1. A uniform circular disc has mass m, centre O and radius $2a$. It is free to rotate about a fixed smooth horizontal axis L which lies in the same plane as the disc and which is tangential to the disc at the point A. The disc is hanging at rest in equilibrium with O vertically below A when it is struck at O by a particle of mass m. Immediately before the impact the particle is moving perpendicular to the plane of the disc with speed $3\sqrt{(ag)}$. The particle adheres to the disc at O.

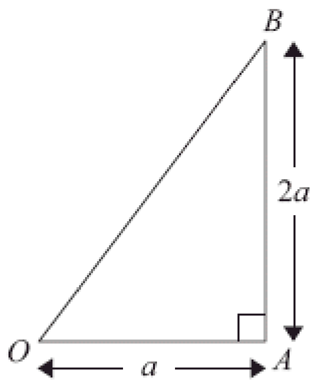
(a) Find the angular speed of the disc immediately after the impact.

(5)

(b) Find the magnitude of the force exerted on the disc by the axis immediately after the impact.

(6)
(Total 11 marks)

2.



A uniform lamina of mass M is in the shape of a right-angled triangle OAB. The angle OAB is 90°, $OA = a$ and $AB = 2a$, as shown in the diagram above.

(a) Prove, using integration, that the moment of inertia of the lamina OAB about the edge OA is $\frac{2}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through one end and perpendicular to the rod is $\frac{4}{3}ml^2$.)

(6)

The lamina *OAB* is free to rotate about a fixed smooth horizontal axis along the edge *OA* and hangs at rest with *B* vertically below *A*. The lamina is then given a horizontal impulse of magnitude *J*. The impulse is applied to the lamina at the point *B*, in a direction which is perpendicular to the plane of the lamina. Given that the lamina first comes to instantaneous rest after rotating through an angle of 120°,

(b) find an expression for *J*, in terms of *M*, *a* and *g*.

(7)
(Total 13 marks)

3. Particles *P* and *Q* have mass 3*m* and *m* respectively. Particle *P* is attached to one end of a light inextensible string and *Q* is attached to the other end. The string passes over a circular pulley which can freely rotate in a vertical plane about a fixed horizontal axis through its centre *O*. The pulley is modelled as a uniform circular disc of mass 2*m* and radius *a*. The pulley is sufficiently rough to prevent the string slipping. The system is at rest with the string taut. A third particle *R* of mass *m* falls freely under gravity from rest for a distance *a* before striking and adhering to *Q*. Immediately before *R* strikes *Q*, particles *P* and *Q* are at rest with the string taut.

(a) Show that, immediately after *R* strikes *Q*, the angular speed of the pulley is $\frac{1}{3}\sqrt{\frac{g}{2a}}$.

(5)

When *R* strikes *Q*, there is an impulse in the string attached to *Q*.

(b) Find the magnitude of this impulse.

(3)

Given that *P* does not hit the pulley,

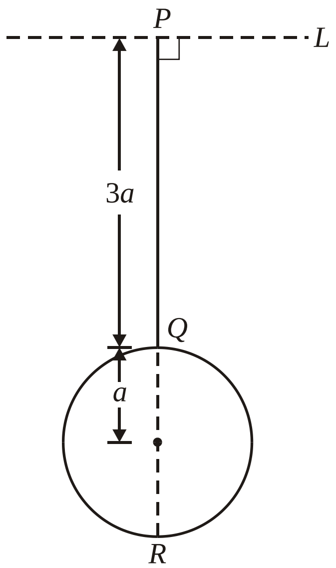
(c) find the distance that *P* moves upwards before first coming to instantaneous rest.

(6)
(Total 14 marks)

4. A uniform circular disc has mass m and radius a. The disc can rotate freely about an axis that is in the same plane as the disc and tangential to the disc at a point A on its circumference. The disc hangs at rest in equilibrium with its centre O vertically below A. A particle P of mass m is moving horizontally and perpendicular to the disc with speed $\sqrt{(kga)}$, where k is a constant. The particle then strikes the disc at O and adheres to it at O. Given that the disc rotates through an angle of 90° before first coming to instantaneous rest, find the value of k.

(Total 10 marks)

5.



A thin uniform rod PQ has mass m and length $3a$. A thin uniform circular disc, of mass m and radius a, is attached to the rod at Q in such a way that the rod and the diameter QR of the disc are in a straight line with $PR = 5a$. The rod together with the disc form a composite body, as shown in the diagram. The body is free to rotate about a fixed smooth horizontal axis L through P, perpendicular to PQ and in the plane of the disc.

(a) Show that the moment of inertia of the body about L is $\dfrac{77ma^2}{4}$.

(7)

When PR is vertical, the body has angular speed ω and the centre of the disc strikes a stationary particle of mass $\frac{1}{2}m$. Given that the particle adheres to the centre of the disc,

(b) find, in terms of ω, the angular speed of the body immediately after the impact.

(4)

(Total 11 marks)

6. A uniform rod AB, of mass m and length $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis through A. The rod is hanging in equilibrium with B below A when it is hit by a particle of mass m moving horizontally with speed v in a vertical plane perpendicular to the axis. The particle strikes the rod at B and immediately adheres to it.

(a) Show that the angular speed of the rod immediately after the impact is $\frac{3v}{8a}$.

(5)

Given that the rod rotates through 120° before first coming to instantaneous rest,

(b) find v in terms of a and g.

(6)

(c) find, in terms of m and g, the magnitude of the vertical component of the force acting on the rod at A immediately after the impact.

(5)
(Total 16 marks)

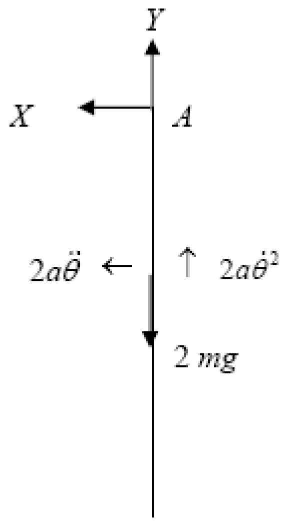
7. A rod AB has mass m and length $4a$. It is free to rotate about a fixed smooth horizontal axis through the point O of the rod, where $AO = a$. The rod is hanging in equilibrium with B below O when it is struck by a particle P, of mass $3m$, moving horizontally with speed v. When P strikes the rod, it adheres to it. Immediately after striking the rod, P has speed $\frac{2}{3}v$.

Find the distance from O of the point where P strikes the rod.

(Total 7 marks)

1.	(a)	MI of disc about $L = \frac{1}{4}m(2a)^2 + m(2a)^2 = 5ma^2$	M1 A1	
		CAM: $m3\sqrt{ag}.2a = (5ma^2 + m(2a)^2)\omega$	M1 A1 ft	
		$\omega = \frac{2}{3}\sqrt{\frac{g}{a}}$	A1	5

(b)



$M(A),\ 0 = I\ddot{\theta}$	B1	
$\ddot{\theta} = 0$		
$R(\leftarrow), X = 2m2a\ddot{\theta} = 0$	B1	
$R(\uparrow), Y - 2mg = 2m2a\dot{\theta}^2$	M1 A1	
$Y = 2mg + 4ma\frac{4g}{9a}$	DM1	
$= \frac{34mg}{9}$	A1	6

[11]

2.	(a)	$\delta m = \frac{2Mx\delta x}{a^2}$	M1 A1
		$\delta I = \frac{1}{3}\frac{2Mx\delta x}{a^2}(2x)^2$	M1 A1
		$I = \int_0^a \frac{8Mx^3 \mathrm{d}x}{3a^2}$	DM1
		$= \frac{8M}{3a^2}\left[\frac{x^4}{4}\right]_0^a$	

$= \frac{2}{3}Ma^2$ * A1 6

(b) $J.2a = \frac{2}{3}Ma^2\omega$ M1 A1

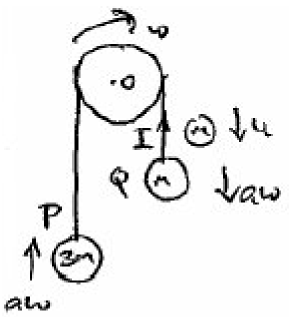
$\frac{1}{2}\frac{2}{3}Ma^2\omega^2 = Mg\frac{2a}{3}(1 + \cos 60°)$ M1 A2

solving for *J* DM1

$J = M\sqrt{\frac{ag}{3}}$ A1 7

[13]

3. (a) $u = \sqrt{2ag}$ B1



<u>CAM about 0</u>:

$m\sqrt{2ag}\,a = 2ma^2\omega + 3ma^2\omega + \frac{1}{2}2ma^2\omega$ M1 A2

$\frac{\sqrt{2ag}}{6a} = \omega$

$\frac{1}{3}\sqrt{\frac{g}{2a}} = w$ A1 5

(b) For Q:

$-\mathrm{I} = 2maw - mu$ M1 A1

$\Rightarrow \mathrm{I} = 6maw - 2maw = 4maw$

$= \frac{4ma}{3}\sqrt{\frac{g}{2a}} \;\; = \frac{m}{3}\sqrt{8ag}$ A1 3

(c) PE gain of P = KE loss of P + KE loss of Q + KE loss of pulley + PE loss of Q

$$3mgd = \frac{1}{2}3ma^2w^2 + \frac{1}{2}2ma^2w^2 + \frac{1}{2}ma^2w^2 + 2mgd$$ M1 A3

$$\not{m}gd = 3\not{m}a^2w^2$$

$$gd = 3a^2 . \frac{1}{9}\frac{g}{2a} \qquad = \frac{a}{6}$$ M1 A1 6

[14]

4. MI of disc abt diameter = $\frac{1}{4}ma^2$ M1

$\therefore$ MI of disc about axis = $\frac{1}{4}ma^2 + ma^2 = \frac{5}{4}ma^2$ M1 A1

CAM: $m.a.\sqrt{kga} = (\frac{5}{4}ma^2 + ma^2)\omega$ M1 A1

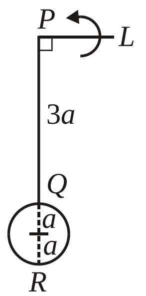
$\Rightarrow \omega = \frac{4}{9}\sqrt{\frac{kg}{a}}$ A1

Energy : $\frac{1}{2} \;.\; \frac{9ma^2}{4} \;.\; \frac{16\,kg}{81a} = 2mga$ M1 A1 A1

$\underline{\Rightarrow k = 9}$ A1

[10]

5. (a)



DISC: $I_{\text{diam}} = \frac{1}{2}\left(\frac{1}{2}ma^2\right) = \frac{1}{4}ma^2$ M1 A1

$I_L = \frac{1}{4}ma^2 + m(4a)^2$ M1 A1

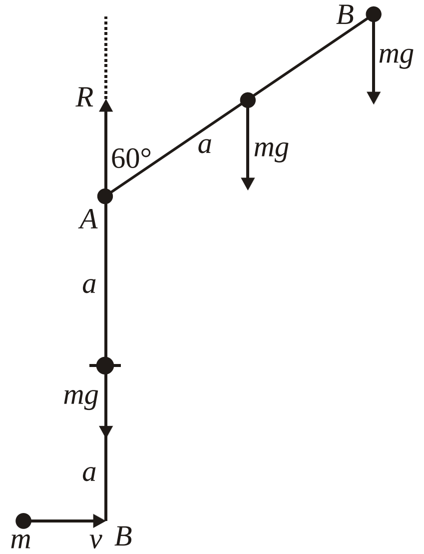
$= \frac{65}{4}ma^2$

ROD: $I_L = \frac{4}{3}\left(\frac{3a}{2}\right)^2 = 3ma^2$ B1

	$I_{\text{TOTAL}} = \frac{65}{4}ma^2 + 3ma^2 = \underline{\frac{77}{4}ma^2}$ (*)	M1 A1	7
(b)	CAM: $\frac{77}{4}ma^2w = \left[\frac{77}{4}ma^2 + \frac{1}{2}m(4a)^2\right]w'$	M1 A1 A1	
	$\Rightarrow \underline{w' = \frac{77}{109}w}$	A1	4

[11]

6. (a)

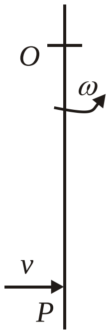


$I_A = \{\frac{4}{3}ma^2 + m(2a)^2\}$	M1 A1	
$mv(2a) = I_A\omega = \frac{16ma^2}{3}\omega$	M1 A1 ft	
$\omega = \frac{3v}{8a}$ (*) no wrong working seen	A1 cso	5
Gain in PE = $mg\,3a(1 + \cos 60°)$	M1 A1	
Attempt at $\frac{1}{2}I\omega^2$ = gain in PE	M1	
$\frac{1}{2}\left(\frac{16ma^2}{3}\right)_c\left(\frac{3v}{8a}\right)^2 = mg\,3a(1 + \cos 60°)$	A1 ft	
Finding v $\quad v = \sqrt{12ga}$	M1 A1	6

(c)	Acceleration of C of G = $(\frac{3}{2}a\omega^2)$	B1	
	$R - 2mg$ = "$mr\omega^2$"; $= 2m(\frac{3}{2}a\omega^2)$	M1 A1	
	Substitution of ω (and v) and finding $R = \ldots$	M1	
	$R = \frac{113}{16}mg$	A1	5

[16]

7.



MI of rod about O is $\frac{1}{3}m(2a)^2 + ma^2 = \frac{7}{3}ma^2$	B1	
Moment of momentum:		
$3mv \times x = (\frac{7}{3}ma^2 + mx^2)\omega$	M1 A1 A1 ft	
$x\omega = \frac{2}{3}v$	M1	
$\Rightarrow 3m \times \frac{3x\omega}{2} \times x = (\frac{7}{3}ma^2 + mx^2)\omega$		
$\Rightarrow \frac{7}{2}x^2 = \frac{7}{3}a^2 \Rightarrow x = a\sqrt{\frac{2}{3}}$	M1 A1	7

[7]

1. A fairly high number of good attempts to this question were seen.

In part (a) most candidates used the correct moment of inertia for an axis tangential to the disc and some did then use the perpendicular axes rule followed by the parallel axes rule. Only a few, erroneously, used an axis perpendicular to the disc. Most of the errors in this part then resulted from trying to use energy conservation for an inelastic impact where energy was *not* conserved. Conservation of angular momentum was required here and those who used this were generally successful in scoring most of the available marks.

Very few completely correct solutions were seen to part (b). Some attempted to use energy conservation again. Of those who correctly realised that equations of motion were required, there were very many incomplete solutions with only a very small number correctly justifying the fact that the horizontal component of the force required was zero. Of those who correctly followed through a vertical equation of motion, quite a number failed to include the mass of the particle to make the total mass $2m$.

2. The simplest way to split the lamina into strips for part (a) was to use the hint in the question and have the strips parallel to *AB*. Those who used strips parallel to *OA* made the question more complicated and more difficult but were, nevertheless, often successful.

The commonest errors in the second part were either to get the position of the centre of mass wrong or to forget to multiply J by $2a$ when applying the rotational impulse-momentum principle.

3. Many candidates tried to use conservation of energy (or the equivalent route using Newton's second law etc) in part (a) in spite of the fact that a collision had occurred. Even those who used conservation of angular momentum often missed out a term and so had to do some interesting fudges to get the printed answer. In part (b) most candidates did not realise that the linear momentum of both *R* and *Q* had to be considered and there were very few correct solutions seen. In the third part most realised that an energy method gave the easiest solution but did not always include all of the particles and the pulley. Some tried to write down equations of motion for the particles and the pulley to find an acceleration. Few however successfully reached the end of this long method.

4. Able candidates scored well here, though the question was found to be more challenging for others, with a number failing to realise that they would need both an energy equation (for the motion after the impact) and a momentum equation (for the impact itself). A number thought that they would only need the energy equation. Some arithmetical slips also occurred in the processing of the equations.

5. This proved to be the most straightforward question on the paper with many fully correct solutions. The majority found the moment of inertia of the body correctly – the error for the minority was to use the moment of inertia of the disc about an axis through the centre rather than about the diameter. In part (b), a few candidates thought that energy was conserved, rather than angular momentum.

6. Part (a) was generally well answered although, as usual, the given answer sometimes emerged from incorrect work. Many good candidates gained full marks in part (b) but there were two fairly common errors: (i) omitting a term in the potential energy element and, more importantly, (ii) using $\frac{1}{2}mv^2$ instead of $\frac{1}{2}\mathrm{I}\omega^2$ for the kinetic energy element.

The final part proved a discriminator even among the better candidates; there was much confused thinking, with a correct equation of motion being very rare.

7. No Report available for this question.