

1.



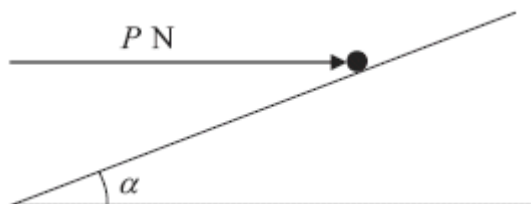
A small box is pushed along a floor. The floor is modelled as a rough horizontal plane and the box is modelled as a particle. The coefficient of friction between the box and the floor is  $\frac{1}{2}$ .

The box is pushed by a force of magnitude 100 N which acts at an angle of  $30^\circ$  with the floor, as shown in the diagram above.

Given that the box moves with constant speed, find the mass of the box.

(Total 7 marks)

2.



A particle of mass 0.4 kg is held at rest on a fixed rough plane by a horizontal force of magnitude  $P$  newtons. The force acts in the vertical plane containing the line of greatest slope of the inclined plane which passes through the particle. The plane is inclined to the horizontal at an angle  $\alpha$ , where  $\tan \alpha = \frac{3}{4}$ , as shown in the diagram above.

The coefficient of friction between the particle and the plane is  $\frac{1}{3}$ .

Given that the particle is on the point of sliding up the plane, find

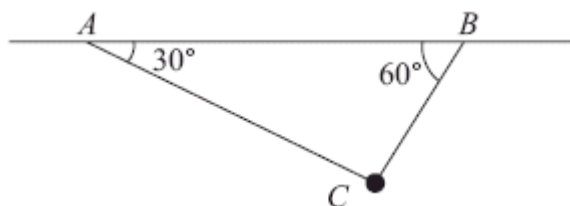
(a) the magnitude of the normal reaction between the particle and the plane,

(5)

- (b) the value of  $P$ .

(5)  
(Total 10 marks)

3.



A particle of mass  $m$  kg is attached at  $C$  to two light inextensible strings  $AC$  and  $BC$ . The other ends of the strings are attached to fixed points  $A$  and  $B$  on a horizontal ceiling. The particle hangs in equilibrium with  $AC$  and  $BC$  inclined to the horizontal at  $30^\circ$  and  $60^\circ$  respectively, as shown in the diagram above.

Given that the tension in  $AC$  is 20 N, find

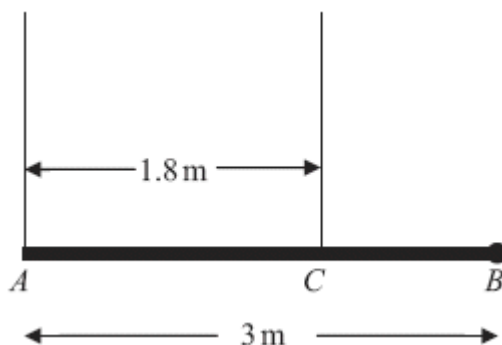
- (a) the tension in  $BC$ ,

(4)

- (b) the value of  $m$ .

(4)  
(Total 8 marks)

4.



A pole  $AB$  has length 3 m and weight  $W$  newtons. The pole is held in a horizontal position in equilibrium by two vertical ropes attached to the pole at the points  $A$  and  $C$  where  $AC = 1.8$  m, as shown in the diagram above. A load of weight 20 N is attached to the rod at  $B$ . The pole is modelled as a uniform rod, the ropes as light inextensible strings and the load as a particle.

(a) Show that the tension in the rope attached to the pole at  $C$  is  $\left(\frac{5}{6}W + \frac{100}{3}\right)$  N. (4)

(b) Find, in terms of  $W$ , the tension in the rope attached to the pole at  $A$ . (3)

Given that the tension in the rope attached to the pole at  $C$  is eight times the tension in the rope attached to the pole at  $A$ ,

(c) find the value of  $W$ . (3)  
(Total 10 marks)

5. A particle is acted upon by two forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$ , given by

$$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j}) \text{ N,}$$

$$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j}) \text{ N, where } p \text{ is a positive constant.}$$

(a) Find the angle between  $\mathbf{F}_2$  and  $\mathbf{j}$ . (2)

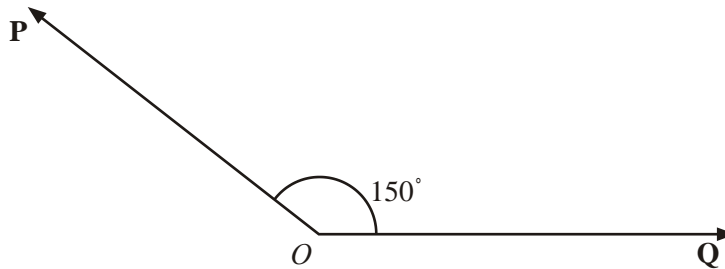
The resultant of  $\mathbf{F}_1$  and  $\mathbf{F}_2$  is  $\mathbf{R}$ . Given that  $\mathbf{R}$  is parallel to  $\mathbf{i}$ ,

(b) find the value of  $p$ .

(4)

(Total 6 marks)

6.



Two forces  $\mathbf{P}$  and  $\mathbf{Q}$  act on a particle at a point  $O$ . The force  $\mathbf{P}$  has magnitude 15 N and the force  $\mathbf{Q}$  has magnitude  $X$  newtons. The angle between  $\mathbf{P}$  and  $\mathbf{Q}$  is  $150^\circ$ , as shown in the diagram above. The resultant of  $\mathbf{P}$  and  $\mathbf{Q}$  is  $\mathbf{R}$ .

Given that the angle between  $\mathbf{R}$  and  $\mathbf{Q}$  is  $50^\circ$ , find

(a) the magnitude of  $\mathbf{R}$ ,

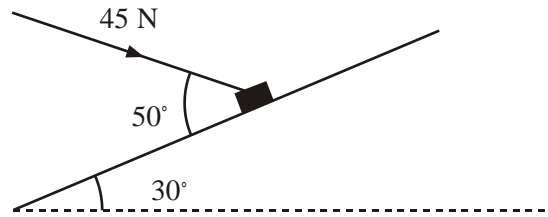
(4)

(b) the value of  $X$ .

(5)

(Total 9 marks)

7.



A package of mass 4 kg lies on a rough plane inclined at  $30^\circ$  to the horizontal. The package is held in equilibrium by a force of magnitude 45 N acting at an angle of  $50^\circ$  to the plane, as shown in the diagram above. The force is acting in a vertical plane through a line of greatest slope of the plane. The package is in equilibrium on the point of moving up the plane. The package is modelled as a particle. Find

- (a) the magnitude of the normal reaction of the plane on the package,

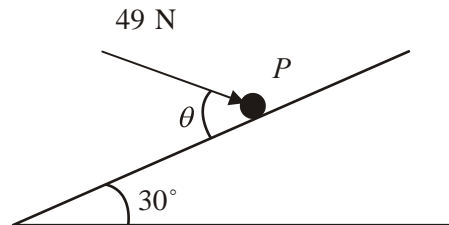
(5)

- (b) the coefficient of friction between the plane and the package.

(6)

(Total 11 marks)

8.



A particle  $P$  of mass 6 kg lies on the surface of a smooth plane. The plane is inclined at an angle of  $30^\circ$  to the horizontal. The particle is held in equilibrium by a force of magnitude 49 N, acting at an angle  $\theta$  to the plane, as shown in the diagram above. The force acts in a vertical plane through a line of greatest slope of the plane.

- (a) Show that  $\cos \theta = \frac{3}{5}$

(3)

- (b) Find the normal reaction between  $P$  and the plane.

(4)

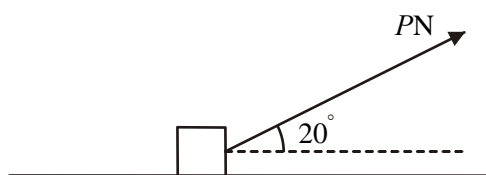
The direction of the force of magnitude 49 N is now changed. It is now applied horizontally to  $P$  so that  $P$  moves up the plane. The force again acts in a vertical plane through a line of greatest slope of the plane.

- (c) Find the initial acceleration of  $P$ .

(4)

(Total 11 marks)

9.



A box of mass 30 kg is being pulled along rough horizontal ground at a constant speed using a rope. The rope makes an angle of  $20^\circ$  with the ground, as shown in the diagram above. The coefficient of friction between the box and the ground is 0.4. The box is modelled as a particle and the rope as a light, inextensible string. The tension in the rope is  $P$  newtons.

- (a) Find the value of  $P$ .

(8)

The tension in the rope is now increased to 150 N.

- (b) Find the acceleration of the box.

(6)

(Total 14 marks)

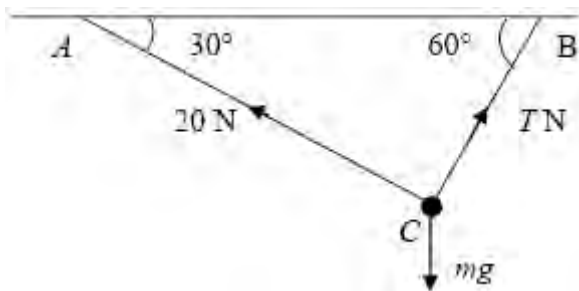
1. (→)  $100\cos 30 = F$  A1  
 $F = 0.5 R$  seen A1 (B1)
- (↓)  $mg + 100\cos 60 = R$  A1  
 $m = 13 \text{ kg or } 12.6 \text{ kg}$  DM1 A1

[7]

2. (a)  $F = \frac{1}{3}R$  B1
- (↑)  $R\cos \alpha - F\sin \alpha = 0.4g$  A1  
 $R = \frac{2}{3}g = 6.53 \text{ or } 6.5$  A1 5
- (b) (→)  $P - F\cos \alpha - R\sin \alpha = 0$  A2  
 $P = \frac{26}{45}g = 5.66 \text{ or } 5.7$  A1 5

[10]

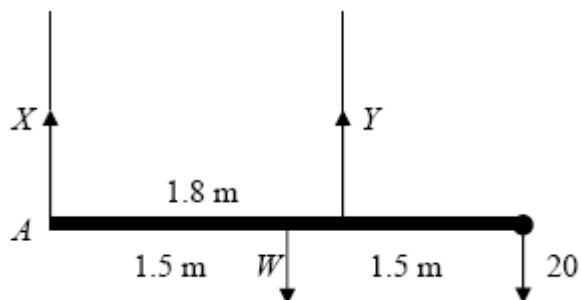
3.



- (a) R(→)  $20\cos 30^\circ = T\cos 60^\circ$  A2 (1,0)  
 $T = 20\sqrt{3}, 34.6, 34.64, \dots$  A1 4
- (b) R(↑)  $mg = 20\sin 30^\circ + T\sin 60^\circ$  A2 (1,0)  
 $m = \frac{40}{8} (\approx 4.1), 4.08$  A1 4

[8]

4. (a)



|      |   |               |           |   |
|------|---|---------------|-----------|---|
| M(A) | $W \times 1.5 + 20 \times 3 = Y \times 1.8$ |               | A2 (1, 0) |   |
|      | $Y = \frac{5}{6}W + \frac{100}{3}$ *        | cso           | A1        | 4 |
| (b)  | $\uparrow \quad X + Y = W + 20$             | or equivalent | A1        |   |
|      | $X = \frac{1}{6}W - \frac{40}{3}$           |               | A1        | 3 |

Alternative

|      |  |  |       |   |
|------|--|--|-------|---|
| M(C) | $X \times 1.8 + 20 \times 1.2 = W \times 0.3$                              |  | A1    |   |
|      | $X = \frac{1}{6}W - \frac{40}{3}$  |  | A1    |   |
| (c)  | $\frac{5}{6}W + \frac{100}{3} = 8\left(\frac{1}{6}W - \frac{40}{3}\right)$ |  | A1 ft |   |
|      | $W = 280$  |  | A1    | 3 |

[10]

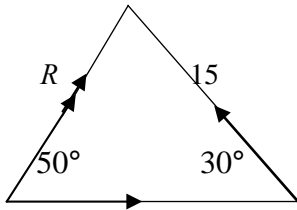
|    |     |  |  |     |   |
|----|-----|--|--|-----|---|
| 5. | (a) | $\tan \theta = \frac{p}{2p} \Rightarrow \theta = 26.6^\circ$   |  | A1  | 2 |
|    | (b) | $\mathbf{R} = (\mathbf{i} - 3\mathbf{j}) + (p\mathbf{i} + 2p\mathbf{j}) = (1 + p)\mathbf{i} + (-3 + 2p)\mathbf{j}$ |  | A1  |   |
|    |     | $\mathbf{R}$ is parallel to $\mathbf{i} \Rightarrow (-3 + 2p) = 0$   |  | DM1 |   |
|    |     | $\Rightarrow p = \frac{3}{2}$  |  | A1  | 4 |

[6]

|    |     |  |            |         |   |
|----|-----|--|------------|---------|---|
| 6. | (a) |  |            | M1A1    |   |
|    |     | $(\uparrow) 15 \sin 30^\circ = R \sin 50^\circ$        |            | DM1A1   | 4 |
|    |     | $R \approx 9.79$ (n)                                   |            |         |   |
|    | (b) | $(\rightarrow) X - 15 \cos 30^\circ = R \cos 50^\circ$ | ft their R | M1A2 ft |   |
|    |     | $X \approx 19.3$ (n)                                   |            | DM1A1   | 5 |



Alternatives using sine rule in (a) or (b); cosine rule in (b)



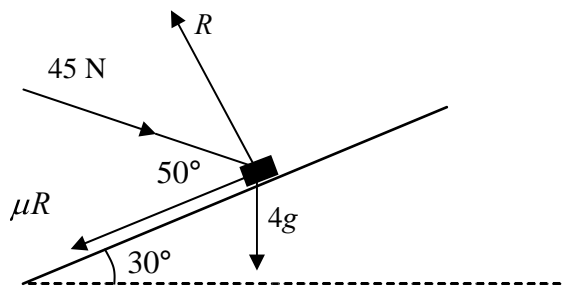
(a)  $\frac{R}{\sin 30^\circ} = \frac{15}{\sin 50^\circ}$  M1A1  
 $r \approx 9.79 \text{ (N)}$  DM1A1 4

(b)  $\frac{X}{\sin 100^\circ} = \frac{15}{\sin 50^\circ} = \frac{R}{\sin 30^\circ}$  M1A2ft on R  
 $X \approx 19.3 \text{ (n)}$  DM1A1 5

**OR:** cosine rule; any of  $X^2 = R^2 + 15^2 - 2 \times 15 \times R \cos 100^\circ$   
 $R^2 = X^2 + 15^2 - 2 \times 15 \times X \cos 30^\circ$  M1A2ft on R  
 $15^2 = R^2 + X^2 - 2 \times X \times R \cos 50^\circ$   
 $X \approx 19.3 \text{ (n)}$  DM1A1 5

[9]

7. (a)



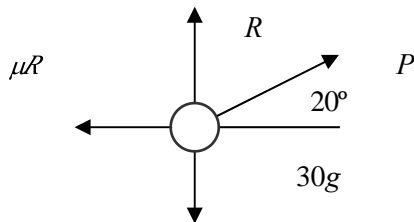
$R = 45 \cos 40^\circ + 4g \cos 30^\circ$  M1A2(1,0)  
 $R \approx 68$  accept 68.4 DM1A1 5

(b) Use of  $F = \mu R$  M1A2(1,0)  
 $F + 4g \sin 30 = 45 \cos 50^\circ$  DM1A1 6  
 Leading to  $\mu \approx 0.14$  accept 0.136

[11]

8. (a) R (// plane):  $49 \cos \theta = 6g \sin 30$   
 $\Rightarrow \cos \theta = 3/5$  \* M1A1  
A1 3
- (b) R (perp to plane):  $R = 6g \cos 30 + 49 \sin \theta$   
 $R \approx \underline{90.1 \text{ or } 90 \text{ N}}$  M1A1  
DM1 A1 4
- (c) R (// to plane):  $49 \cos 30 - 6g \sin 30 = 6a$   
 $\Rightarrow a \approx 2.17 \text{ or } 2.2 \text{ m s}^{-2}$  M1A2,1,0  
A1 4
- [11]**

9. (a)



- Use of  $F = \mu R$  B1  
 $\overline{\phi} P \cos 20^\circ = \mu R$  M1A1  
 $\downarrow R + P \sin 20^\circ = 30g$  A1  
 $P \cos 20^\circ = \mu(30g - P \sin 20^\circ)$   
 $P = \frac{0.4 \times 30g}{\cos 20^\circ + 0.4 \sin 20^\circ}$   
 $\approx 110 \text{ (N)}$  accept 109 A1 8
- (b) i  $R + 150 \sin 20^\circ = 30g$  M1A1  
 $(R \approx 242.7)$   
 N2L  $\overline{\phi} 150 \cos 20^\circ - \mu R = 30a$  M1A1  
 $a \approx \frac{150 \cos 20^\circ - 0.4 \times 242.7}{30}$   
 $= 1.5 \text{ (m s}^{-2}\text{)}$  accept 1.46 A1 6

**[14]**

1. Many candidates scored well on this question. Most resolved in the horizontal direction correctly, with the occasional introduction of an acceleration term at first. In most cases this was equated to zero and most candidates were able to score the method marks at least. The mark for use of limiting friction was obtained by most. For the vertical resolution, the most common errors were either for mistakes in sign or else to state that  $R = mg$ , omitting the component of the 100 N force. Very few candidates solved for the weight rather than the mass. Accuracy errors were common, with 12.57 being a common, too accurate, answer.
2. This question proved to be a good discriminator. The most popular approach was to resolve parallel and perpendicular to the plane (rather than horizontally and vertically which was much easier and avoided having to use simultaneous equations). The majority of candidates used  $F = \mu R$  appropriately. Some, however, just equated the reaction to a weight component thereby simplifying the equations considerably and losing a significant number of marks. Candidates who did set up simultaneous equations correctly sometimes had difficulty in solving them to find the correct values for  $R$  and  $P$ , with poor use of brackets and algebraic manipulation contributing to this. A fairly common error was to give  $R$  in terms of  $P$  instead of calculating a numerical value for it. The final answers were required to be rounded to 2 or 3 significant figures for consistency with the use of  $g = 9.8$  but this was not always observed and incurred a one mark penalty for the question.
3. Far too many candidates worked with the triangle as given in the diagram, rather than with a (vector) triangle of forces. Use of incorrect trig. ratios was the main source of error for those who chose to resolve horizontally and vertically. Relatively few chose to exploit the fact that the tensions were at right angles by resolving along the strings. Some did successfully work with a (right-angled) triangle of forces and a tiny minority used the 'old-fashioned' Lami's Theorem. In part (a), since  $g$  was not involved, the answer needed to be given to at least 2 sf but otherwise there was no limit to the number of figures accepted. However, in the second part, since the answer was dependent on  $g$ , decimal answers needed to be given to either 2 or 3 sf and more accurate versions were penalised.
4. This question was well done by the majority of candidates and was the next best answered question after 1 and 2. Most made valid attempts at taking moments, in part (a) about  $A$  and often also about  $C$  in part (b). The printed answer was an additional help to the less able students who were able to score the marks in part (b) by using it in a vertical resolution. There was some confusion in the last part over the interpretation and use of the information given. Correct statements of simply  $Y = 8X$  or else  $8X + X = W + 20$  were seen but also  $X = 8Y$  was common as were the more surprising  $X + 8Y = W + 20$  and  $8X + Y = W + 20$ , both of which scored nothing.
5. In part (a) the majority of candidates were able to find the correct angle. For those that didn't the most common error was to find the complementary angle. In the second part, provided that it was realised that the sum of the  $\mathbf{j}$  components was equal to zero, full marks were usually achieved. A significant number of candidates equated the sum of the  $\mathbf{i}$  components to zero.

6. This question, involving a resultant force, was not well answered by many candidates, although there were also a fair number of full marks seen. There were two possible approaches (resolving and sine/cosine rule). Some treated it as an equilibrium problem and used correct terms in resolving but with sign errors. Some who used the triangle approach used a triangle with **R** (the resultant) opposite an angle of  $150^\circ$  rather than  $30^\circ$ . This enabled them to find the correct numerical value for the magnitude of **R** by the sine rule but then it was difficult to achieve any more marks since it was impossible to find a third angle. A number of candidates made no significant progress in answering this question; there was a lot of crossed out working seen.
7. It was good to see so many fully correct solutions to this question which was best solved by resolving parallel and perpendicular to the plane. Only the weakest candidates failed to include all the relevant forces. Those candidates who attempted vertical and horizontal resolution often fell victim to inaccuracies in angles or more costly, to missing forces. Since  $g = 9.8$  had been used, the final mark in part (a) was often lost for an answer of 68.42. Virtually all tried to use  $F = \mu R$  appropriately in part (b) although occasionally  $F$  was acting in the wrong direction. Other errors in both parts included incorrect signs, confusion over which angles to use and sine/cosine applied the wrong way round.
8. Throughout this question candidates' answers were marred by confusion between  $30^\circ/\theta$ , cos/sin, and even horizontal/ parallel to the plane.
- Part (a) caused a few problems and sometimes it was not attempted, even though parts (b) and (c) were fully correct. An exact fraction, using  $g = 9.8$ , was required so that recourse to inexact decimals lost marks. In part (b) a significant number of candidates lost the final mark by leaving their answer as 90.12. In the final part many candidates treated the 49 as a force up the slope, rather than horizontal, so failed to resolve up the slope thus failing to score any marks here.
9. Good candidates found this question reasonably straightforward, but many of the weaker ones lost significant numbers of marks because they thought that  $R = 30g$ . It was odd that many candidates could get part (a) completely correct but then were unable to make any progress at all with part (b) and didn't appreciate the similarity between them. Some marks were again lost due to over-accurate answers. A clearly labelled diagram in each part made a huge difference.