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3. A spaceship is moving in a straight line in deep space and needs to increase its speed. This is done by ejecting fuel backwards from the spaceship at a constant speed c relative to the spaceship. When the speed of the spaceship is v , its mass is m .

(a) Show that, while the spaceship is ejecting fuel,

$$\frac{dv}{dm} = -\frac{c}{m} \tag{5}$$

The initial mass of the spaceship is m_0 and at time t the mass of the spaceship is given by $m = m_0(1 - kt)$, where k is a positive constant.

(b) Find the acceleration of the spaceship at time t . (4)



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Question 3 continued

A series of horizontal lines for writing the answer to Question 3.



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4.

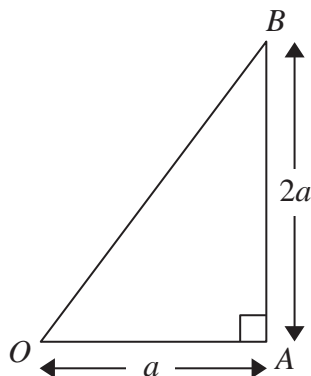


Figure 1

A uniform lamina of mass M is in the shape of a right-angled triangle OAB . The angle OAB is 90° , $OA = a$ and $AB = 2a$, as shown in Figure 1.

- (a) Prove, using integration, that the moment of inertia of the lamina OAB about the edge OA is $\frac{2}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through one end and perpendicular to the rod is $\frac{4}{3}ml^2$.)

(6)

The lamina OAB is free to rotate about a fixed smooth horizontal axis along the edge OA and hangs at rest with B vertically below A . The lamina is then given a horizontal impulse of magnitude J . The impulse is applied to the lamina at the point B , in a direction which is perpendicular to the plane of the lamina. Given that the lamina first comes to instantaneous rest after rotating through an angle of 120° ,

- (b) find an expression for J , in terms of M , a and g .

(7)



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6. A pendulum consists of a uniform rod AB , of length $4a$ and mass $2m$, whose end A is rigidly attached to the centre O of a uniform square lamina $PQRS$, of mass $4m$ and side a . The rod AB is perpendicular to the plane of the lamina. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through B . The axis L is perpendicular to AB and parallel to the edge PQ of the square.

(a) Show that the moment of inertia of the pendulum about L is $75ma^2$. (4)

The pendulum is released from rest when BA makes an angle α with the downward vertical through B , where $\tan \alpha = \frac{7}{24}$. When BA makes an angle θ with the downward vertical through B , the magnitude of the component, in the direction AB , of the force exerted by the axis L on the pendulum is X .

(b) Find an expression for X in terms of m , g and θ . (9)

Using the approximation $\theta \approx \sin \theta$,

(c) find an estimate of the time for the pendulum to rotate through an angle α from its initial rest position. (6)



