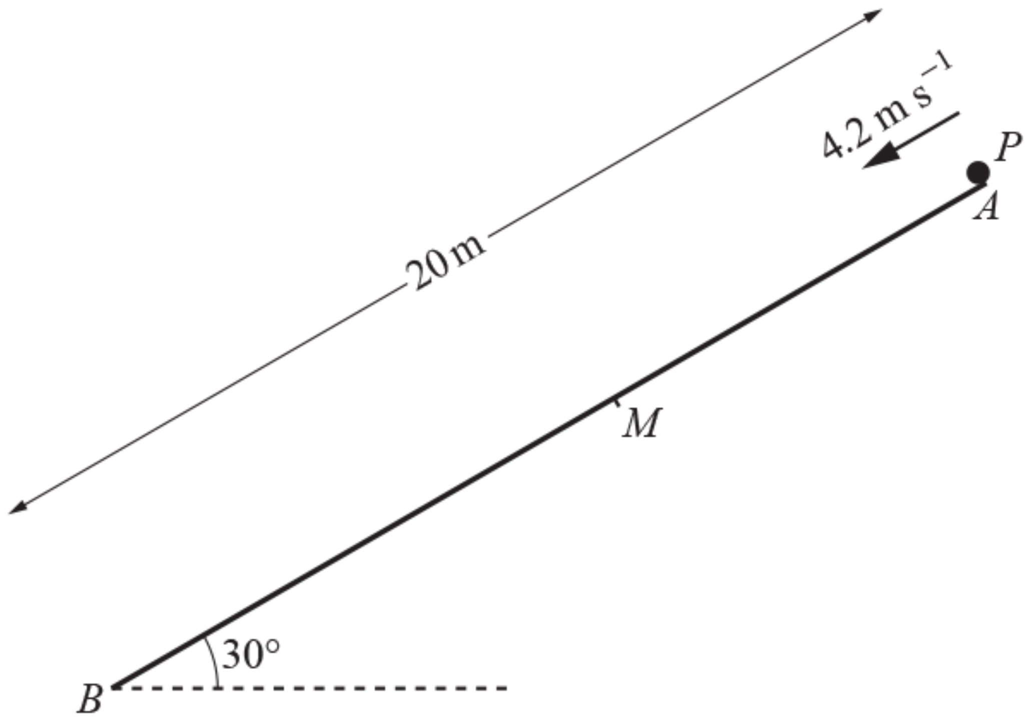
$$4 \sin \theta - 5 \cos \theta = 1.$$

[8]

(b) By expressing $4 \sin \theta - 5 \cos \theta$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0 > \alpha > 90°$, find, correct to 3 significant figures, the tension in the string.

[7]

14.



A and *B* are points at the upper and lower ends, respectively, of a line of greatest slope on a plane inclined at 30° to the horizontal. The distance *AB* is 20 m. *M* is a point on the plane between *A* and *B*. The surface of the plane is smooth between *A* and *M*, and rough between *M* and *B*.

A particle *P* is projected with speed 4.2ms^{-1} from *A* down the line of greatest slope (see diagram). *P* moves down the plane and reaches *B* with speed 12.6ms^{-1}. The coefficient of friction between *P* and the rough part of the plane is $\frac{\sqrt{3}}{6}$.

(a) Find the distance *AM*. [8]

(b) Find the angle between the contact force and the downward direction of the line of greatest slope when *P* is in motion between *M* and *B*. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	Fr = 0.2 × 0.4$g$cos45	M1	Fr = 0.554(37..)	
		i	0.4a = 0.4$g$sin45 – 0.554(37..) (= 2.21748..)	M1	N2L, their Fr value and cmpt wt, opposite signs	
		i	a = 5.54(37..)	A1	May be implied	
		i	$v^2 = 5^2 + 2 \times 5.54 \times 2$	M1	$v^2 = u^2 + 2as$, a is not 0.2g. $0 < a < g$. Consistent signs	
		i	v = 6.87 m s^{-1}	A1		
		i	6.87 = 5 + 5.54t	M1	$2 = 5t + 5.54t^2/2$, a is not 0.2g. $0 < a < g$ **Examiner's Comments**	
		i	t = 0.337 s	A1	This was well answered in the main, but candidates who found that P had an acceleration down the plane greater than g did not look for an error in their calculation. There were a significant number of scripts in which only one of v or t were found.	
		ii	*(a)* +/–0.4a = –0.4$g$sin45 – 0.55437 (= 3.3262..)	M1	N2L, Fr and cmpt wt same sign (accept +ve)	
		ii	a = +/–8.31(557..)	A1	Accept +ve value	
		ii	$0^2 = 5^2 - 2 \times 8.32 \times s$		$5^2 = 2 \times 8.32 \times s$, a is not g or 0.2g. Consistent signs.	
		ii	s = 1.5(0) (so does not reach B)	A1	cso	
		ii	*OR*			
		ii	$v^2 = 5^2 - 2 \times 8.32 \times 2$			
		ii	v^2 = –ve (–8.28) so does not reach B	A1	Some comment on impossibility **Examiner's Comments**	

					There was a noticeable reluctance on the part of candidates to create a Newton's Second Law equation in which all the force terms were negative. This lead to the sly insertion of a minus sign into a calculation of the distance travelled by P before coming to rest.	Friction
		ii	(b) $v^2 = 2 \times 5.54(37) \times 1.5$	M1*	No A1 to be given for $s = 1.5$ (if last A1 not given in iia), a is not g or $0.2g$ or their a in 7iia allow $a > g$	
		ii	$v = +/- 4.08$	A1		
		ii	Momentum change = +/−0.4(4.08 + 5)	D*M1	Must be a sum of 5 and a speed meaningfully less than 5	
		ii	Change = +/−3.63 kg m s^{-1}	A1	**Examiner's Comments** This proved too demanding for most candidates, hinging as it did on using the acceleration found in (i), and a distance calculated in (ii)(a). Some candidates who did this introductory work correctly then overlooked the vector nature of momentum. A significant minority used either the velocity calculated in (i) or the acceleration found in (ii)(a) in this part.	
			Total	**14**		
2		i	$s = 0.6 \times 2 + 0.9 \times 2^2/2$	M1	Uses $s = ut + at^2/2$, $u \neq 0$, $a \neq g$ or gCorS30	
		i	$s = 3$	A1		
		i	$AB = 6$ m	A1	**Examiner's Comments** Many good solutions were seen for three of the four parts of this question.	
		ii	$V_M = 0.6 + 0.9 \times 2$ *OR*		2.4	Award if found in (i) and used in (ii)
		ii	$V_M^2 = 0.6^2 + 2 \times 0.9 \times 3$	B1	5.76	
		ii	$a = g\sin 30$	B1	4.9	

		ii	$V_B^2 = 2.4^2 + 2(9.8\sin30) \times 3$	M1	Uses $v^2 = u^2 + 2as$, $u \neq 0$ or 0.6, $a \neq g$ or 0.9, $s \neq AB$(i) Accept 5.9 **Examiner's Comments** It was part (ii) which presented the greatest challenge. The analysis of the motion of *P* rarely reflected its having two different accelerations, 0.9 m s^{-2} before *Q* reaches *B*, but 4.9 m s^{-2} subsequently.	If *AB* (i) = 3, allow its use for final M1A1
		ii	$V_B = 5.93$ ms^{-1}	A1		
		iii		M1	N2L, $0.3 \times 0.9 = +/-\ (0.3g\text{CorS}30 - T)$	$a = 0.9$ essential, $m = 0.3$ but if 0.4 used in (iii) AND 0.3 used in (iv), treat as a single mis-read
		iii	$0.3 \times 0.9 = 0.3g\sin30 - T$	A1		
		iii	$T = 1.2$ N	A1	**Examiner's Comments** It was pleasing that the correct forces were used in the Newton's Second Law equations in (iii) and (iv). Perhaps it was coincidence, but candidates who drew clear diagrams and included the forces and accelerations scored particularly well. A common error among candidates who left out a diagram was to have 0.6 as an acceleration.	
		iv		M1*	N2L, 3 forces inc $+/-\ (0.4g\text{CorS}30 + T)$	$a = 0.9$ or value used in (iii), $m = 0.4$ but if 0.4 used in (iii) AND 0.3 used in (iv), treat as a single mis-read
		iv	$0.4 \times 0.9 = 0.4g\sin30 + 1.2 - Fr$	A1ft	ft cv(T) in (iii)	
		iv	$Fr = 2.8$	A1	May be shown by mu calculation	
		iv	$R = 0.4g\cos 30$	B1	May be implied, 3.39(48...)	

		iv	$\mu = 2.8/3.39$	D*M1	$2.8 = 3.39(48)\ \mu$, both forces positive Accept 0.82, not 0.83 or 0.826 **Examiner's Comments**	Awarded only if M1 forN2L equation
		iv	$\mu = 0.825$	A1	It was pleasing that the correct forces were used in the Newton's Second Law equations in (iii) and (iv). Perhaps it was coincidence, but candidates who drew clear diagrams and included the forces and accelerations scored particularly well. A common error among candidates who left out a diagram was to have 0.6 as an acceleration.	
			Total	16		
3		i	$0.5g - T = \pm\ 0.5 \times 1.4$	M1	N2L for Q, difference of 2 force terms **Examiner's Comments** Was routine, though some solutions involved the mass of *P* and were given no credit.	
		i	$0.5g - T = 0.5 \times 1.4$	A1		
		i	$T = 4.2$ N	A1		
		ii	$4.2 - F - 0.6g\sin 30 = 0.6 \times 1.4$ *OR* $4.2 - \mu R - 0.6g\sin 30 = 0.6 \times 1.4$	M1	N2L for *P*, 3 forces including a component of weight of *P* and cv(4.2)	
		ii	Friction (= $4.2 - 0.6g\sin 30 - 0.6 \times 1.4$) = 0.42	A1	May be implied	
		ii	Reaction = 0.6gcos30	B1	May be implied	
		ii	$0.42 = 0.6g\cos30\mu$ *OR* $\mu = 0.42/\ 0.6g\cos30$	M1	$F = \mu R$, *R* a component of weight of *P* and *F* has been found using a component of the weight of *P*. Tolerate *F* –ve and |–veF|.	
		ii	$\mu = 0.0825$	A1	Accept 0.082, not 0.083. **Examiner's Comments**	

					The best candidates were able to answer correctly. The main error was the omission of one of the four terms from the Newton's Second Law equation for *P*.	Friction
		iii	$R = (0.6g + 7) \cos 30$	M1	Includes weight cmpts of *P* and *B*, allow $7g$	
		iii	$R = 11.2$	A1	11.154… May be implied	
		iii	Fr = 7sin30 – 0.42	M1*	Wt cmpt *B* (allow $7g$) – Fr(ii) must be difference.	
		iii	Fr = 3.08	A1	May be implied.	
		iii	$\mu = 3.08/11.2$	D*M1	Both quantities +ve, *F* and *R* both from 2 term equations	
		iii	$\mu = 0.276$	A1	Value of μ, accept 0.28, disregard inequality sign	
		iii	$\mu \geq 0.276$	B1 ft	ft cv (μ found in (iii)) direction of greater than or equal to sign; isw any work relating to an upper limit for μ **Examiner's Comments** Involved many candidates in more work than was necessary, as most tried to find both an upper and a lower bound for μ. The part of the solution best attempted was the normal component of the force between *B* and the plane, and the least successful was the magnitude of the frictional force between the two. That a component of the weight of *P* is included in the former but not the latter was the major difficulty. Some scripts offer a variety of approaches, and it seems that many candidates believe that a "range" must have upper and lower bounds. Some additional solutions are inspired by *P* having left the surface. Give credit for the single attempt which addresses the correct scenario, whether it is presented first or second. The B1 ft mark is assigned or withheld for the correct inequality sign attached to the mu value which has been marked. If in doubt about which of two alternatives to mark, please contact your team leader.	

			Total	15		Friction
4		i	$3 = 8\mu$	M1	Uses $F = \mu R$, Allow R is 8 or $8g$, $Fr = 3$ only 3/8 (fraction), not 3÷8 (division)	
		i	$\mu = 0.375$	A1	**Examiner's Comments** This part was invariably correct.	
		ii	$C^2 = 3^2 + 8^2$	M1	Uses Pythagoras with 3 and 8 or $8g$	
		ii	$C = 8.54$ N	A1	Accept 8.5 or $\sqrt{73}$	
		ii	$\tan\theta = 3/8$ or $\tan\theta = 8/3$	M1	Uses tan with 3 and 8 or $8g$ Accept 21 or 69, direction clear by words or diagram.	Or CorS with answer for C
		ii	$\theta = 20.6°$ with vertical or $69.4°$ with horizontal	A1	**Examiner's Comments** This part was done satisfactorily by candidates who knew the term "contact force" mentioned in the specification. The valid solutions offered had only one common weakness, the specification of the direction of the force. An angle without a reference line or diagram is not specific. Many candidates gave their answer as a bearing, even though the contact force lies in a vertical plane. Too often, candidates' answers were "8 N" and "vertical".	isw work after correct angle magnitude found
		iii	**(a)** $T(\cos 0) - 3 = +/-3$	M1	$T(\cos 0) - 3 = 0$ is M0	$T\cos 0 - 3 = -3$ assumes Fr direction has
		iii	$T = 6$		Answer alone is sufficient for M1A1	not changed
		iii	**(b)** $R = +/-(8 - T \times \text{SorC}30)$	M1	Accept $8g$ with cmpt T	(This is required also in the SC case)
		iii	$R = 8 - T\sin 30$	A1	oe	

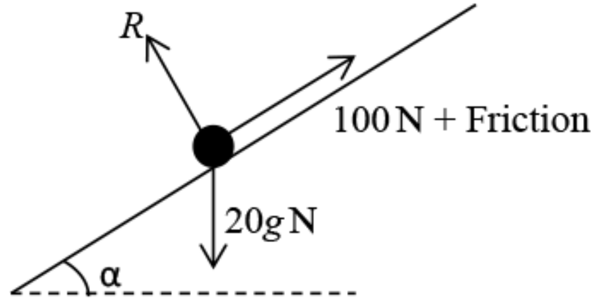
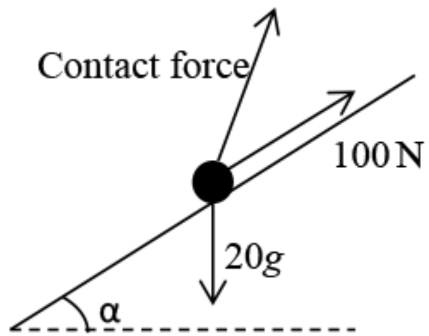
		iii	$Fr = +/- (T \times \text{CorS}30 - 3)$	M1	Accept 3 with cmpt T, not $T \times \text{CorS}30 +/- 3 = 0$	SC Does not allow for change in direction of Friction
		iii	$Fr = T\cos 30 - 3$	A1	oe	$Fr = 3 - T\cos 30$ A1
		iii	$0.375 = (T\cos 30 - 3) / (8 - T\sin 30)$	M1	Accept use of μ from (i). For forming an equation in T alone.	$0.375 = (3 - T\cos 30) / (8 - T\sin 30)$ M1
		iii	$T = 5.70$	A1		$T = 0$ A0
		iii	*OR* Alternative for last 4 marks			SC (Alternative)
		iii	$Fr = 0.375(8 - T\sin 30)$		Accept use of μ from (i).	$Fr = 0.375(8 - T\sin 30)$
		iii	$Fr = +/- (T \times \text{CorS}30 - 3)$	M1		$Fr = +/- (T \times \text{CorS}30 - 3)$ M1
		iii	$Fr = T\cos 30 - 3$	A1	oe	$Fr = 3 - T\cos 30$ A1
		iii	$0.375(8 - T\sin 30) = T\cos 30 - 3$	M1	For forming an equation in T alone.	$0.375(8 - T\sin 30) = (3 - T\cos 30)$ M1
		iii	$T = 5.70$	A1	**Examiner's Comments** The initial situation was extended by the introduction of an additional force, pulling towards "the left". Very many solutions implied that this force meant the particle was still on the verge of moving to "the right". Solutions based on this notion led to candidates finding $T = 0$ after having equations which erred only in the sign of the frictional force. A simpler common misunderstanding was that the horizontal effect of T was to eliminate friction.	$T = 0$ A0
			Total	14		
5		i	$T - 0.3g\sin 30 = 0.3a$ *OR* $0.4g\sin 30 - T = 0.4a$	B1	Either correct N2L for one particle May be awarded later in (i)	Putting $a = 0.7$ into correct equation for a single particle and working out T correctly gets B1M0A0M1A1.

		i	$0.4g\sin30 - 0.3g\sin30 = 0.7a$	M1	Allow combined approach as "method", must be components of weight, allow mg(cos / sin)30	Consult TL if this is done for both particles.
		i	$a = 0.7$ m s^{-2} AG	A1		
		i	$T = 0.3g\sin30 + 0.3 \times 0.7$	M1	Allow $0.3g$(cos / sin)30. Accept cv(0.7)	May use the other equation.
		i	$T = 1.68$ N	A1	**Examiner's Comments** There were many fully correct answers to this question although some failed to find both the quantities required in (i). Candidates should be reminded to check that they have answered a question in full. Use of the combined approach is still quite common and some credit was given if the value of a was found this way. To gain full marks a correct Newton's second law equation had to be obtained for at least one of the particles so that a could be used to find T. A number of candidates forgot or neglected to find T having found a.	
		ii	$V = 1.8 \times 0.7$	M1	Accept cv(0.7)	
		ii	$V = 1.26$ m s^{-1}	A1	**Examiner's Comments** Parts (ii) and (iii) did not depend on work from (i) but blank spaces on some scripts suggest some candidates did not appreciate this. When attempted these parts were well done and most candidates scored full marks on both parts.	
		iii	Dec = 1.26 /1.4	M1	Accept 1.8 × 0.7/ 1.4 Or a = +/− 0.9	cv(1.26)
		iii	Dec = 0.9 m s^{-2}	A1	**Examiner's Comments** Parts (ii) and (iii) did not depend on work from (i) but blank spaces on some scripts suggest some candidates did not appreciate this. When attempted	

					these parts were well done and most candidates scored full marks on both parts.	Friction
		iv	(a)	M1	N2L, 2 forces including cmpt of weight	Allow mg(cos / sin)30
		iv	$T - 0.3g\sin30 = -0.3 \times 0.9$	A1ft	cv(0.9) but signs must be consistent with the direction of motion	
		iv	$T = 1.2$	A1		
			(b)			
		iv	$-0.4 \times 0.9 = 0.4g\sin30 - T - F_r$	M1	N2L, 3 forces including cmpt of weight	Allow mg(cos / sin)30
		iv	$-0.4 \times 0.9 = 0.4g\sin30 - 1.2 - F_r$	A1ft	cv(0.9) and cv(1.2) but signs must be consistent with the direction of motion	
		iv	$F_r = 1.12$ N	A1	**Examiner's Comments** Here it was necessary to use Newton's second law on each particle separately. In both parts the signs used needed to be consistent with the direction of motion in order to gain accuracy marks. In (a) the motion of particle P should have been used; most candidates realised this but a few used the wrong mass or had different masses on the 2 sides of the equation. In (b), to find the friction, the motion of Q should have been used but candidates who used the combined approach were also given credit. Again there were some muddles with the masses and signs but most scored well here.	
			Total	**15**		
6		i	$Fr = 0.2 \times 0.4g$	B1		
		i	$1.2 - 0.2 \times 0.4g = 0.4a$	M1	N2L, 2 forces	
		i	$a = 1.04$ m s^{-2}	A1	**Examiner's Comments** Good solutions from many candidates particularly in (i) and (ii) but (iii) and (iv) proved a challenge for some who failed to think through the mechanics of	

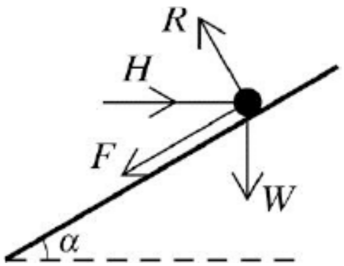
					the varying situations. There was some misunderstanding of the phrase 'above the horizontal' which indicated that the force was acting upwards away from the surface. Credit was given, in the parts affected, if the force was taken to be acting downwards with a small penalty applied in (iv) since this part was simplified by the misunderstanding. Part (i) was not always completed as successfully as (ii) but was generally well done.	Friction
		ii	$R = 0.4g - 1.2\sin 20$	B1		SC $R = 0.4g + 1.2\sin 20$
		ii	$1.2\cos 20 - 0.2(0.4g - 1.2\sin 20) = 0.4a$	M1	N2L, 2 forces, cmpt of 1.2 and *Fr* not *Fr*(i)	$1.2\cos 20 - 0.2(0.4g + 1.2\sin 20) = 0.4a$
		ii	$a = 1.06\ \text{m s}^{-2}$	A1	**Examiner's Comments** Most candidates understood that the normal contact force and hence the friction would now change. No credit was given if the value of friction from (i) was used.	$a = 0.654\ \text{m s}^{-2}$ Give B1M1A1
		iii	$1.2\cos 70 - 0.2(0.4g - 1.2\sin 70)$	M1	Difference of two relevant forces, neither used earlier (or find and compare)	SC $1.2\cos 70 - 0.2(0.4g + 1.2\sin 70)$ Mark as correct case
		iii	(Total is negative,) friction not overcome by (tractive) force	A1		Only finding a negative acceleration scores maximum M1 in both cases.
		iii	$a = 0\ \text{m s}^{-2}$	A1	**Examiner's Comments** Too many simply followed the same routine application of Newton's second law used in (ii) and gave an negative value for the acceleration having failed to appreciate that the tractive force was no longer enough to overcome friction so the acceleration would be zero.	
		iv	$1.2 < 0.4g$ (oe, soi)	M1	Comparison of weight and 1.2 without involving R	SC Sum of weight and 1.2

					Only finding a negative acceleration scores M0 **Examiner's Comments**	Friction
		iv	P cannot rise from table *or* $a = 0$ m s^{-2}	A1	The applied force was now vertical so no credit was given for work which only considered horizontal motion even if this produced the answer $a = 0$. There was a general lack of understanding that the crucial question now was whether or not the applied force was sufficient to lift the object off the table.	P can't go through the table *or* a = 0 B1 only
			Total	**11**		
7		i	Perpendicular components of ($2P$) and +/−($5 - P$)	B1		
		i	$(P - 5)^2 + (2P)^2 = 25^2$	M1	Uses appropriate Pythagoras	
		i	$5P^2 - 10P - 600 = 0$	M1	Attempt to solve 3 term QE "=0"	
		i	$P = 12$	A1		
		i		M1	Targets any relevant angle appropriately	
		i	$\cos\theta = (2 \times 12)/25$, $\tan\theta = (12 - 5)/ 2 \times 12$ etc.	A1√	ft cv(P)	
		i	Angle with vertical = 16.3°	A1	Must be angle with vertical **Examiner's Comments** A good diagram showing all forces was beneficial to those who included it. The only common error was that the squaring of the $2P$ N force when using Pythagoras theorem was written $2P^2$. Too often candidates failed to see their "invisible brackets", and obtained a quadratic equation starting $3P^2$ which would fail to give a useful answer. Several candidates found the angle the resultant makes with the horizontal at the end of part (i).	
		ii	R = +/−(3 × 9.8 − 2 × 12) *OR* R = +/−(3 × 9.8 − 25cos(Ans(i)))	M1*	Bracketed terms must have opposite signs	
		ii	R = 5.4 N (may be implied)	A1√	ft 29.4 − 2 × cv(P(i)) *OR* 29.4 −25cos(cv(θ(i)))	

		ii	$12 - 5 - 0.15 \times 5.4 = 3a$ *OR* $25\sin(\text{cv}(\theta(\text{i}))) - 0.15 \times 5.4 = 3a$	D*M1	N2L, cv(12) cv(5.4) should be acceptable	Friction
		ii	$a = 2.06 \text{ m s}^{-2}$	A1		
		ii	Direction East	B1	Allow bearing (0)90° **Examiner's Comments** Many candidates made a reasonable attempt, but fully developed solutions were rare. Using the vertical component of the resultant to find the normal component of reaction (often ignored), and then using Newton's Second Law with the correct forces was demanding. The direction of motion was presented in many ways, but only "East" or "Bearing 090°" (using the context set in the question) were deemed acceptable.	
			Total	12		
8	a		 Resolve parallel to the slope: $100 + F - 20g\sin\alpha = 0$ (*) Resolve perpendicular to the slope and friction force is maximum: $R = 20g\cos\alpha$ and $F = \mu R$ Substitute and obtain $20g\sin\alpha = 20g\mu\cos\alpha + 100$	B1(AO 2.1) M1(AO 3.3) M1(AO 3.3) E1(AO 1.1)	Any equivalent which makes clear the relationships between: Reaction, 100 N force, friction acting upwards, weight of 20 g N A diagram is OR 	

				[4]	not *necessary* provided that sufficient explanation is given. AG	Friction
		b	All forces shown on diagram of inclined plane Resolve parallel to the slope: $150 - F - 20g\sin\alpha = 0$ (**) From * and ** $250 - 40g\sin\alpha = 0$ $\alpha = \sin^{-1}\dfrac{25}{4g}$	B1(AO 3.3) M1(AO 3.4) A1(AO 1.1) [3]	Reaction, 150 N force, friction acting downwards, weight of 20 g N Eliminate μ and attempt to solve for α. — One valid step after elimination required	
			Total	7		
9		a	Resolving vertically to the plane for Particle A $R = mg\cos\alpha = \frac{4}{5}mg$ Since A is in motion,	B1(AO1.1) B1(AO2.2a)	Obtain $\frac{4}{5}mg$	

			$F_s = \mu R = \frac{1}{3}\left(\frac{4}{5}\right)mg = \frac{4}{15}mg$ Resolving horizontally to the plane for both particles: $T - \frac{13mg}{15} = ma$ $-T + \frac{16mg}{5} = 4ma$ $a = \frac{7g}{15}$	M1(AO3.1b) A1(AO2.1) M1(AO1.1) E1(AO2.4) [6]	Obtain $\frac{4}{15}mg$ Must obtain two equations in T and a Particle A: Attempt resolution as far as stating $T - Fs - mg \sin \alpha = ma$ Particle B: Attempt resolution as far as stating $-T + 4mg \sin \beta\ \alpha$ $4ma$ Solve their simultaneous equations to find a in terms of g. AG Solution must include clear diagrams or explanation for Fs and for horizontal resolutions.	Friction
		b	$\frac{7g}{30} = 2 \times \frac{7g}{15} \times s$	M1(AO 1.1) E1(AO2.1)	Use $v^2 = 0^2 + 2as$	

			$s=\frac{1}{4}$	[2]	AG Must include sufficient working to justify the given answer from the constant acceleration formula		Friction
			Total	8			
10		a		B1(AO1.1) [1]	Correct four forces shown; F could be labelled as μR or $\frac{1}{2}R$		
		b	Resolve || to plane: $F+W\sin\alpha = H\cos\alpha$ Resolve perp. to plane: $R = W\cos\alpha + H\sin\alpha$ $H\cos\alpha - W\sin\alpha = \mu(W\cos\alpha + H\sin\alpha)$ $\frac{3}{5}H-\frac{4}{5}W=\frac{1}{2}\left(\frac{3}{5}W+\frac{4}{5}H\right)$ $\frac{1}{5}H=\frac{11}{10}W \Rightarrow H=\frac{11}{2}W$	M1(AO3.3) M1(AO3.3) M1(AO3.3) M1(AO2.1) A1(AO2.2a) M1 M1	Three terms with resolving attempted Three terms with resolving attempted Use of $F=\mu R$; dep on first two M marks oe; equation in H and W only, in any form **AG**; sufficient working must be shown	$\sin\alpha$ and $\cos\alpha$ may be numerical	

			Alternative solution Resolve vertically: $W + F\sin\alpha = R\cos\alpha$ $W = \frac{3}{5}R - \frac{4}{5} \times \frac{1}{2}R = \frac{1}{5}R \Rightarrow R = 5W$ Resolve horizontally: $H = F\cos\alpha + R\sin\alpha$ Substitute $H = \frac{1}{2} \times 5W \times \frac{3}{5} + 5W \times \frac{4}{5}$ $H = \frac{11}{2}W$	M1 M1 A1 [5]	Use $F = \mu R$ and obtain R in terms of W AG; sufficient working must be shown		Friction
		c	$R = W\cos\alpha$ $W\sin\alpha - F = \frac{W}{g}a$ $W\sin\alpha - \mu W\cos\alpha = \frac{W}{g}a$ $a = \frac{1}{2}g$	B1(AO1.1) M1(AO3.3) M1(AO3.4) A1(AO1.1) [4]	For attempt at N2L || to plane, with F in the opposite direction to that seen in (b) Using $F = \mu R$ and eliminating R and F	Trig ratios may be numerical Do not allow W for the mass	
			Total	10			
11		i	$\sin\theta = 0.4/0.5$ or $\cos\theta = 0.3/0.5$	B1	θ is angle between string and horizontal		

			$T = 0.1g$ (=0.98) N $Fr = T\cos\theta$ (= 0.588) $R = 0.4g - T\sin\theta$ (= 3.136) μ (= 0.588 /3.136) = 3/16 or 0.1875 $C^2 = 0.588^2 + 3.136^2$ $C = 3.19$ N	B1 M1 M1 A1 M1 A1 [7]	CorS. T, angle do not have to be numerical — If two values of T are employed, award B1 for 0.1g associated with Q. SorC. T, angle do not have to be numerical with 0.4g — R must be a difference of forces 0.187 or 0.188 Must have two non-zero numerical values **Examiner's Comments** Part (i) was often incomplete as candidates were unfamiliar with the term "contact force". The initial part was well answered by many, and candidates who could not find a relevant angle were still able to gain a majority of marks.	Friction
		ii	$0.4g - T = 0.4a$ $T - 0.1g = 0.1a$ $0.3g = 0.5a$ *OR* 0.4g - 0.1g = 0.4a + 0.1a	M1 A1 M1	N2L for either particle, no components Both equations correct Solves two simultaneous — Finding a correctly from the combined equation gets M1A1. Using a in an N2L equation for P or Q can get M1, and obtaining the correct value of T gets A1,	

			$a = 5.88$ m s^{-2} $T = 1.568$ N = 1.57 N	A1 A1 [5]	equations | hence 4 marks out of 5 **Examiner's Comments** In part (ii) many fully correct solutions were found, and the most common error was to omit an answer for the tension.	Friction
		iii	P descends = x m (= (2x0.4 - l) m) v^2 = 2x5.88x (=11.76x) 0 = $v^2 - 2g(0.4-x)$ $x = 0.25$ String is 0.8-0.25 m long $l = 0.55$ m *OR* (P starts d m below pulley) v^2 = 2x5.88(0.4 – d) $v^2 = 2gd$ $d = 0.15$ String is 0.4+0.15 m long $l = 0.55$ m	 M1 M1 A1 M1 A1 [5] M1 M1 A1 M1 A1	P and Q moving together | Eqn has two unknowns Q rising alone | Eqn has two unknowns P and Q moving together | Eqn has two unknowns Q rising alone | Eqn has two unknowns	

					Examiner's Comments Part (iii) was challenging for most, setting up and solving two simultaneous $v^2 = u^2 + 2as$ equations being unfamiliar.	
			Total	17		
12		a	(i) $R = mg\cos 30$ $T = \frac{1}{4}mg$ $T + F - mg\sin 30 = 0$ $F = \mu(mg\cos 30)$ $\frac{1}{4}mg + \mu\left(\frac{mg\sqrt{3}}{2}\right) - \frac{1}{2}mg = 0 \Rightarrow \mu = \ldots$	B1(AO 3.3)E B1(AO 1.1)E M1(AO 3.3)E A1(AO 1.1)C M1(AO 3.3)E M1(AO 2.1)A A1(AO 2.2a)A [7]	Resolving perpendicular to the plane Resolving vertically for B Resolving parallel to the plane – three terms – allow signs and sin/cos confusion Use of $F = \mu R$ Deriving equation in μ (and m and g) and attempt to solve for μ – dependent on previous M marks and second B mark	

			<table><tr><td></td><td>$\mu = \frac{\sqrt{3}}{6}$</td></tr></table>		**Examiner's Comments** This question was answered extremely well with most candidates setting out their working clearly and explaining what they were doing at each step. As mentioned in the overview candidates are strongly advised to deal with each particle separately and many correctly resolved vertically for *B*, and resolved both parallel and perpendicular to the plane for *A* using $F \le \mu R$ to find the value of μ when $\lambda = \frac{1}{4}$. <table><tr><td>Resolving parallel to the plane with λmg</td><td></td></tr></table>	Friction
		a	(ii) <table><tr><td></td><td>$F = mg\sin 30 - \lambda mg (> 0)$ $F > 0 \Rightarrow \lambda < \frac{1}{2}$</td></tr></table>	M1(AO 3.1a)A A1(AO 2.2a)A [2]	**Examiner's Comments** This part differentiated well with very few candidates showing clearly why $\lambda < \frac{1}{2}$. Only the most able realised that resolving parallel to the plane and then considering the fact that frictional force had to be positive was the easiest way of tackling this part. A number of candidates started by assuming that $\lambda = \frac{1}{2}$ and then concluding that the magnitude of the frictional force would therefore be zero. It was rarely felt by examiners that in these situations candidates made it clear why it was therefore less than (and not greater than) a half. In general, candidates are reminded that 'show that' questions are just that; the information in the question should be used to show a given result and therefore the result (or a limiting case of it) should not be used in an attempt to show that the given result is indeed true.	
		b	$T - F - mg\sin 30 = m\left(\frac{1}{4}g\right)$ $2mg - T = 2m\left(\frac{1}{4}g\right)$	M1(AO 3.3)C B1(AO 3.3)C A1(AO	<table><tr><td>N, II parallel to the plane – four terms N, II for B</td><td>Allow a Allow a</td></tr></table>	

			$2mg - F - mg\sin 30 = \frac{3}{4}mg$ $2mg - \mu(mg\cos 30) - mg\sin 30 = \frac{3}{4}mg$ $\mu = \frac{\sqrt{3}}{2}$	1.1)C A1(AO 2.1)A A1(AO 2.2a)A [5]	Correct method for eliminating T Correct use of $F = \mu R$ and $R = mg\cos 30$ **Examiner's Comments** Like part (b) this part differentiated well with many candidate unaware that if the value of λ was now greater than a half then particle A would move up the plane; many candidates assumed that the motion of A was down the plane. Of those that did have the correct direction of motion most correctly applied Newton's second law to both particles separately and obtained the correct value for the coefficient of friction. Clear diagrams, with arrows showing the direction of motion and/or acceleration help to reduce the risk of sign errors when identifying the direction that any frictional force will act.	Friction
			Total	14		
13	a		$R_1 = 2g\cos 30$ $F = \frac{\sqrt{3}}{3} \times g\sqrt{3}$	B1 (AO1.1) M1 (AO3.3) M1 (AO3.3) A1 (AO1.1) M1 (AO3.3)	Resolve perpendicular to Π_1 where R_1 is the normal contact force on P — $R_1 = g\sqrt{3}$ Use of $F = \mu R$ — $F = g$ Applying N II parallel — The M marks for N II parallel to a plane	

			$T - F - 2g\sin 30 = 2(g\cos\theta)$ $8g\sin\theta - T = 8(g\cos\theta)$ $8g\sin\theta - T = g - 2g\sin 30 = 10g\cos\theta$ $8\sin\theta - 1 - 1 = 10\cos\theta \Rightarrow 4\sin\theta = 1$	A1 (AO1.1) M1 (AO3.4) A1 (AO2.2a) [8]	to the plane for *P* Allow with *a* Applying N II parallel to the plane for *Q* Allow with *a* Combining simultaneous equations to eliminate *T* (dependent on all previous M marks) AG	require the correct number of terms and the weight resolved; allow sign errors and sin/cos confusion	Friction
		b	$R = \sqrt{41}$ $R\cos\alpha = 4$, $R\sin\alpha = 5$ $\tan\alpha = \frac{5}{4} \Rightarrow \alpha = 51.3$ $\theta - \alpha = \sin^{-1}\left(\frac{1}{R}\right)$ $\theta = 60.3$	B1 (AO1.1) M1 (AO1.1) A1 (AO1.1) M1 (AO1.1) A1 (AO1.1) M1 (AO3.4) A1	Forming two equations in *R* and α (allow sign errors or sin/cos confusion) Correct method for finding θ If θ is obtained by calculator with none of the above marks	6.403124… This mark is implied if correct α seen 51.340191… $\theta - 51.34\ldots = 8.9848\ldots$ 60.325068…	

			$T = 8g \sin 60.3 - 8g \cos 60.3$ $T = 29.3$N	(AO2.2a) [7]	earned, allow SC **B2** for 60.3 or better Using their θ to evaluate T	29.303539…	Friction
			Total	15			
14		a	Acceleration component = $g \sin 30°$ $v_M^2 = 4.2^2 + 2(g \sin 30°)x$ $R = mg \cos 30°$ $F = \frac{\sqrt{3}}{6} mg \cos 30°$ $mg \sin 30° - F = ma$ $12.6^2 = v_M^2 + 2g\left(\sin 30° - \frac{\sqrt{3}}{6}\cos 30°\right)(20 - x)$	B1 (AO 1.2) M1 (AO 3.3) B1 (AO 3.3) M1 (AO 3.4) M1* (AO 3.3) M1dep* (AO 3.4) M1 (AO 2.1)	Correct acceleration component seen Use of $v^2 = u^2 + 2as$ for the motion from A to M Resolving perpendicular to the plane Use of $F = \mu R$ for the motion of P between M and B Use of Newton's 2nd Law for the motion of P between M and B Correct use of $v^2 = u^2 + 2as$ for the motion from M to B with their a and	x is the distance AM and v_M is the speed of P at M R is the normal contact force between P and the plane, m is the mass of P	

			$12.6^2 = 4.2^2 + 2(g\sin 30°)x$ $+2g(20-x)\left(\sin 30° - \frac{\sqrt{3}}{6}\cos 30°\right)$ $x = 8.8$ so the distance AM is 8.8m	A1 (AO 2.2a) [8]	correct s Substitute their expression for v_M to obtain an equation in x only BC	Friction
		b	$\tan\alpha = \frac{R}{F} = \frac{mg\cos 30°}{\frac{\sqrt{3}}{6}mg\cos 30°}$ angle = 180° − α = 106.1°	M1* (AO 3.1b) M1dep* (AO 1.1) M1 (AO 1.1) [3]	Equates ratio of contact forces to tan Correct answer (to at least 3 sf) — 106.102 113…	
			Total	11		