Applications of Forces Cheat Sheet

In this chapter, you will learn to solve a greater range of problems involving static particles, where tension, inclined planes and limiting friction need to be accounted for. You will also learn how to deal with static rigid bodies and connected particles on inclined planes.

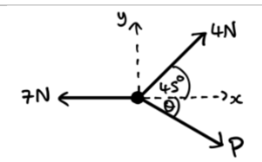
Static equilibrium

If a particle is in static equilibrium, then the resultant force in any direction is equal to 0 and the particle is at rest.

To solve problems involving particles in static equilibrium you can use a three-step procedure:

1. Draw a detailed diagram showing all the forces acting on the particle.
2. Resolve forces in the horizontal and vertical direction. If the particle is on an inclined plane however, then you should resolve parallel and perpendicular to the plane instead.
3. Set the resultant force in each direction equal to 0, then solve for anything unknown.

Example 1: The following diagram shows a particle in static equilibrium. Find the magnitude of P.



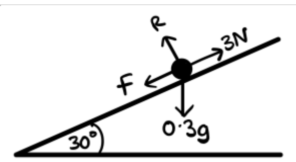
Resolving in the horizontal direction:	$4\cos(45)+P\cos(\theta)-7=0$ $\therefore P\cos(\theta)=7-2\sqrt{2}$ [1]
Resolving in the vertical direction:	$4\sin(45)-P\sin(\theta)=0$ $\therefore P\sin(\theta)=2\sqrt{2}$ [2]
Squaring equations [1] and [2] and adding together:	$[1]^2 \Rightarrow P^2\cos^2\theta=17.402$ $[2]^2 \Rightarrow P^2\sin^2\theta=8$ $[1]^2+[2]^2$: $P^2\sin^2(\theta)+P^2\cos^2(\theta)=25.402$
Factoring out P^2:	$\Rightarrow P^2(\sin^2(\theta)+\cos^2(\theta))=25.402$
But since $\sin^2(\theta)+\cos^2(\theta)=1$	$\Rightarrow P^2(1)=25.402$ $\therefore P=5.04N$ to 3 s.f.

Friction with static particles

When considering a particle in static equilibrium on a rough surface, you need to be able to model the frictional force acting upon the particle. Recall from Chapter 5 that the coefficient of friction, μ, is a constant specific to a pair of surfaces that tells us how rough the two surfaces are. μ can take any value between 0 and 1. The larger μ is, the greater the roughness of the two objects.

- For a particle at rest on a rough surface, the frictional force F is such that $F \le \mu R$, where μ is the coefficient of friction and R is the reaction force normal to the surface.
- The maximum value of the frictional force is reached when the particle is on the point of moving. This is when the particle is said to be in limiting equilibrium, where $F_{max}=\mu R$.
- Remember that the frictional force will always oppose the direction the particle would move in if the frictional force was not there.

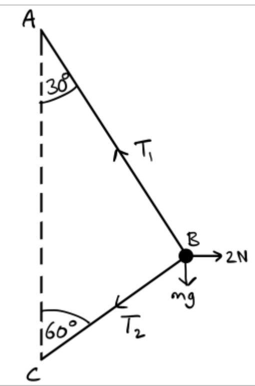
Example 2: A box of mass 0.3kg lies on a rough plane inclined at 30° to the horizontal. The box is held in equilibrium by a force of magnitude 3 N acting up the plane, in a direction parallel to the line of greatest slope of the plane. The particle is on the point of slipping up the plane. Find the coefficient of friction between the particle and the plane.

We start by drawing a detailed diagram. Note that the frictional force, F, acts down the slope since the box is on the point of moving up the plane.	
Resolving parallel to the plane:	$3-0.3g\sin 30-F=0$ $\therefore F=1.53\ N$
Resolving perpendicular to the plane:	$R-0.3g\cos 30=0$ $\therefore R=2.546\ N$
We are told that the particle is on the point of slipping up the plane, so we know that $F_{max}=\mu R$ applies here.	$F_{max}=\mu R$ $\therefore 1.53=2.546\times\mu$ $\Rightarrow \mu=\frac{1.53}{2.546}=0.60$ to 2 d.p

Harder statics

Harder problems will involve weight, tension, pulleys and inclined planes. The same 3-step procedure can be applied to these questions, but there are typically more forces to model.

Example 2: A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C where A is vertically above C. The bead is held in equilibrium by a horizontal force of magnitude 2 N. The sections AB and BC of the string make angles of 30° and 60° with the vertical respectively. Find: a) The tension in the string. b) The mass of the bead.

We start with a detailed diagram:	
Resolving in the vertical direction:	$T\cos 30-mg-T\cos 60=0$
Making T the subject:	$\therefore T(\cos 30-\cos 60)=mg$ $T=\frac{mg}{\cos 30-\cos 60}=(1+\sqrt{3})mg$
Now resolving in the horizontal direction:	$T\sin 30+T\sin 60-2=0$
Making T the subject:	$T(\sin 30+\sin 60)=2$ $\therefore T=\frac{2}{\sin 30+\sin 60}=2\sqrt{3}-2$
Use $T=(1+\sqrt{3})mg$ to find m:	$\Rightarrow 2\sqrt{3}-2=(1+\sqrt{3})mg$ $\Rightarrow m=\frac{2\sqrt{3}-2}{(1+\sqrt{3})g}=0.0547\ kg$ (3 s.f.)

Static rigid bodies

Sometimes you might need to consider the rotational forces acting on an object in static equilibrium. In such cases you can model the object as a rigid body and use moments as well as your knowledge of frictional forces and resolving forces to solve problems.

If you are told that a rigid body is in equilibrium, then you can assume:

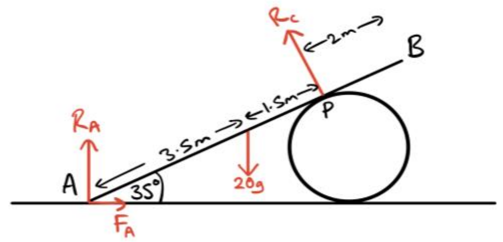
- The resultant force in any direction is 0
- The resultant moment is 0. (i.e. if you take moments about any point, the sum of the moments will be 0)
- The body is at rest.

Questions involving static rigid bodies often involve rods and ladders resting on rough surfaces. It is important that you are able to accurately model all of the forces acting on the rod. There are four steps which you will need to carry out for such questions:

- Take moments about a point and set the resultant moment to be 0. Usually but not always, the best point to take moments about is the point of contact of the rod with the ground.
- Resolve in the vertical direction and set the resultant force to be 0.
- Resolve in the horizontal direction and set the resultant force to be 0.
- If you are told that the rod is in limiting equilibrium with any of the surface it is in contact with, you can use the fact that $F_{max}=\mu R$. Otherwise you can use $F \le \mu R$.

Note that you won't necessarily need to use every one of these steps for a particular question.

Example 3: A uniform ladder AB has length 7m and mass 20kg. The ladder is resting against a smooth cylindrical drum at P, where AP is 5m, with end A in contact with rough horizontal ground. The ladder is inclined at 35° to the horizontal. Find the normal and frictional components of the contact force at A, and hence find the least possible value of the coefficient of friction between the ladder and the ground.

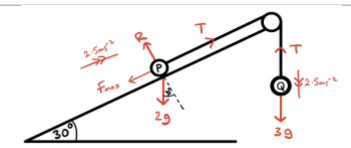
We start with a detailed diagram. R is used to denote the reaction forces while F denotes the frictional forces.	
Taking moments about A, taking the clockwise direction to be positive:	$20g\cos(35)(3.5)-R_C(5)=0$ $\therefore R_C=\frac{20g\cos(35)(3.5)}{5}=112.4N$
Resolving in the vertical direction. Since we know the value of R_C, we can find R_A.	$R_C\cos(35)+R_A-20g=0$ $R_A=20g-112.4\cos(35)=103.9N$
To find F_A, we can resolve horizontally:	$F_A-R_C\sin(35)=0$ $\therefore F_A=112.4\sin(35)=64.5N$
Recall that in general, we have that $F\le\mu R$. This means that $\mu\ge\frac{F}{R}$. Hence the least possible value of μ is when $\mu=\frac{F}{R}$ (i.e. $F=\mu R$). So letting $F_A=\mu R_A$:	$F_A=\mu R_A$ $64.5=\mu(103.9)$
Solving for μ:	$\mu=\frac{64.5}{103.9}=0.621$. This is the least possible value as required.

Dynamics and connected particles

You may be expected to solve problems involving connected particles on inclined planes and rough surfaces. Remember that:

- If both particles are not moving along the same straight line, then you must consider them separately.
- If a particle is moving along a rough surface, then the frictional force acting is maximum (limiting). Therefore, $F_{max}=\mu R$ applies.

Example 4: Two particles P and Q of mass 2kg and 3kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle of 30°. Particle P is held at rest on the inclined plane and Q hangs freely with the string vertical and taut. Particle P is released and it accelerates up the plane at $2.5ms^{-2}$. Find: a) the tension in the string and b) the coefficient of friction between P and the plane.

We start with a detailed diagram.	
Using $F=ma$ for Q in the vertical direction taking downwards to be positive:	$3g-T=3(2.5)$ $\therefore T=3g-3(2.5)=21.9N$
Resolving forces acting on P perpendicular to the plane:	$R-2g\cos 30=0$ $\therefore R=2g\cos 30=\frac{49\sqrt{3}}{5}=16.97$
Using $F=ma$ for P along the slope:	$T-2g\sin 30-F_{max}=(2)(2.5)$
Making F_{max} the subject and substituting $T=21.9$:	$F_{max}=T-2g\sin 30-5=7.1$ $\Rightarrow F_{max}=7.1$
Using $F_{max}=\mu R$:	$\mu R=7.1$
We already found R so we can solve for μ :	$\mu=\frac{7.1}{R}=\frac{7.1}{16.97}=0.418.$