Questions

Q1.

Unless otherwise stated, whenever a numerical value of g is required, take g = 9.8 m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

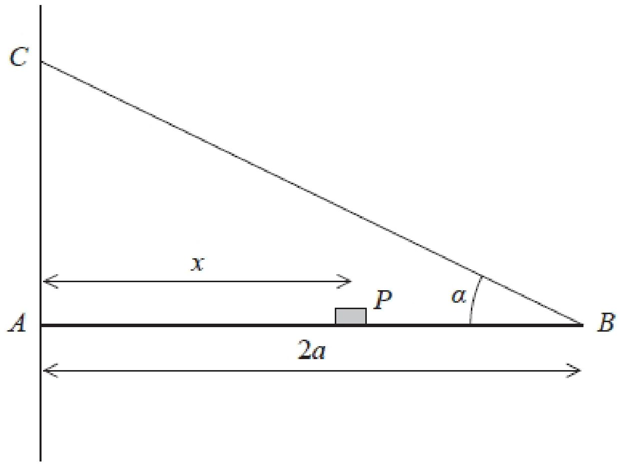


Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall.
The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$.
The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \dfrac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x+a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2\,Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5\,Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

(Total for question = 13 marks)

Q2.

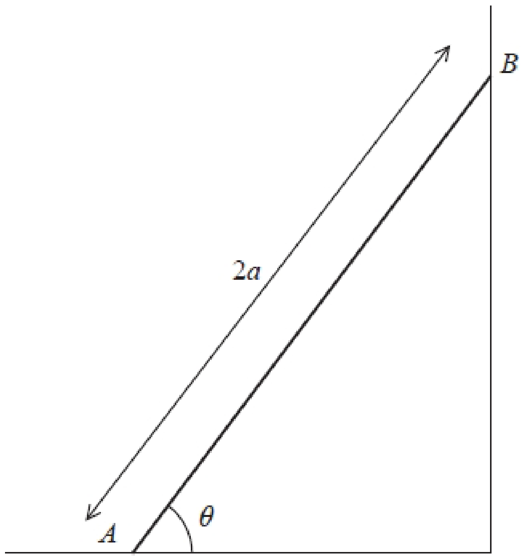


Figure 2

A beam *AB* has mass *m* and length 2*a*.

The beam rests in equilibrium with *A* on rough horizontal ground and with *B* against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geq \frac{1}{2} \cot \theta$

(5)

A horizontal force of magnitude *kmg*, where *k* is a constant, is now applied to the beam at *A*.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan \theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of *k*.

(5)

(Total for question = 10 marks)

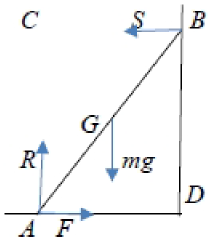
Mark Scheme

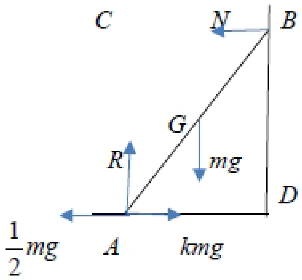
Q1.

Question	Scheme	Marks	AOs
(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \dfrac{Mg(a+3x)}{2a \times \frac{3}{5}} = \dfrac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\dfrac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \dfrac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \dfrac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\dfrac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1ft	1.1b
	$Y = \dfrac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \dfrac{Y}{X}$ or $\dfrac{R\sin\beta}{R\cos\beta}$ $= \dfrac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \dfrac{5}{4}$	A1	2.2a
		(5)	
(d)	$\dfrac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \dfrac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\dfrac{5a}{3}$ from A oe Or just: $x \le \dfrac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 A1	2.4
		(3)	
			(13 marks)

Notes:
(a) **M1**: Using M(A), with usual rules, or any other complete method to obtain an equation in a, M, x and T only. **A1**: Correct equation **A1***: Correct PRINTED ANSWER, correctly obtained, need to see $\sin\alpha = \frac{3}{5}$ used.
(b) **M1:** Using an appropriate strategy to find x. e.g. Resolve horizontally with usual rules applying OR Moments about C. Must use the given expression for T. **A1**: Accept $0.67a$ or better
(c) **M1**: Using a complete method to find Y (or $R\sin\beta$) e.g. resolve vertically or Moments about B, with usual rules **A1 ft:** Correct equation with their x substituted in T expression or using $T = \frac{2Mg}{\cos\alpha}$ **A1**: Y (or $R\sin\beta$) $= \frac{5Mg}{2}$ or $2.5Mg$ or $2.50Mg$ **M1**: For finding an equation **in tan** β **only** using $\tan\beta = \frac{Y}{X}$ or $\tan\beta = \frac{X}{Y}$ This is independent but must have found a Y. **A1**: Accept $\frac{-5}{4}$ if it follows from their working.
(d) **M1**: Allow $T = 5Mg$ or $T < 5Mg$ and solves for x, showing all necessary steps (M0 for $T > 5Mg$) **A1**: Allow $x = \frac{5a}{3}$ or $x < \frac{5a}{3}$. Accept $1.7a$ or better. **B1**: **Treat as A1.** For any appropriate equivalent fully correct comment or statement. E.g. maximum value of x is $\frac{5a}{3}$

Q2.

Question	Scheme	Marks	AOs
	Part (a) is a 'Show that..' so equations need to be given in full to earn A marks		
(a)			
	Moments equation: (M1A0 for a moments inequality)	M1	3.3
	M(A), $mga\cos\theta = 2Sa\sin\theta$ M(B), $mga\cos\theta + 2Fa\sin\theta = 2Ra\cos\theta$ M(C), $F \times 2a\sin\theta = mga\cos\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + 2Sa\sin\theta$ M(G), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$.	A1	1.1b
	($\updownarrow$) $R = mg$ **OR** ($\leftrightarrow$) $F = S$	B1	3.4
	Use their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ **only** (allow DM1 for use of $F = \mu R$ to give an *equation* in μ and θ only)	DM1	2.1
	$\mu \ge \frac{1}{2}\cot\theta$ *	A1*	2.2a
		(5)	

(b)			
	Moments equation:	M1	3.4
	M(A), $mga\cos\theta = 2Na\sin\theta$ M(B), $mga\cos\theta + 2kmga\sin\theta = 2Ra\cos\theta + \frac{1}{2}mg2a\sin\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + N2a\sin\theta$ M(G), $kmga\sin\theta + Na\sin\theta = \frac{1}{2}mga\sin\theta + Ra\cos\theta$ S.C. M(C), $mga\cos\theta + \frac{1}{2}mg2a\sin\theta = kmg2a\sin\theta$ **M1A1B1** $1 + \frac{5}{4} = \frac{5k}{2}$ **M1** $k = 0.9$ **A1**	A1	1.1b
	$N = kmg - F$ **OR** $R = mg$	B1	3.3
	Use their equations <u>(they must have enough)</u> to solve for k (numerical)	DM1	3.1b
	$k = 0.9$ oe	A1	1.1b
		(5)	
			(10 marks)

Notes:		
a	M1	Any moments equation with correct terms, condone sign errors and sin/cos confusion
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ only (allow M1 for use of $F = \mu R$ to give an equation in μ and θ only)
	A1*	Given answer correctly obtained with no wrong working seen (e.g. if they use $F = \mu R$ anywhere, A0)
b	M1	Any moments equation with correct terms, condone sign errors
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) with trig substituted, to solve for k, which must be numerical.
	A1	cao