1. At time $t = 0$, the position vector of a particle P is $-3\mathbf{j}$ m. At time t seconds, the position vector of P is $\mathbf{r}$ metres and the velocity of P is $\mathbf{v}$ m s^{-1}. Given that

$$\mathbf{v} - 2\mathbf{r} = 4e^{t}\,\mathbf{j}\ ,$$

find the time when P passes through the origin.

(Total 7 marks)

2. At time t seconds, the position vector of a particle P is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the vector differential equation

$$\frac{d^2\mathbf{r}}{dt^2} + 4\mathbf{r} = e^{2t}\,\mathbf{j}.$$

When $t = 0$, P has position vector $(\mathbf{i} + \mathbf{j})$ m and velocity $2\mathbf{i}$ m s^{-1}.

Find an expression for $\mathbf{r}$ in terms of t .

(Total 11 marks)

3. The velocity $\mathbf{v}$ m s^{-1} of a particle P at time t seconds satisfies the vector differential equation

$$\frac{d\mathbf{v}}{dt} + 4\mathbf{v} = 0\ .$$

The position vector of P at time t seconds is $\mathbf{r}$ metres.

Given that at $t = 0$, $\mathbf{r} = (\mathbf{i} - \mathbf{j})$ and $\mathbf{v} = (-8\mathbf{i} + 4\mathbf{j})$, find $\mathbf{r}$ at time t seconds.

(Total 7 marks)

4. At time t seconds, the position vector of a particle P is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the differential equation

$$\frac{d^2r}{dt^2} + 3\frac{dr}{dt} = 0.$$

When $t = 0$, the velocity of P is $(8\mathbf{i} - 12\mathbf{j})$ m s^{-1}.

Find the velocity of P when $t = \frac{2}{3}\ln 2$.

(Total 7 marks)

5. A particle P moves in the x-y plane and has position vector $\mathbf{r}$ metres at time t seconds. It is given that $\mathbf{r}$ satisfies the differential equation

$$\frac{d^2\mathbf{r}}{dt^2} = 2\frac{d\mathbf{r}}{dt}$$

When $t = 0$, P is at the point with position vector $3\mathbf{i}$ metres and is moving with velocity $\mathbf{j}$ m s^{-1}.

(a) Find $\mathbf{r}$ in terms of t.

(8)

(b) Describe the path of P, giving its cartesian equation.

(2)
(Total 10 marks)

6. The position vector $\mathbf{r}$ of a particle P at time t satisfies the vector differential equation

$$\frac{d\mathbf{r}}{dt} + 2\mathbf{r} = 4\mathbf{i}$$

Given that the position vector of P at time $t = 0$ is $2\mathbf{j}$, find the position vector of P at time t.

(Total 6 marks)

7. At time t seconds the position vector of a particle P, relative to a fixed origin O, is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + 2\mathbf{r} = 3\mathrm{e}^{-t}\mathbf{j}.$$

Given that $\mathbf{r} = 2\mathbf{i} - \mathbf{j}$ when $t = 0$, find $\mathbf{r}$ in terms of t.

(Total 7 marks)

8. A particle P of mass 2 kg moves in the x-y plane. At time t seconds its position vector is $\mathbf{r}$ metres. When $t = 0$, the position vector of P is $\mathbf{i}$ metres and the velocity of P is $(-\mathbf{i} + \mathbf{j})$ m s^{-1}.

The vector $\mathbf{r}$ satisfies the differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 2\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + 2\mathbf{r} = \mathbf{0}.$$

(a) Find $\mathbf{r}$ in terms of t.

(8)

(b) Show that the speed of P at time t is $\mathrm{e}^{-t}\sqrt{2}$ m s^{-1}.

(5)

(c) Find, in terms of e, the loss of kinetic energy of P in the interval $t = 0$ to $t = 1$.

(2)
(Total 15 marks)

9. With respect to a fixed origin O, the position vector, $\mathbf{r}$ metres, of a particle P at time t seconds satisfies

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \mathbf{r} = (\mathbf{i} - \mathbf{j})\mathrm{e}^{-2t}.$$

Given that P is at O when $t = 0$, find

(a) **r** in terms of *t*,

(7)

(b) a cartesian equation of the path of *P*.

(2)
(Total 9 marks)

10. At time *t* seconds, the position vector **r** metres of a particle *P*, relative to a fixed origin *O*, satisfies the differential equation

$$\frac{d^2\mathbf{r}}{dt^2} + 4\frac{d\mathbf{r}}{dt} + 3\mathbf{r} = \mathbf{0}.$$

At time $t = 0$, *P* is at the point with position vector 2**i** m and is moving with velocity 2**j** m s^{-1}.

Find the position vector of *P* when $t = \ln 2$.

(Total 10 marks)

1.

$\frac{d\mathbf{r}}{dt} - 2\mathbf{r} = 4e^{t}\,\mathbf{j}$

IF = e^{-2t}

$e^{-2t}(\frac{d\mathbf{r}}{dt} - 2\mathbf{r}) = e^{-2t}.4e^{t}\,\mathbf{j}$ M1

$\frac{d(\mathbf{r}e^{-2t})}{dt} = 4e^{-t}\,\mathbf{j}$

$\mathbf{r}e^{-2t} = \int 4e^{-t}\,\mathbf{j}\,dt$ DM1

$= -4e^{-t}\,\mathbf{j}\ (+\ \mathbf{C})$ A1

$t = 0, \mathbf{r} = -3\mathbf{j} \Rightarrow \mathbf{C} = \mathbf{j}$ DM1

$e^{-2t}\mathbf{r} = (1 - 4e^{-t})\mathbf{j}$ or $\mathbf{r} = (e^{2t} - 4e^{t})\mathbf{j}$ A1

$(1 - 4e^{-t}) = 0$ or $(e^{2t} - 4e^{t}) = 0$ DM1

$t = \ln 4$, 1.4 or better A1

[7]

2.

C.F. is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t$ B1

P.I. is $\mathbf{r} = \mathbf{p}e^{2t}$ B1

$\dot{\mathbf{r}} = 2\mathbf{p}e^{2t}$ B1 ft

$\ddot{\mathbf{r}}\ \ 4\mathbf{p}e^{2t}$

$4\mathbf{p}e^{2t} + 4\mathbf{p}e^{2t} = \mathbf{j}e^{2t}$ M1

so, (PI is) $\mathbf{r} = \frac{1}{8}\mathbf{j}e^{2t}$ A1

GS is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$ A1 ft

$t = 0$, $\mathbf{r} = \mathbf{i} + \mathbf{j} \Rightarrow \mathbf{i} + \mathbf{j} = \mathbf{A} + \frac{1}{8}\mathbf{j} \Rightarrow \mathbf{i} + \frac{7}{8}\mathbf{j} = \mathbf{A}$ DM1 A1

$\dot{\mathbf{r}} = -2\mathbf{A}\sin 2t + 2\mathbf{B}\cos 2t + \frac{1}{4}\mathbf{j}e^{2t}$ M1A1

$t = 0$, $\dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow 2\mathbf{i} = 2\mathbf{B} + \frac{1}{4}\mathbf{j} \Rightarrow \mathbf{i} - \frac{1}{8}\mathbf{j} = \mathbf{B}$

$\mathbf{r} = (\mathbf{i} + \frac{7}{8}\mathbf{j})\cos 2t + (\mathbf{i} - \frac{1}{8}\mathbf{j}\ \sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$ A1

[11]

3. Aux Equn: $m^2 + 4m = 0 \Rightarrow m = 0$ or -4 M1

$\mathbf{r} = \mathbf{A} + \mathbf{B}e^{-4t}$

$t = 0, \mathbf{r} = \mathbf{i} - \mathbf{j}$: $\mathbf{A} + \mathbf{B} = \mathbf{i} - \mathbf{j}$ A1

$\mathbf{v} = -4\mathbf{B}e^{-4t}$ M1

$t = 0$, $v = -8\mathbf{i} + 4\mathbf{j}$: $-4\mathbf{B} = -8\mathbf{i} + 4\mathbf{j}$ M1

$\mathbf{B} = 2\mathbf{i} - \mathbf{j} \Rightarrow \mathbf{A} = -\mathbf{i}$

so, $\mathbf{r} = -\mathbf{i} + (2\mathbf{i} - \mathbf{j})e^{-4t}$ A1A1

$= (2e^{-4t} - 1)\mathbf{i} - e^{-4t}\mathbf{j}$ A1 7

[7]

4. $\frac{\mathrm{d}v}{\mathrm{d}t} + 3v = 0$ B1

IF = $\mathrm{e}^{3t} \Rightarrow \frac{\mathrm{d}(\mathrm{ve}^{3t})}{\mathrm{d}t} = 0$ M1

$\Rightarrow \mathrm{ve}^{3t} = \mathbf{A}$ A1

$t = 0$, v = 8**i** – 12**j** ⇒ **v** = (8**i** – 12**j**)e^{-3t} M1A1

$t = \frac{2}{3}\ln 2 \Rightarrow \mathbf{v} = (8\mathbf{i} - 12\mathbf{j})\mathrm{e}^{-2\ln 2} = (2\mathbf{i} - 3\mathbf{j})\ \mathrm{m\ s^{-1}}$ DM1 A1 7

[7]

5.	(a)	$m^2 - 2m = m(m-2) = 0$	M1	
		$\Rightarrow m = 0$ or $m = 2$	A1	
		$\Rightarrow \underline{r} = \underline{A} + \underline{B}e^{2t}$	A1	
		$t = 0, \underline{r} = 3\underline{i} \Rightarrow \underline{A} + \underline{B} = 3\underline{i}$	M1 A1	
		$\dot{\underline{r}} = 2\underline{B}e^{2t}$	M1	
		$t = 0, \dot{\underline{r}} = \underline{j} \Rightarrow \underline{B} = \frac{1}{2}\underline{j}$	A1	
		$\Rightarrow \underline{r} = (3\underline{i} + \frac{1}{2}\underline{j}) + \frac{1}{2}\underline{j}e^{2t} = 3\underline{i} + \frac{1}{2}\underline{j}(e^{2t} - 1)$	A1	8
	(b)	Particle moves in a straight line	B1	
		Equation of line is $x = 3$	B1	2

[10]

6.	Either CF: $\dot{\mathbf{r}} + 2\mathbf{r} = O$		*OR*	1F = e^{2t}	B1
	$\Rightarrow \mathbf{r} = \mathbf{A}e^{-2t}$	M1 A1		$\frac{d}{dt}(\mathbf{r}e^{2t}) = 4\mathbf{i}\, e^{2t}$	M1
	PI $\mathbf{r} = 2\mathbf{i}$	B1		$\mathbf{r}\, e^{2t} = 2\mathbf{i}\, e^{2t} + \mathbf{A}$	A1
	GS $\mathbf{r} = \mathbf{A}e^{-2t} + 2\mathbf{i}$	A1		$\mathbf{r} = 2\mathbf{i} + \mathbf{A}e^{-2t}$	A1
	t = o $\mathbf{r} = 2\mathbf{j} \Rightarrow \mathbf{A} = 2\mathbf{j} - 2\mathbf{i}$				M1

$\mathbf{r} = (2\mathbf{j} - 2\mathbf{i})e^{-2t} + 2\mathbf{i}$ or $2\mathbf{i}(1 - e^{-2t}) + 2\mathbf{j}e^{-2t}$ A1 6

7.	I.F. = $e^{\int 2dt} = e^{2t}$	B1	
	$\Rightarrow e^{2t}\frac{dr}{dt} + 2e^{2t}\mathbf{r} = 3e^{t}\mathbf{j}$	M1	
	$\Rightarrow \frac{d}{dt}(\mathbf{r}e^{2t}) = 3e^{t}\mathbf{j}$		
	$\underline{e^{2t}\mathbf{r} = 3e^{t}\mathbf{j}\ (+C)}$	M1 A1	
	$t = 0$, $\mathbf{r} = 2\mathbf{i} - \mathbf{j} \Rightarrow 2\mathbf{i} - \mathbf{j} = 3\mathbf{j} + C$	M1	
	$2\mathbf{i} - 4\mathbf{j} = C$	A1	
	$\Rightarrow \mathbf{r} = 3e^{-t}\mathbf{j} + (2\mathbf{i} - 4\mathbf{j})e^{-2t}$	A1	7

[7]

8	(a)	Aux equ. $m^2 + 2m + 2 = 0$		
		$\Rightarrow m = -1 \pm \mathbf{i}$	M1 A1	
		G. soln.: $\mathbf{r} = e^{-t}(\mathbf{A}\cos t + \mathbf{B}\sin t)$	A1 ft	
		$t = 0$, $\mathbf{r} = \mathbf{i} \Rightarrow \mathbf{A} = \mathbf{i}$	M1 A1	
		$\dot{\mathbf{r}} = -e^{-t}(\mathbf{A}\cos t + \mathbf{B}\sin t) + e^{-t}(-\mathbf{A}\sin t + \mathbf{B}\cos t)$	M1	
		$t = 0$, $\dot{\mathbf{r}} = (-\mathbf{i} + \mathbf{j}) \Rightarrow -\mathbf{i} + \mathbf{j} = -\mathbf{i} + \mathbf{B} \Rightarrow \mathbf{B} = \mathbf{j}$	M1	
		$\therefore \mathbf{r} = \underline{e^{-t}(\cos t\,\mathbf{i} + \sin t\,\mathbf{j})}$	A1	8
	(b)	$\dot{\mathbf{r}} = -e^{-t}(\cos t\,\mathbf{i} + \sin t\,\mathbf{j}) + e^{-t}(-\sin t\,\mathbf{i} + \cos t\,\mathbf{j})$	M1	
		$= e^{-t}\{-(\cos t + \sin t)\mathbf{i} + (\cos t - \sin t)\mathbf{j}\}$		
		Speed $= e^{-t}\sqrt{\{(\cos t + \sin t)^2 + (\cos t - \sin t)^2\}}$	M1 A1	
		$= e^{-t}\sqrt{\{1 + 2\cos t\sin t + 1 - 2\cos t\sin t\}}$	M1	
		$= \underline{e^{-t}\sqrt{2}}$ (*)	A1	5
	(c)	Loss of KE $= \frac{1}{2} \times 2 \times 2(1 - e^{-2})$	M1	
		$= \underline{2(1 - \frac{1}{e^2})}$ ($\approx$ 1.73 (AWRT))	A1	2

[15]

9.	(a)	*Integrating factor approach:*		
		I F $= e^{\int 1 dt} = e^t$	B1	
		Multiplying through $\Rightarrow \frac{d}{dt}(\mathbf{r}\,e^t) = (\mathbf{i} - \mathbf{j})\,e^{-t}$	M1 A1	
		Integrating $\Rightarrow \mathbf{r}\,e^t = -(\mathbf{i} - \mathbf{j})\,e^{-t}$ (+ $\mathbf{c}$)	M1 A1 ft	
		Using $\mathbf{r} = \mathbf{0}$, $t = 0$ to find $\mathbf{c}$ [$\mathbf{c} = \mathbf{i} - \mathbf{j}$]	M1	
		$\Rightarrow \mathbf{r} = -(\mathbf{i} - \mathbf{j})e^{-2t} + (\mathbf{i} - \mathbf{j})\,e^{-t}$ any form	A1	7
	Alternative:			
	(a)	AE $m + 1 = 0 \Rightarrow \mathbf{r} = \mathbf{A}\,e^{-t}$ [Form of PI: $\mathbf{r} = \mathbf{B}\,e^{-2t}$]	B1	
		Equation for PI: $-2\,e^{-2t}\,\mathbf{B} + \mathbf{B}\,e^{-2t} = (\mathbf{i} - \mathbf{j})e^{-2}\ \mathbf{B} = -(\mathbf{i} - \mathbf{j})$	M1 A1, A1 ft	
		General Solution: $\mathbf{r} = \mathbf{A}e^{-t} + (-\mathbf{i} + \mathbf{j})\,e^{-2t}$	M1	
		Using $\mathbf{r} = \mathbf{0}$, $t = 0$ to find $\mathbf{A}$	M1	
		$\mathbf{r} = (\mathbf{i} - \mathbf{j})e^{-t} + (-\mathbf{i} + \mathbf{j})e^{-2t}$	A1	7
	(b)	Writing r = f(t)$\mathbf{i}$ + g(t)$\mathbf{j}$ or x = f(t), y = g(t) and attempt to eliminate t	M1	
		$y = -x$ A1	2	

[9]

10.	Auxiliary equation $m^2 + 4m + 3 = 0 \Rightarrow m = -1$ or -3	M1	
	$\Rightarrow \mathbf{r} = \mathbf{A}e^{-t} + \mathbf{B}e^{-3t}$	A1	
	$t = 0,\ \dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow \mathbf{A} + \mathbf{B} = 2\mathbf{i}$	M1 A1	
	$t = 0,\ \dot{\mathbf{r}} = 2\mathbf{j} \Rightarrow -\mathbf{A} - 3\mathbf{B} = 2\mathbf{j}$		
	Solving: $-2\mathbf{B} = 2\mathbf{i} + 2\mathbf{j}$		
	$\Rightarrow \mathbf{B} = -(\mathbf{i} + \mathbf{j})$ $\mathbf{A} = (3\mathbf{i} + \mathbf{j})$	M1 A1	
	$\mathbf{r} = (3\mathbf{i} + \mathbf{j})e^{-t} - (\mathbf{i} + \mathbf{j})e^{-3t}$		
	$t = \ln 2 \Rightarrow e^{-t} = \frac{1}{2}$, $e^{-3t} = \frac{1}{8}$	B1	
	$\mathbf{r} = \frac{1}{2}(3\mathbf{i} + \mathbf{j}) - \frac{1}{8}(\mathbf{i} + \mathbf{j}) = \frac{11}{8}\mathbf{i} + \frac{3}{8}\mathbf{j}$	M1 A1	10

[10]

1. This question was very well answered. Most candidates scored full marks. Those who lost marks generally had no idea how to start and failed to use a correct method in their attempt to solve the differential equation. Just occasionally candidates attempted to use the initial conditions before attempting to solve the differential equation. There were very few algebraic errors and almost all remembered to include the constant of integration.

2. This question was a good source of marks for many candidates. There were two common errors: finding the values of the arbitrary constants before adding in the particular integral to obtain the complete general solution; calculating the value of the constant for the particular integral as $\frac{1}{8}\mathbf{j}$ but then forgetting to put the e^{2t} back in. Some candidates unnecessarily doubled their workload by considering separate differential equations for the **i** and **j** components.

3. There were three approaches used to tackle this question: treating it as a second order differential equation in **r**, which was the neatest and safest way, working in **v** and multiplying by an integrating factor, which involved solving two differential equations, or lastly, separating the variables and again solving two differential equations. Since the last approach involved dividing by a vector, no marks were awarded for the first stage of this process. Some candidates tried to get round this obstacle by replacing the vector differential equation by two scalar differential equations. These candidates sometimes came to grief if they tried to find the values of constants at too early a stage, as they were faced with ln(–8).

4. There were many correct solutions to this question, with the majority of candidates treating it as a second order equation in **r** rather than a first order equation in **v**. A few candidates treated it as a separable equation, dividing by the vector **v**, which was not considered to be an acceptable method. (Interestingly, if you find the value of **r** instead of **v** you get the same final answer) Many candidates insisted on using components which makes the problem much more difficult and much longer and, moreover, is not necessary.

5. Many candidates found part (a) a source of easy marks. The majority used the standard auxiliary equation method for solving a second order differential equation and some of those wasted a lot of time and energy using components. Others successfully started from a first order equation in **v** and some integrated both sides with respect to time first. A few candidates (mostly overseas) tried to divide a vector by a vector, which was not accepted. In the second part many candidates gave the equation $x = 3$ but many did not go on to give any description implying that the particle moved in a straight line.

6. Most could make good attempts at this question. All the general principles were well known with most gaining either full marks or making tiny slips in accuracy. All succeeded in working with vectors throughout in an appropriate way, which was very encouraging to see.

7. Candidates using an integrating factor were usually successful and many gained full marks. Two other methods were seen – splitting the differential equation into two components and using a complementary function and particular integral. Correct solutions were sometimes seen but many of these candidates failed to use the initial conditions properly leading to incorrect constants.

8. Part (a) cause few problems for most candidates and most could solve the vector differential equation; some however made life longer and more complicated for themselves by splitting the problem up into separate equations for each component and solving each one separately. Solutions in part (b) were not always so accurate, with many failing to collect the relevant components in i and j before squaring and adding to find the speed. Also a number of candidates failed to justify their working in omitting cross-product terms when expanding squared brackets.

9. This was a well answered question by the majority of candidates. In part (a) the most common approach was to use the integrating factor and most candidates correctly found this to be e^t. Errors tended to be either leaving the right hand side as (i – j) e^{-2t}, or making a mistake in dividing by e^t in the final stage to give r. Candidates who found the complementary function and the particular integral were less frequent but they were often successful; the main error here tended to be omission of a constant in the complementary function.

In part (c) the majority of candidates knew what to do although some spent considerable time and space in producing an equation which still contained *t*.

10. No Report available for this question.