1. Two forces $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ N and $\mathbf{F}_2 = (3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ N act on a rigid body. The force $\mathbf{F}_1$ acts through the point with position vector $(2\mathbf{i} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts through the point with position vector $(\mathbf{j} + 2\mathbf{k})$ m.

(a) If the two forces are equivalent to a single force $\mathbf{R}$, find

(i) $\mathbf{R}$,

(2)

(ii) a vector equation of the line of action of $\mathbf{R}$, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$.

(6)

(b) If the two forces are equivalent to a single force acting through the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m together with a couple of moment $\mathbf{G}$, find the magnitude of $\mathbf{G}$.

(5)
(Total 13 marks)

2. Two forces $\mathbf{F}_1 = (2\mathbf{i} + \mathbf{j})$ N and $\mathbf{F}_2 = (–2\mathbf{j} – \mathbf{k})$ N act on a rigid body. The force $\mathbf{F}_1$ acts at the point with position vector $\mathbf{r}_1 = (3\mathbf{i} + \mathbf{j} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts at the point with position vector
$\mathbf{r}_2 = (\mathbf{i} – 2\mathbf{j})$ m. A third force $\mathbf{F}_3$ acts on the body such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are in equilibrium.

(a) Find the magnitude of $\mathbf{F}_3$.

(4)

(b) Find a vector equation of the line of action of $\mathbf{F}_3$.

(8)

The force $\mathbf{F}_3$ is replaced by a fourth force $\mathbf{F}_4$, acting through the origin O, such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ are equivalent to a couple.

(c) Find the magnitude of this couple.

(4)
(Total 16 marks)

3. Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ act on a rigid body, where

$\mathbf{F_1} = (3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$ N and

$\mathbf{F_2} = (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N.

The force $\mathbf{F_1}$ acts at the point with position vector $(\mathbf{i} - 2\mathbf{j})$m, and the force $\mathbf{F_2}$ acts at the point with position vector $(3\mathbf{i} - \mathbf{k})$m. The two forces are equivalent to a single force $\mathbf{F}$ acting at the point with position vector $(\mathbf{i} - \mathbf{k})$m, together with a couple $\mathbf{G}$.

(a) Find $\mathbf{F}$.

(1)

(b) Find the magnitude of $\mathbf{G}$.

(8)
(Total 9 marks)

4. A particle of mass 0.5 kg is at rest at the point with position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$ m. The particle is then acted upon by two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$. These are the only two forces acting on the particle. Subsequently, the particle passes through the point with position vector $(4\mathbf{i} + 5\mathbf{j} - 5\mathbf{k})$ m with speed 12 m s^{-1}. Given that $\mathbf{F}_1 = (\mathrm{i} + 2\mathbf{j} - \mathbf{k})$ N, find $\mathbf{F}_2$.

(Total 9 marks)

5. A force system consists of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a rigid body.

$\mathbf{F}_1 = (\mathbf{i} + \mathbf{2j})$ N and acts at the point with position vector $(-\mathbf{i} + 4\mathbf{j})$ m.
$\mathbf{F}_2 = (-\mathbf{j} + \mathbf{k})$ N and acts at the point with position vector $(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ m.
$\mathbf{F}_3 = (3\mathbf{i} - \mathbf{j} + \mathbf{k})$ N and acts at the point with position vector $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m.
It is given that this system can be reduced to a single force $\mathbf{R}$.

(a) Find $\mathbf{R}$.

(2)

(b) Find a vector equation of the line of action of $\mathbf{R}$, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a parameter.

(10)
(Total 12 marks)

6. The vertices of a tetrahedron $PQRS$ have position vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ and $\mathbf{s}$ respectively, where

$$\mathbf{p} = -3\mathbf{i} + 4\mathbf{j} - \mathbf{k}, \quad \mathbf{q} = 4\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}, \quad \mathbf{r} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}, \quad \mathbf{s} = 4\mathbf{i} + \mathbf{k}.$$

Forces of magnitude 20 N and $2\sqrt{13}$ N act along SQ and SR respectively. A third force acts at P.

Given that the system of three forces reduces to a couple $\mathbf{G}$, find

(a) the third force,

(6)

(b) the magnitude of $\mathbf{G}$.

(6)
(Total 12 marks)

7. A system of forces acting on a rigid body consists of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at a point A of the body, together with a couple of moment $\mathbf{G}$. $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ N and $\mathbf{F}_2 = (-2\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ N. The position vector of the point A is $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m and $\mathbf{G} = (7\mathbf{i} - 3\mathbf{j} + 8\mathbf{k})$ Nm.

Given that the system is equivalent to a single force $\mathbf{R}$,

(a) find $\mathbf{R}$,

(2)

(b) find a vector equation for the line of action of $\mathbf{R}$.

(7)
(Total 9 marks)

8. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body. $\mathbf{F}_1 = (12\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$ N and acts at the point with position vector $(2\mathbf{i} - 3\mathbf{j})$ m, $\mathbf{F}_2 = (-3\mathbf{j} + 2\mathbf{k})$ N and acts at the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m. The force $\mathbf{F}_3$ acts at the point with position vector $(2\mathbf{i} - \mathbf{k})$ m.

Given that this set of forces is equivalent to a couple, find

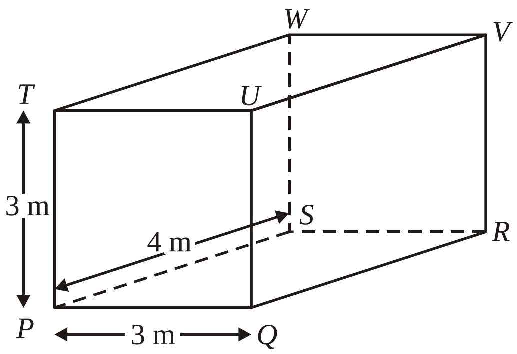
(a) $\mathbf{F}_3$,

(2)

(b) the magnitude of the couple.

(5)
(Total 7 marks)

9.



The diagram above shows a box in the shape of a cuboid *PQRSTUVW* where $\overrightarrow{PQ}$ = 3**i** metres, $\overrightarrow{PS}$ = 4**j** metres and $\overrightarrow{PT}$ = 3**k** metres. A force (4**i** – 2**j**) N acts at *Q*, a force (4**i** + 2**j**) N acts at *R*, a force (–2**j** + **k**) N acts at *T*, and a force (2**j** + **k**) N acts at *W*. Given that these are the only forces acting on the box, find

(a) the resultant force acting on the box,

(2)

(b) the resultant vector moment about *P* of the four forces acting on the box.

(5)

When an additional force **F** acts on the box at a point *X* on the edge *PS*, the box is in equilibrium.

(c) Find **F**.

(1)

(d) Find the length of *PX*.

(5)
(Total 13 marks)

10. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a rigid body, where $\mathbf{F}_1 = (2\mathbf{j} + 3\mathbf{k})$ N and $\mathbf{F}_2 = (\mathbf{i} + 4\mathbf{k})$ N. The force $\mathbf{F}_1$ acts through the point with position vector $(\mathbf{i} + \mathbf{k})$ m relative to a fixed origin O. The force $\mathbf{F}_2$ acts through the point with position vector $(2\mathbf{j})$ m. The two forces are equivalent to a single force $\mathbf{F}$.

(a) Find the magnitude of $\mathbf{F}$.

(3)

(b) Find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, a vector equation of the line of action of $\mathbf{F}$.

(8)
(Total 11 marks)

1.	(a)	(i)	$\mathbf{R} = (\mathbf{i}+2\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+\mathbf{j}+2\mathbf{k})$	M1	
			$=(4\mathbf{i}+3\mathbf{j}+5\mathbf{k})$	A1	2
		(ii)	$(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\text{x}(4\mathbf{i}+3\mathbf{j}+5\mathbf{k}) = (2\mathbf{i}+\mathbf{k})\text{x}(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+(\mathbf{j}+2\mathbf{k})\text{x}(3\mathbf{i}+\mathbf{j}+2\mathbf{k})$	M1	
			$(5y-3z)\mathbf{i}+(4z-5x)\mathbf{j}+(3x-4y)\mathbf{k} = (-2\mathbf{i}-5\mathbf{j}+4\mathbf{k})+(6\mathbf{j}-3\mathbf{k})$	A2	
			$(5y-3z)\mathbf{i}+(4z-5x)\mathbf{j}+(3x-4y)\mathbf{k} = (-2\mathbf{i}+\mathbf{j}+\mathbf{k})$		
			a solution is $x=0, y=-\frac{1}{4}, z=\frac{1}{4}$;		
			$x=\frac{1}{3}, y=0, z=\frac{2}{3}; x=-\frac{1}{5}, y=-\frac{2}{5}, z=0$	B1	
			$\mathbf{r} = -\frac{1}{4}\mathbf{j}+\frac{1}{4}\mathbf{k}+\lambda(4\mathbf{i}+3\mathbf{j}+5\mathbf{k})$	M1 A1 ft	6
	(b)		$(\mathbf{i}+2\mathbf{j}+\mathbf{k})\text{x}(4\mathbf{i}+3\mathbf{j}+5\mathbf{k})+\mathbf{G} = (-2\mathbf{i}+\mathbf{j}+\mathbf{k})$	M1A1	
			$(7\mathbf{i}-\mathbf{j}-5\mathbf{k})+\mathbf{G} = (-2\mathbf{i}+\mathbf{j}+\mathbf{k})$		
			$\mathbf{G} = (-9\mathbf{i}+2\mathbf{j}+6\mathbf{k})$	A1	
			$\lvert\mathbf{G}\rvert = \sqrt{(-9)^2+2^2+6^2}$	M1	
			$=11$ (Nm)	A1 ft	5
					[13]
2.	(a)		$(2\mathbf{i}+\mathbf{j}) + (-2\mathbf{j}-\mathbf{k}) + \mathbf{F}_3 = \mathbf{0}$	M1	
			$\mathbf{F}_3 = -2\mathbf{i}+\mathbf{j}+\mathbf{k}$	A1	
			$\lvert\mathbf{F}_3\rvert = \sqrt{(-2)^2+1^2+1^2} = \sqrt{6}\text{N}$	M1 A1	4
	(b)		$(3\mathbf{i}+\mathbf{j}+\mathbf{k}) \times (2\mathbf{i}+\mathbf{j}) + (\mathbf{i}-2\mathbf{j}) \times (-2\mathbf{j}-\mathbf{k})$ + $(x\mathbf{i}+\text{y}\mathbf{j}+z\mathbf{k}) \times (-2\mathbf{i}+\mathbf{j}+\mathbf{k})$	M1	
			$(-\mathbf{i}+2\mathbf{j}+\mathbf{k}) + (2\mathbf{i}+\mathbf{j}-2\mathbf{k}) + ((y-z)\mathbf{i}$ + $(-2z-x)\mathbf{j}+(x+2y)\mathbf{k})$	A3	
			$y-z=-1, -x-2z=-3, x+2y=1$	DM1	
			$x=1, y=0, z=1$ is a solution	DM1	
			so, $\mathbf{r} = (\mathbf{i}+\mathbf{k}) + \lambda(-2\mathbf{i}+\mathbf{j}+\mathbf{k})$ is a vector equn of line of action of $\mathbf{F}_3$	M1 A1	8
	(c)		$(3\mathbf{i}+\mathbf{j}+\mathbf{k}) \times (2\mathbf{i}+\mathbf{j}) + (\mathbf{i}-2\mathbf{j}) \times (-2\mathbf{j}-\mathbf{k}) = \mathbf{G}$	M1	
			$(-\mathbf{i}+2\mathbf{j}+\mathbf{k}) + (2\mathbf{i}+\mathbf{j}-2\mathbf{k}) = (\mathbf{i}+3\mathbf{j}-\mathbf{k}) = \mathbf{G}$	A1	
			$\lvert\mathbf{G}\rvert = \sqrt{1^2+3^2+=(-1)^2} = \sqrt{11}$ N m	M1 A1	4
					[16]

3.	(a)	$\mathbf{F} = \Sigma\, \mathbf{F}_i = (8\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$N	B1	1
	(b)	$\Sigma \mathrm{r}_i \times \mathbf{F}_i = (\mathbf{i} - 2\mathbf{j}) \times (3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}) + (3\mathbf{i} - \mathbf{k}) \times (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$	M1	
		$= (12\mathbf{i} + 6\mathbf{j} + 10\mathbf{k}) + (-\mathbf{i} - 11\mathbf{j} - 3\mathbf{k})$ $(=(11\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$	A2,1,0	
		$\mathbf{G} + (\mathbf{i} - \mathbf{k}) \times (8\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) = (11\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$	M1	
		$(3\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$, $\mathbf{G} = (8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$	B1, A1	
		$\mathbf{G} = \sqrt{8^2 + (-1)^2 + 4^2} = 9$ Nm.	M1A1	8

[9]

4.	$\underline{d} = \begin{pmatrix} 4 \\ 5 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = 2\underline{i} + 2\underline{j} - \underline{k}$	B1	
	$\underline{\mathrm{F}}.(2\underline{i} + 2\underline{j} - \underline{k}) = \frac{1}{2} \times \frac{1}{2} \times 12^2 = 36$	M1 A2	
	but $\underline{\mathrm{F}} = \lambda(2\underline{i} + 2\underline{j} - \underline{k})$ (particle starts at rest)	M1	
	$\Rightarrow \lambda(2\underline{i} + 2\underline{j} - \underline{k}) - (2\underline{i} + 2\underline{j} - \underline{k}) = 36$	M1	
	$\Rightarrow 9\lambda = 36$		
	$\Rightarrow \lambda = 4$	A1	
	$\mathrm{F}_2 = 4\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 7\underline{i} + 6\underline{j} - 3\underline{k}$	M1 A1	

[9]

5.	(a)	$\underline{\mathrm{R}} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = (4\underline{i} + 2\underline{k})N$	M1 A1	2

(b) $\begin{pmatrix}-1\\4\\0\end{pmatrix}\times\begin{pmatrix}1\\2\\0\end{pmatrix}+\begin{pmatrix}2\\1\\1\end{pmatrix}\times\begin{pmatrix}0\\-1\\1\end{pmatrix}+\begin{pmatrix}1\\-1\\2\end{pmatrix}\times\begin{pmatrix}3\\-1\\1\end{pmatrix}$ M1

$=\begin{pmatrix}0\\0\\-6\end{pmatrix}+\begin{pmatrix}2\\-2\\-2\end{pmatrix}+\begin{pmatrix}1\\5\\2\end{pmatrix}$ A1 A1 A1

$=\begin{pmatrix}3\\3\\-6\end{pmatrix}$ A1

$\begin{pmatrix}x\\y\\z\end{pmatrix}\times\begin{pmatrix}4\\0\\2\end{pmatrix}=\begin{pmatrix}3\\3\\-6\end{pmatrix}$ M1

$\begin{pmatrix}2y\\4z-2x\\-4y\end{pmatrix}=\begin{pmatrix}3\\3\\-6\end{pmatrix}$ A1 ft

$e.g.\quad x=-\frac{3}{2}, y=\frac{3}{2}, z=0$ B1

$\underline{r}=\begin{pmatrix}-3/2\\3/2\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\0\\1\end{pmatrix}$ M1 A1 10

[12]

6. (a) $\mathbf{SQ}=\mathbf{q}-\mathbf{s}=4\mathbf{j}-3\mathbf{k} \Rightarrow \mathbf{F}_1=16\mathbf{j}-12\mathbf{k}$ M1 A1

$\mathbf{SR}=-3\mathbf{i}-2\mathbf{j} \Rightarrow \mathbf{F}_2=-6\mathbf{i}-4\mathbf{j}$ M1 A1

Net couple alone $\Rightarrow \Sigma\mathbf{F_i}=\mathbf{0}$

$\Rightarrow \mathbf{F}_3=6\mathbf{i}-12\mathbf{j}+12\mathbf{k}$ M1 A1 6

(b) M(s) $\mathbf{G} = \mathbf{sp} \times \mathbf{F}_3$ M1

$= (-7\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}) \times (6\mathbf{i} - 12\mathbf{j} + 12\mathbf{k})$ A1

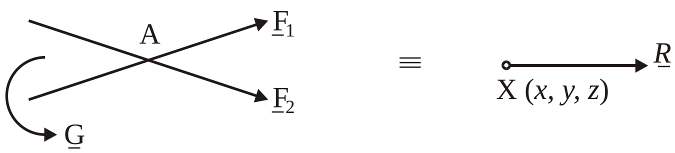
(or complete expression if about another pt.)

$\mathbf{G} = 24\mathbf{i} + 72\mathbf{j} + 60\mathbf{k}$ M1 A1

$= 12\,(2\mathbf{i} + 6\mathbf{j} + 5\mathbf{k})$

$|\mathbf{G}| = \underline{12\sqrt{65}}$ M1 A1 6

7. (a)



$R = F_1 + F_2$ M1

$R = (-\mathbf{I} + 3\mathbf{j} + 2\mathbf{k})$N A1 2

(b) M(0): $\begin{pmatrix}1\\1\\1\end{pmatrix} \times \begin{pmatrix}-1\\3\\2\end{pmatrix} + \begin{pmatrix}7\\-3\\8\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix} \times \begin{pmatrix}-1\\3\\2\end{pmatrix}$ 1 A2ft on R

$\begin{pmatrix}-1\\-3\\4\end{pmatrix} + \begin{pmatrix}7\\-3\\8\end{pmatrix} = \begin{pmatrix}2y-3z\\-z-2x\\3x+y\end{pmatrix}$ A1 A1

$\Rightarrow \begin{pmatrix}6\\-6\\12\end{pmatrix} = \begin{pmatrix}2y-3z\\-z-2x\\3x+y\end{pmatrix}$

Take $z = 0$, one solution is $x = 3$, $y = 3$, $z = 0$ M1

$\underline{\therefore \underline{r} = \begin{pmatrix}3\\3\\0\end{pmatrix} + \lambda\begin{pmatrix}-1\\3\\2\end{pmatrix}}$ is an equation of the line of action of R A1 7

[9]

8. (a) $\begin{pmatrix}12\\-4\\6\end{pmatrix} + \begin{pmatrix}0\\-3\\2\end{pmatrix} + \underline{\mathbf{F}}_3 = \underline{\mathbf{0}} \Rightarrow \underline{\mathbf{F}}_3 = \underline{\begin{pmatrix}-12\\7\\-8\end{pmatrix}}$ $\underline{\text{N}}$ M1 A1 2

(b) $\mathbf{G} = \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 12 \\ -4 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} -12 \\ 7 \\ -8 \end{pmatrix}$ M1

$= \begin{pmatrix} -18 \\ -12 \\ 28 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \\ -3 \end{pmatrix} + \begin{pmatrix} 7 \\ 28 \\ 14 \end{pmatrix}$ A2, 1,0 f.t

$= \begin{pmatrix} -6 \\ 14 \\ 39 \end{pmatrix} \Rightarrow \underline{|\mathbf{G}|} = \sqrt{(6^2 + 14^2 + 39^2)} \approx \underline{41.9}$ Nm M1 A1 5

[7]

9. (a) $\mathbf{R} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 2 \end{pmatrix}$ or $8\mathbf{i} + 2\mathbf{k}$ M1 A1 2

(b) Finding one of $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ M1

$= \begin{pmatrix} 0 \\ 0 \\ -6 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix}, \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}$ A2, 1, 0

[A1 one correct, A2 at least three correct]

Resultant $= \begin{pmatrix} 4 \\ 0 \\ -16 \end{pmatrix}$ any form M1 A1 5

(c) $\mathbf{F} = -8\mathbf{i} - 2\mathbf{k}$ B1 ft 1

(d) For equilibrium $\mathbf{r} \times \begin{pmatrix} -8 \\ 0 \\ -2 \end{pmatrix}_{cand} = -\begin{pmatrix} 4 \\ 0 \\ -16 \end{pmatrix}_{cand}$ or equivalent — M1

$\mathbf{PX} = \begin{pmatrix} 0 \\ \lambda \\ 0 \end{pmatrix} \Rightarrow \mathbf{r} \times \begin{pmatrix} -8 \\ 0 \\ -2 \end{pmatrix}_{cand} = \begin{pmatrix} -2\lambda \\ 0 \\ 8\lambda \end{pmatrix}$ — M1 A1 ft

Finding λ; $PX = 2$. — M1; A1 — 5

[13]

10. (a) $\mathbf{F} = (2\mathbf{j} + 3\mathbf{k}) + (\mathbf{i} + 4\mathbf{k}) = \mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$ — M1

$|\mathbf{F}| = \sqrt{(1 + 4 + 49)} = \sqrt{54}$ N — M1 A1 — 3

(b) **F** acts through point with p.v. **r**

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$ — M1

$\begin{pmatrix} 7y - 2z \\ z - 7x \\ 2x - y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 0 \\ -2 \end{pmatrix}, = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$ — A1 A1, A1

so $7y - 2z = 6$, $z - 7x = -3$, $2x - y = 0$

$\Rightarrow y = 2x \Rightarrow$ e.g. $x = 0, y = 0, z = -3$ — M1 A1

Hence equation of line of action of **F** is

$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$ — M1 A1 — 8

[11]

1. Candidates had more success with this question and many good attempts were seen. A few candidates seemed to have little knowledge of vectors and failed to score any marks apart from the marks for (a)(i) where very few errors were seen.

Most candidates were able to answer (a)(ii) successfully. It was pleasing to see few arithmetical errors in vector products and all but a few used their vector products consistently. Problems occasionally arose in attempts to find a. Some candidates were attempting to solve their equations to find a *unique* solution for the components of a when one did not exist. Some candidates simply found the intersection of the lines of the action of the two forces to obtain a.

Part (b) proved more difficult and quite a few candidates failed to read the question properly and hence failed to include the vector moment of the single force they found in (a)(i) or indeed failed to include the moments of the other two forces in their equation to find G. Many failed to realise that if they had used the vector product method in (a)(ii) they already had the answer to the moments of the other two forces. A significant number failed to find the *magnitude* of **G** as required in the question.

2. Many candidates scored well on the first part of this question, although there were many sign errors. It was common for even good candidates to throw away 2 marks by not finding the magnitude of $\mathbf{F_3}$.

In part (b), most candidates used vector products to find the moments and then put the sum of these equal to zero but some did not know how to proceed when they found that their equations were not independent. A surprising number forgot to put "**r** =" at the beginning of their line equation.

The final part of the question was usually well answered but again some forgot to find the magnitude of the couple.

3. Most candidates were able to find **F** and then went on to find the sum of the moments of $\mathbf{F}_1$ and $\mathbf{F}_2$ about *O*. A number then thought that they had found **G**. Others mistakenly added the moments of $\mathbf{F}_1$, $\mathbf{F}_2$ and **F** about *O*. Many, however, did succeed in finding the correct value of **G**, but some then forgot to find its magnitude. The most common accuracy error was in the signs when using cross products.

4. Most candidates found the vector **AB** and then equated the Work done to the KE gained, using the dot product to calculate the work done. Many candidates, failing to realise that the resultant force, acceleration and final velocity all had to be a scalar multiple of **AB**, made little further progress. Any of these quantities could be used to complete the question. A few candidates tried to use a cross product to find the work done by a force and a few thought that **F2** had to be a multiple of **AB**.

5. This was a good source of marks for many candidates, with almost all getting part (a) correct although a few thought that $\Sigma\mathbf{F} + \mathbf{R} = \mathbf{0}$. In part (b) the method was well known but some gave up, however, when they reached the stage where they had one equation in two unknowns, not realising that any pair of values that satisfied the equation would do. A few candidates tried to use dot products and some used $\mathbf{F} \times \mathbf{r}$ but this could lead to full marks if they were consistent.

6. Some good answers were seen. Most could obtain the forces as vectors and realised which equation was appropriate to use in part (a). In part (b), there was some confusion about which position vectors to use in finding the total moment of the system; some also failed to understand the meaning of the word 'magnitude', finding only **G** as a vector.

7. Nearly everyone gained the first two marks but correct solutions to part (b) were very rare. Most candidates did not know how to derive an equation for the equivalent systems of forces – they knew that a vector product was required but could not combine all the relevant terms correctly, omitting either the couple or the moments of the original two forces. Those who took moments about (1,1,1) saved themselves some time.

8. This proved to be a good for the vast majority of candidates with several fully correct solutions seen. Most clearly knew what to do to find the vector couple (with just a few slips in finding the vector products). However, some candidates failed to find the magnitude of the couple, leaving their answer in vector form.

9. This was another well answered question with the majority of candidates scoring high marks. Solutions to the first three parts were usually very good. A few candidates used $F \times r$ for the moment of a force, some confused the axes and there were a few misreads, inevitable with the quantity of information given in the one paragraph, but these were not too costly. Vector products were usually correct although it was not uncommon to see $(3i + 4j) \times (4i + 2j)$ given as $-2k$. Full marks in part (d) were less frequent than in the rest of the question; PX given as 2j lost the final mark, as did the correct answer from inconsistent equations.

10. No Report available for this question.