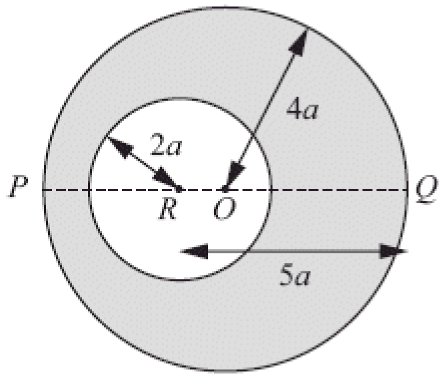
1.



A uniform circular disc has mass $4m$, centre O and radius $4a$. The line POQ is a diameter of the disc. A circular hole of radius $2a$ is made in the disc with the centre of the hole at the point R on PQ where $QR = 5a$, as shown in the diagram above.

The resulting lamina is free to rotate about a fixed smooth horizontal axis L which passes through Q and is perpendicular to the plane of the lamina.

(a) Show that the moment of inertia of the lamina about L is $69ma^2$.

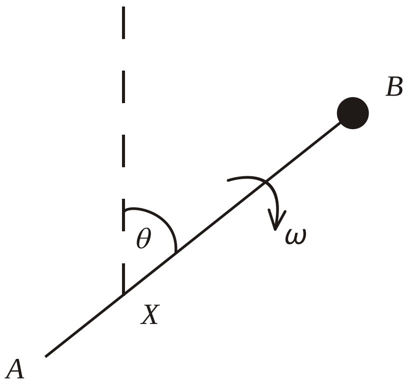
(7)

The lamina is hanging at rest with P vertically below Q when it is given an angular velocity Ω. Given that the lamina turns through an angle $\frac{2\pi}{3}$ before it first comes to instantaneous rest,

(b) find Ω in terms of g and a.

(6)
(Total 13 marks)

2.



A uniform rod AB has mass $3m$ and length $2a$. It is free to rotate in a vertical plane about a smooth fixed horizontal axis through the point X on the rod, where $AX = \frac{1}{2}a$. A particle of mass m is attached to the rod at B. At time $t = 0$, the rod is vertical, with B above A, and is given an initial angular speed $\sqrt{\frac{g}{a}}$. When the rod makes an angle θ with the upward vertical, the angular speed of the rod is ω, as shown in the diagram above.

(a) By using the principle of the conservation of energy, show that

$$\omega^2 = \frac{g}{2a}(5 - 3\cos\theta)$$

(8)

(b) Find the angular acceleration of the rod when it makes an angle θ with the upward vertical.

(3)

When $\theta = \phi$, the resultant force of the axis on the rod is in a direction perpendicular to the rod.

(c) Find $\cos\phi$.

(5)
(Total 16 marks)

3. Four uniform rods, each of mass m and length $2a$, are joined together at their ends to form a plane rigid square framework $ABCD$ of side $2a$. The framework is free to rotate in a vertical plane about a fixed smooth horizontal axis through A. The axis is perpendicular to the plane of the framework.

(a) Show that the moment of inertia of the framework about the axis is $\frac{40ma^2}{3}$.

(5)

The framework is slightly disturbed from rest when C is vertically above A. Find

(b) the angular acceleration of the framework when AC is horizontal,

(3)

(c) the angular speed of the framework when AC is horizontal,

(3)

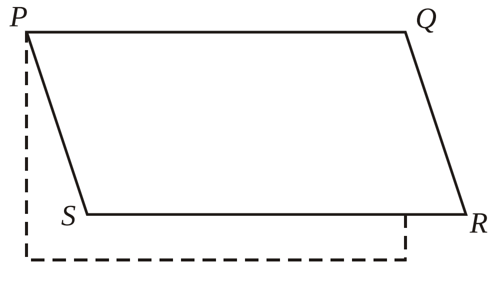
(d) the magnitude of the force acting on the framework at A when AC is horizontal.

(6)
(Total 17 marks)

4. A uniform lamina of mass m is in the shape of a rectangle $PQRS$, where $PQ = 8a$ and $QR = 6a$.

(a) Find the moment of inertia of the lamina about the edge PQ.

(2)



The flap on a letterbox is modelled as such a lamina. The flap is free to rotate about an axis along its horizontal edge *PQ*, as shown in the diagram above. The flap is released from rest in a horizontal position. It then swings down into a vertical position.

(b) Show that the angular speed of the flap as it reaches the vertical position is $\sqrt{\left(\frac{g}{2a}\right)}$.

(3)

(c) Find the magnitude of the vertical component of the resultant force of the axis *PQ* on the flap, as it reaches the vertical position.

(4)
(Total 7 marks)

5. A uniform rod *AB*, of mass *m* and length 2*a*, is free to rotate in a vertical plane about a fixed smooth horizontal axis through *A*. The rod is hanging in equilibrium with *B* below *A* when it is hit by a particle of mass *m* moving horizontally with speed *v* in a vertical plane perpendicular to the axis. The particle strikes the rod at *B* and immediately adheres to it.

(a) Show that the angular speed of the rod immediately after the impact is $\frac{3v}{8a}$.

(5)

Given that the rod rotates through 120° before first coming to instantaneous rest,

(b) find *v* in terms of *a* and *g*.

(6)

(c) find, in terms of *m* and *g*, the magnitude of the vertical component of the force acting on the rod at *A* immediately after the impact.

(5)
(Total 16 marks)

(3)

(c) the angular speed of the framework when *AC* is horizontal,

(3)

(d) the magnitude of the force acting on the framework at *A* when *AC* is horizontal.

(6)
(Total 17 marks)

6. A uniform sphere, of mass m and radius a, is free to rotate about a smooth fixed horizontal axis L which forms a tangent to the sphere. The sphere is hanging in equilibrium below the axis when it receives an impulse, causing it to rotate about L with an initial angular velocity of $\sqrt{\frac{18g}{7a}}$.

Show that, when the sphere has turned through an angle θ,

(a) the angular speed ω of the sphere is given by $\omega^2 = \frac{2g}{7a}(4 + 5\cos\theta)$,

(5)

(b) the angular acceleration of the sphere has magnitude $\frac{5g}{7a}\sin\theta$.

(2)

(c) Hence find the magnitude of the force exerted by the axis on the sphere when the sphere comes to instantaneous rest for the first time.

(10)
(Total 17 marks)

1. (a)

Mass of disc removed = m	B1	
$\frac{1}{2}4m(4a)^2 + 4m(4a)^2$	M1 A1	
$\frac{1}{2}m(2a)^2 + m(5a)^2$	M1 A1	
$I = \frac{1}{2}4m(4a)^2 + 4m(4a)^2 - (\frac{1}{2}m(2a)^2 + m(5a)^2)$	DM1	
$= 69ma^2$ *	A1	7

(b)

$4m.0 = 3m\bar{x} - ma$	M1	
$\bar{x} = \frac{1}{3}a$ (from O)	A1	
$\frac{1}{2}69ma^2\Omega^2 = 3mg(4a - \frac{1}{3}a)(1 - \cos\frac{2\pi}{3})$	M1 A2	
$\Omega = \sqrt{\frac{11g}{23a}}$	A1	6

[13]

2. (a)

MI of rod + particle		
$= \frac{1}{12}3m(2a)^2 + 3m(\frac{1}{2}a)^2, + m(\frac{3}{2}a)^2$	M1, M1	
$= 4ma^2$	A1	
$\frac{1}{2}4ma^2(\omega^2 - \frac{g}{a}) = 3mg\frac{a}{2}(1-\cos\theta) + mg\frac{3a}{2}(1-\cos\theta)$	M1AftA1	
$\omega^2 = \frac{g}{2a}(5 - 3\cos\theta)$	DM1A1	8

(b)

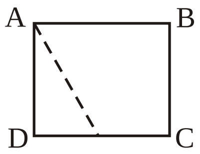
$4ma^2\ddot{\theta} = 3mg\frac{1}{2}a\sin\theta + mg\frac{3}{2}a\sin\theta$	M1A1ft	
$\Rightarrow \ddot{\theta} = \frac{3g\sin\theta}{4a}$	A1	3

(c)

$F + 4mg\cos\theta = 3m\frac{1}{2}a\omega^2 + m\frac{3}{2}a\omega^2$	M1A1	
$= 3ma\frac{g}{2a}(5 - 3\cos\theta)$	DM1	
$\Rightarrow F = \frac{mg}{2}(15 - 17\cos\theta)$	DM1	
When $\theta = \phi$, $F = 0$		
$\Rightarrow \cos\phi = \frac{15}{17}$	A1	5

[16]

3. (a)



$I_{AB} = 2 \times \frac{4}{3}ma^2 + m(2a)^2 = \frac{20}{3}ma^2$ M1 A1 A1

(BC, AD) (CD)

By $\perp$r axis: $I_A = 2 \times \frac{20}{3}ma^2 = \underline{\frac{40}{3}ma^2}$ M1 A1 5

[or $I_A = 2 \times \frac{4}{3}ma^2, +2(\frac{1}{3}ma^2 + 5ma^2)$ M1, M1 A1 A1

(AB, AD) (BC, CD)

$= \underline{\frac{40}{3}ma^2}$ A1

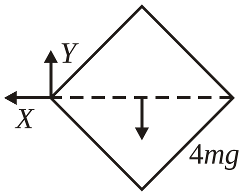
(b) M(A) $\frac{40ma^2}{3}\ddot{\theta} = 4mg.a\sqrt{2}$ M1 A1

$\ddot{\theta} = \underline{\frac{3g\sqrt{2}}{10a}}$ A1 3

(c) $\frac{1}{2}.\frac{40ma^2}{3}\dot{\theta}^2 = 4mg.a\sqrt{2}$ M1 A1

$\dot{\theta} = \sqrt{\frac{3g\sqrt{2}}{5a}}$ A1 3

(d)



R(←) $X = 4ma\sqrt{2}\dot{\theta}^2$ M1

$= 4ma\ \sqrt{2}.\frac{3g\sqrt{2}}{5a} = \frac{24mg}{5}$ A1

R(↑) $4mg - Y = 4ma\sqrt{2}\ddot{\theta}$ M1

$Y = 4mg - 4ma\sqrt{2}.\frac{3g\sqrt{2}}{10a}$

$Y = \frac{8mg}{5}$ A1

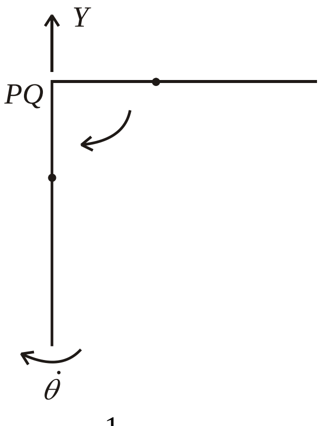
$R = \sqrt{(X^2 + Y^2)} = \frac{8mg}{5}\sqrt{(1^2 + 3^2)}$ M1

$= \underline{\frac{8\sqrt{10}}{5}mg}$ A1 6

[17]

4. (a) $I_{PQ} = \frac{4}{3}m\,(3a)^2 = \underline{12\ ma^2}$ M1 A1 2

(b)



Energy: $\frac{1}{2} \times 12\ ma^2 \times \dot{\theta}^2 = mg \times 3a$ M1 A1 ft

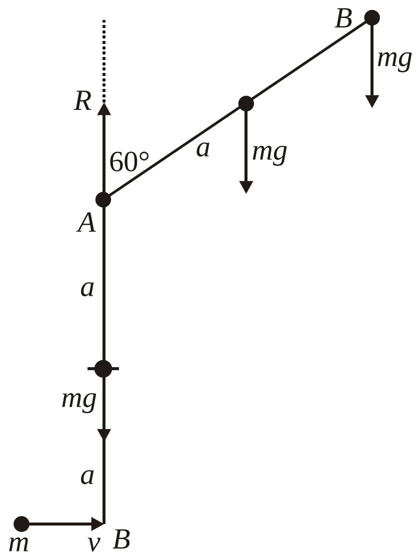
$\Rightarrow \underline{\dot{\theta} = \sqrt{\left(\frac{g}{2a}\right)}}$ (*) A1 3

(c) $R(\uparrow)$: $Y - mg = m \times 3a\dot{\theta}^2$ M1 A1

$Y = \text{mg} + \text{m} \times 3\text{a} \times \dfrac{g}{2a} = \underline{\dfrac{5}{2}mg}$ M1 A1 4

[9]

5. (a)



$I_A = \{\frac{4}{3}ma^2 + m(2a)^2\}$ M1 A1

$mv\,(2a) = I_A\omega = \dfrac{16ma^2}{3}\omega$ M1 A1 ft

$\omega = \dfrac{3v}{8a}$ (*) no wrong working seen A1 cso 5

Gain in PE = $mg\,3a(1 + \cos 60°)$ M1 A1

Attempt at $\frac{1}{2}I\,\omega^2$ = gain in PE M1

$\dfrac{1}{2}\left(\dfrac{16ma^2}{3}\right)_c\left(\dfrac{3v}{8a}\right)^2 = mg\,3a(1 + \cos 60°)$ A1 ft

Finding v $v = \sqrt{12ga}$ M1 A1 6

(c) Acceleration of C of G = $(\frac{3}{2}a\omega^2)$ B1

$R - 2mg$ = "$mr\omega^2$"; $= 2m(\frac{3}{2}a\omega^2)$ M1 A1

Substitution of ω (and v) and finding $R = \ldots$ M1

$R = \dfrac{113}{16}mg$ A1 5

[16]

6. (a) MI of sphere about L = $\frac{2}{5}ma^2 + ma^2 = \frac{7}{5}ma^2$ B1

Energy: $\frac{1}{2}\times\frac{7}{5}ma^2 \times \frac{18g}{7a} - \frac{1}{2}\times\frac{7}{5}ma^2 \times \omega^2 = mga(1 - \cos\theta)$ M1 A2, 1, 0

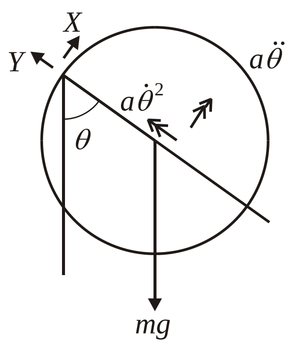
$\Rightarrow \frac{7}{10}a\omega^2 = \frac{8}{10}g + g\cos\theta$

$\omega^2 = \frac{g}{7a}(8 + 10\cos\theta) = \frac{2g}{7a}(4 + 5\cos\theta)$ A1 5

(b) $\frac{7}{5}ma^2\ddot{\theta} = -mga\sin\theta \Rightarrow \ddot{\theta} = -\frac{5g}{7a}\sin\theta$ M1 A1 2

[or $2\omega\dot{\omega} = -\frac{10g}{7a}\sin\theta\times\omega \Rightarrow \dot{\omega} = -\frac{5g}{7a}\sin\theta$]

(c)



$\dot{\theta} = 0$ when $\cos\theta = -\frac{4}{5}$ B1

$Y - mg\cos\theta = ma\dot{\theta}^2$ M1

$\dot{\theta} = 0, \cos\theta = -\frac{4}{5} \Rightarrow Y = -\frac{4mg}{5}$ M1 A1

$X - mg\sin\theta = ma\ddot{\theta}$ M1

$\dot{\theta} = 0$ and $\cos\theta = -\frac{4}{5} \Rightarrow \sin\theta = \frac{3}{5}$ M1

$\Rightarrow \ddot{\theta} = -\dfrac{3g}{7a}$

$\Rightarrow X = \dfrac{3mg}{5} - \dfrac{3mg}{7} = \dfrac{6mg}{35}$ A1

Magnitude of force = $\sqrt{(X^2 + Y^2)}$ M1

$= \text{mg}\left[\left(\dfrac{6}{35}\right)^2 + \left(\dfrac{4}{5}\right)^2\right]^{\frac{1}{2}}$

$\approx 0.818\, mg$ A1 10

[17]

1. In part (a) many correct solutions were seen. Errors arose in the use of the parallel axes rule. Despite distances being quite clearly given and marked on the diagram some candidates attempted to use 3*a*, perhaps misreading the question. A few failed to use the parallel axes rule at all. It was surprising to see such basic errors in what was a fairly standard problem involving a change of axis for a moment of inertia.

In part (b) most candidates realised that an energy equation was required. A surprisingly large number, however, failed to realise that the position of the centre of mass for the lamina was required in the calculation of Potential Energy. Another common error was to use 4*mg* rather than 3*mg* for the weight of the lamina. It was surprising to see a number misquoting the expression for Kinetic Energy – sometimes omitting the $\frac{1}{2}$ or failing to square the Ω.

2. The moment of inertia of the system was generally well done. The energy equation often led to the correct result, although sometimes after a few practice goes. Some candidates missed out the change in PE of either the particle or the rod.
Part (b) was usually successful apart from amongst those candidates who seemed to have little knowledge of rotational motion.
Many candidates made a sensible attempt at part (c), but made slips with the numbers. Some found the centre of mass of the rod and particle while doing this part. Common slips included putting an "*a*" in with the component of the weight or just calling the mass on each side "*m*", which was fine for those candidates using the centre of mass. Some candidates, however, thought that they needed to consider the tangential acceleration.

3. Most could find the moment of inertia in part (a) correctly. However, there were many mistakes in parts (b) and (c). Many failed to consider an equation of rotational motion in part (b); and although most realised that they could do part (c) easiest by considering energy, there were a number of mistakes in dealing with the distances involved. Part (d) was generally approached correctly, but often incorrect previous answers led to an incorrect answer here as well.

4. This proved to be a very straightforward question for the vast majority of candidates and full marks here were regularly obtained. It was very pleasing to note the clear understanding shown of the general principles involved.

5. Part (a) was generally well answered although, as usual, the given answer sometimes emerged from incorrect work. Many good candidates gained full marks in part (b) but there were two fairly common errors: (i) omitting a term in the potential energy element and, more importantly, (ii) using $\frac{1}{2}mv^2$ instead of $\frac{1}{2}I\omega^2$ for the kinetic energy element.

The final part proved a discriminator even among the better candidates; there was much confused thinking, with a correct equation of motion being very rare.

6. No Report available for this question.