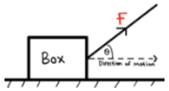
Forces and Friction Cheat Sheet

Edexcel Stats/Mech Year 2

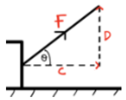
In this chapter, we will learn to resolve more difficult systems of forces including situations where an object is on an inclined plane. We will also learn how to model the frictional force experienced by a body that is at rest or moving on a rough surface.

Resolving forces

You need to be able to find the component of a force acting in a particular direction using basic trigonometry. Let's say that we wish to find the component of F in the direction of motion of the box (shown dotted) in the following example:



Here, we have a force F applied at an angle θ to the direction of motion of the object. If we want to resolve in the direction of motion, we need to find the component of F acting in this direction. Taking a closer look at the components of F:

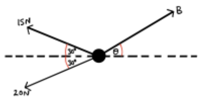


The component that we want to find is C. From basic trigonometry, we know that $cos\theta = \frac{adjacent}{hypotenuse}$, and so $cos\theta = \frac{C}{F}$. Therefore, we have that $C = Fcos\theta$ is the component of F in the direction of motion of the box. To generalise what we have just found:

- The component of a Force F in a particular direction is $Fcos\theta$, where θ is the angle between F and the specified direction.
- The component of the force F perpendicular to the specified direction is $Fsin\theta$, where θ is the angle between F and the specified direction.

We will now go through an example where we will need to use the above ideas:

Example 1: Three forces act upon a particle as shown in the below diagram. Given that the particle is in equilibrium, calculate the magnitude of F and the value of θ.

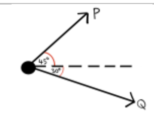
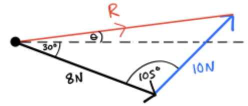
	
We start by resolving horizontally, taking the right to be positive:	$Bcos\theta - 15cos30 - 20cos30 = 0$ $\therefore Bcos\theta = 35cos30 = (17.5)\sqrt{3}$ **[1]**
Resolving vertically now, taking the upwards direction to be positive:	$Bsin\theta + 15sin30 - 20sin30 = 0$ $\therefore Bsin\theta = 2.5$ **[2]**
Squaring **[1]** and **[2]**, then adding them together:	$[1]^2$: $B^2cos^2\theta = 918.75$ $[2]^2$: $B^2sin^2\theta = 6.25$ $[1]^2 + [2]^2$: $B^2cos^2\theta + B^2sin^2\theta = 918.75 + 6.25$
Factoring out B^2 from the LHS and using the identity $cos^2\theta + sin^2\theta = 1$:	$B^2(cos^2\theta + sin^2\theta) = 925$ $B^2(1) = 925$ $\therefore B = \sqrt{925} = 5\sqrt{37}\ N$ $5\sqrt{37}\ sin\theta = 2.5$ $\theta = 4.72$ degrees

Triangle law

The triangle law allows you to use vector addition to find the resultant of two forces without needing to resolve them into components. Using the triangle law can simplify the working for some questions.

- For any two forces P and Q, the resultant force will be the missing side of the triangle formed by the forces P and Q. You can use geometry to figure out any missing angles/sides.

Example 2: Two forces P and Q act on a particle as shown. P has a magnitude of $10N$ and Q has a magnitude of $8N$. Work out the magnitude of the resultant force.

This is the given diagram	
We start by forming a diagram, where we add the vectors P and Q to make up a triangle. The missing side is the resultant force. Remember that when we add two vectors together, one arrow must start where the other ends.	
From the diagram we formed, we can see that since we know the magnitude of the forces P and Q and the angle between them, we can use the cosine rule to find the magnitude of R.	$R^2 = 8^2 + 10^2 - 2(8)(10)\cos(105)$ $R^2 = 205.411 \ldots$ $\therefore R = 14.3N\ (3\ s.f.)$

Friction

Friction is simply a force that opposes the motion of an object moving over a rough surface. You need to be able to model the frictional force experienced by an object at rest or moving on a rough surface.

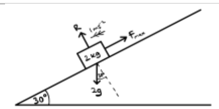
- For a particle at rest on a rough surface, the frictional force F is such that $F \leq \mu R$, where μ is the coefficient of friction and R is the reaction force normal to the surface.
- The maximum value of the frictional force is reached when the particle is on the point of moving. This is when the particle is said to be in limiting equilibrium, where $F_{max} = \mu R$ applies.
- If a particle is moving along a rough surface, then the frictional force acting is maximum (limiting). Therefore, $F_{max} = \mu R$ applies.
- Remember that the frictional force will always oppose the direction the particle would move in if the frictional force was not there.

The example below shows how we apply the concept of limiting friction to a question involving an inclined plane.

Inclined planes

When solving problems involving inclined planes, you should resolve parallel and perpendicular to the plane, instead of vertically and horizontally. You need to be confident in resolving forces for a body on an inclined plane, which may be rough.

Example 3: A box of mass $2kg$ is sliding down a rough slope that is inclined at $30°$ to the horizontal. Given that the acceleration of the particle is $1\ ms^{-2}$, find the coefficient of friction, μ, between the particle and the slope.

We draw a force diagram detailing the given scenario. The dotted line is perpendicular to the plane, and the angle between the weight and the dotted line is found to be 30°. It is helpful to annotate this on your diagram when you are given an inclined plane as you will often have to resolve perpendicular to the plane.	
a) Resolving perpendicular to the plane:, we can sum up the forces and equate to 0 **since there is no movement along the line perpendicular to the plane**.	$R - 2gcos30 = 0$ $\therefore R = 2gcos30 = 17.0N$
b) Resolving parallel to the plane, using $F = ma$ taking the direction of motion to be positive:	$2gsin30 - F_{max} = (2)(1)$
Since the box is moving on a rough surface, the friction is limiting and so $F_{max} = \mu R$ applies. Substituting this into our equation:	$2gsin30 - \mu R = (2)(1)$ $\mu R = 7.8$
Since we found that $R = 17.0$:	$\therefore \mu = \frac{7.8}{17.0} = 0.460$ (3 s.f.)