1. At time $t = 0$, a particle P of mass 3 kg is at rest at the point A with position vector $(\mathbf{j} - 3\mathbf{k})$ m. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ then act on the particle P and it passes through the point B with position vector $(8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ m.

Given that $\mathbf{F}_1 = (4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ N and $\mathbf{F}_2 = (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})$ N and that $\mathbf{F}_1$ and $\mathbf{F}_2$ are the *only* two forces acting on P, find the velocity of P as it passes through B, giving your answer as a vector.

(Total 7 marks)

2. [*In this question* **i** *and* **j** *are horizontal unit vectors*.]

A small bead of mass 0.5 kg is threaded on a smooth horizontal wire. The bead is initially at rest at the point with position vector $(\mathbf{i} - 6\mathbf{j})$ m. A constant horizontal force **P** N then acts on the bead causing it to move along the wire. The bead passes through the point with position vector $(7\mathbf{i} - 14\mathbf{j})$ m with speed $2\sqrt{7}$ m s^{-1}.

Given that **P** is parallel to $(6\mathbf{i} + \mathbf{j})$, find **P**.

(Total 6 marks)

3. A bead of mass 0.5 kg is threaded on a smooth straight wire. The only forces acting on the bead are a constant force $(4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k})$ N and the normal reaction of the wire. The bead starts from rest at the point A with position vector $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ m and moves to the point B with position vector $(4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ m. Find the speed of the bead when it reaches B.

(Total 4 marks)

4. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are the only forces acting on a particle. $\mathbf{F}_1$ has magnitude 9 N and acts in the direction of $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. $\mathbf{F}_2$ has magnitude 18 N and acts in the direction of $\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$.

Find the total work done by the two forces in moving the particle from the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m to the point with position vector $(3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ m.

(Total 6 marks)

5. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are the only forces acting on a particle *P* of mass 2 kg. The particle is initially at rest at the point *A* with position vector $(-2\mathbf{i} - \mathbf{j} - 4\mathbf{k})$ m. Four seconds later, *P* is at the point *B* with position vector $(6\mathbf{i} - 5\mathbf{j} + 8\mathbf{k})$ m.

Given that $\mathbf{F}_1 = (12\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$ N, find

(a) $\mathbf{F}_2$,

(5)

(*b*) the work done on *P* as it moves from *A* to *B*.

(3)

(Total 8 marks)

6. *In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane and* **k** *is a unit vector vertically upwards.*

A small smooth ring of mass 0.1 kg is threaded onto a smooth horizontal wire which is parallel to $(\mathbf{i} + 2\mathbf{j})$. The only forces acting on the ring are its weight, the normal reaction from the wire and a constant force $(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ N. The ring starts from rest at the point *A* on the wire, whose position vector relative to a fixed origin is $(2\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$ m, and passes through the point *B* with speed 5 m s^{-1}. Find the position vector of *B*.

(Total 6 marks)

1.	$\pm(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$	B1		
	$((4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})).(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = \frac{1}{2}3v^2$	M1 A1 f.t.		
	$12 = v$	A1		
	$\mathbf{v} = \dfrac{12}{\sqrt{8^2 + (-4)^2 + 8^2}}\ (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$	M1		
	$\mathbf{v} = (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})\ \text{ms}^{-1}$	DM1 A1		
				[7]

2.	$\mathbf{d} = (7\mathbf{i} - 14\mathbf{j}) - (\mathbf{i} - 6\mathbf{j}) = (6\mathbf{i} - 8\mathbf{j})$	B1		
	$(6ki + k\mathbf{j}).(6\mathbf{i} - 8\mathbf{j}) = \frac{1}{2}\times\frac{1}{2}\times(2\sqrt{7})^2$	M1A2ft		
	$28k = 7 \Rightarrow k = \frac{1}{4}$	DM1		
	$\Rightarrow \mathbf{P} = \frac{3}{2}\mathbf{i} + \frac{1}{4}\mathbf{j}$	A1	6	
				[6]

3.	$\mathbf{d} = \mathbf{AB} = 3\text{i} + \mathbf{j} - 5\mathbf{k}$	B1		
	$\frac{1}{2}.0.5v^2 = \mathbf{F}.\mathbf{d} = (4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}).(3\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	M1A1		
	$v = 6\text{ m s}^{-1}$	A1	4	
				[4]

4.	$\lvert 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}\rvert = 3 \Rightarrow F_1 = 6\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$	B1		
	$\lvert \mathbf{i} + 8\mathbf{j} - 4\mathbf{k}\rvert = 9 \Rightarrow F_2 = 2\mathbf{i} + 16\mathbf{j} - 8\mathbf{k}$	B1		
	WD $= (6\mathbf{i} + 3\mathbf{j} + 6\mathbf{k} + 2\mathbf{i} + 16\mathbf{j} - 8\mathbf{k}).(3\mathbf{i} + 2\mathbf{j} - \mathbf{k} - \mathbf{i} - \mathbf{j} + \mathbf{k})$	M1 A1ft on F		
	$= (8\mathbf{i} + 19\mathbf{j} - 2\mathbf{k}).(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	M1		
	$= \underline{39\text{ J}}$	A1	6	
				[6]

5.	(a) $\mathbf{AB} = 8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$			
	$8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k} = \frac{1}{2} \times \mathbf{a} \times 4^2 \Rightarrow \mathbf{a} = \mathbf{i} - \frac{1}{2}\mathbf{j} + \frac{3}{2}\mathbf{k}$	M1 A1 ft		
	$(12\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) + \mathbf{F}_2 = 2(\mathbf{i} - \frac{1}{2}\mathbf{j} + \frac{3}{2}\mathbf{k})$	M1		
	$\Rightarrow \mathbf{F}_2 = (-10\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})\text{N}$	A1	5	
	(b) Work done $= (\mathbf{F}_1 + \mathbf{F}_2).\mathbf{AB}$			

= (2**i** – **j** + 3k).(8**i** – **4j** + 12**k**) M1

= 16 + 4 + 36 = 56 J M1 A1 3

[8]

6. Work done by force = $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ **. AB** M1

Attempt at equating work done to KE M1

$\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} \lambda \\ 2\lambda \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} x-2 \\ y+2 \\ z+3 \end{pmatrix} = \frac{1}{2}(0.1)5^2$ A1

Solving for λ(λ = 0.25) or forming sufficient equations in *x*, *y* M1

[e.g $x + 2y = -0.75$, $y + 2 = 2(x - 2)$]

Method to find **OB** M1

[**OB** = $\begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ or solving for *x*, *y*]

OB = $2\frac{1}{4}$**i** – $1\frac{1}{2}$**j** – 3**k** any form A1

[6]

Alternative (Non-vector approach):

$|F| \cos \theta = ma$ applied; [$a = 10\sqrt{5}$] M1

Method to find "*s*": $5^2 = 2(10\sqrt{5})s$ [$s = \frac{\sqrt{5}}{4}$] M1 A1

Finding λ M1,

Method to find **OB** M1 A1

1. Most candidates scored at least 4 out of 7 on this question by using the work-energy principle to find the speed of *P* but some did not then go on to find a velocity, using the fact that the velocity is parallel to *AB*. Candidates whose dot product resulted in a vector gained only the first mark for **AB**. There were a number of other approaches which involved finding the acceleration, either as a scalar or as a vector, followed by the use of an appropriate constant acceleration formula.

2. Many candidates answered this correctly, using a scalar product approach. Some were, however, unable to deal with **P** being parallel to 6**i** + **j**. A substantial minority tried to work with magnitudes rather than components and made little progress.

3. This proved to be an easy starter and was mostly well done, although a few candidates insisted on including the weight, in spite of being told that "the only forces...".

4. Some candidates were unable to find the two forces in vector form but then went on to use a scalar product correctly. There were many fully correct solutions.

5. Better candidates were successful in this question, though others found it harder. Many effectively assumed that the situation was one of constant velocity. Also many simply assumed that motion was one-dimensional (which it in fact was, but needed justification) and so wrote down that work done = magnitude of force x distance without justifying this further or going via the scalar product to find the work done.

6. Candidates who equated work done to gain in kinetic energy, after finding work done using the scalar product were more successful than those who used a "speed equation" approach; the latter usually showed little appreciation of the vector nature of the given data and rarely produced anything of significance.

Candidates who worked with the scalar product, using AB = λ(i + 2j), usually produced the correct answer succinctly but those candidates who worked with the position vector of *B* to give AB in the form $(x - 2)$ i + $(y + 2)$ j + $(z + 3)$ k were generally much less successful, often not able to solve the resulting equations.