

June 2009
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
Q1	$\pm(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$ $((4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})) \cdot (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = \frac{1}{2}3v^2$ $12 = v$ $\mathbf{v} = \frac{12}{\sqrt{8^2 + (-4)^2 + 8^2}}(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$ $\mathbf{v} = (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) \text{ ms}^{-1}$	B1 M1 A1 f.t. A1 M1 DM1 A1 [7]
Q2	<p>C.F. is $\mathbf{r} = \mathbf{A} \cos 2t + \mathbf{B} \sin 2t$</p> <p>P.I. is $\mathbf{r} = \mathbf{p}e^{2t}$</p> $\dot{\mathbf{r}} = 2\mathbf{p}e^{2t}$ $\ddot{\mathbf{r}} = 4\mathbf{p}e^{2t}$ $4\mathbf{p}e^{2t} + 4\mathbf{p}e^{2t} = \mathbf{j}e^{2t}$ <p>so, (PI is) $\mathbf{r} = \frac{1}{8}\mathbf{j}e^{2t}$</p> <p>GS is $\mathbf{r} = \mathbf{A} \cos 2t + \mathbf{B} \sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$</p> $t = 0, \mathbf{r} = \mathbf{i} + \mathbf{j} \Rightarrow \mathbf{i} + \mathbf{j} = \mathbf{A} + \frac{1}{8}\mathbf{j} \Rightarrow \mathbf{i} + \frac{7}{8}\mathbf{j} = \mathbf{A}$ $\dot{\mathbf{r}} = -2\mathbf{A} \sin 2t + 2\mathbf{B} \cos 2t + \frac{1}{4}\mathbf{j}e^{2t}$ $t = 0, \dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow 2\mathbf{i} = 2\mathbf{B} + \frac{1}{4}\mathbf{j} \Rightarrow \mathbf{i} - \frac{1}{8}\mathbf{j} = \mathbf{B}$ $\mathbf{r} = (\mathbf{i} + \frac{7}{8}\mathbf{j}) \cos 2t + (\mathbf{i} - \frac{1}{8}\mathbf{j}) \sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$	B1 B1 B1 ft M1 A1 A1 ft DM1 A1 M1A1 A1 [11]

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Q3 (a)	$mv = (m + \delta m)(v + \delta v) - (-\delta m)(c - v)$ $mv = mv + m\delta v + v\delta m + c\delta m - v\delta m$ $-m\delta v = c\delta m$ $\frac{dv}{dm} = -\frac{c}{m} *$	M1 A2 DM1 A1 (5)
(b)	$\frac{dm}{dt} = -m_0 k$ $\frac{dv}{dt} = \frac{dv}{dm} \times \frac{dm}{dt}$ $= -\frac{c}{m} \times -m_0 k$ $= \frac{cm_0 k}{m_0(1 - kt)}$ $= \frac{ck}{(1 - kt)}$	B1 M1 DM1 A1 (4) [9]

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Q4 (a)	$\delta m = \frac{2Mx\delta x}{a^2}$ $\delta I = \frac{1}{3} \frac{2Mx\delta x}{a^2} (2x)^2$ $I = \int_0^a \frac{8Mx^3 dx}{3a^2}$ $= \frac{8M}{3a^2} \left[\frac{x^4}{4} \right]_0^a$ $= \frac{2}{3} Ma^2 *$	M1 A1 M1 A1 DM1 A1 (6)
(b)	$J.2a = \frac{2}{3} Ma^2 \omega$ $\frac{1}{2} \frac{2}{3} Ma^2 \omega^2 = Mg \frac{2a}{3} (1 + \cos 60^\circ)$ <p>solving for J</p> $J = M \sqrt{\frac{ag}{3}}$	M1 A1 M1 A2 DM1 A1 (7) [13]

Question Number	Scheme	Marks
Q5 (a)	$(2\mathbf{i} + \mathbf{j}) + (-2\mathbf{j} - \mathbf{k}) + \mathbf{F}_3 = \mathbf{0}$ $\mathbf{F}_3 = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$ $ \mathbf{F}_3 = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6} \text{ N}$	M1 A1 M1 A1 (4)
(b)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) + (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + ((y - z)\mathbf{i} + (-2z - x)\mathbf{j} + (x + 2y)\mathbf{k})$ $y - z = -1, -x - 2z = -3, x + 2y = 1$ $x = 1, y = 0, z = 1 \text{ is a solution}$ <p>so, $\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ is a vector equn of line of action of \mathbf{F}_3</p>	M1 A3 DM1 DM1 M1 A1 (8)
(c)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) = \mathbf{G}$ $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = (\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \mathbf{G}$ $ \mathbf{G} = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{11} \text{ N m}$	M1 A1 M1 A1 (4)
		[16]

Question Number	Scheme	Marks
Q6 (a)	$\frac{1}{3}2m(4a)^2 + \frac{1}{12}4ma^2 + 4m(4a)^2$ $= \frac{32}{3}ma^2 + \frac{1}{3}ma^2 + 64ma^2$ $= 75ma^2 \quad *$	B1 M1 A1 A1 (4)
(b)	$\frac{1}{2}75ma^2\omega^2 = 2mg2a(\cos\theta - \cos\alpha) + 4mg4a(\cos\theta - \cos\alpha)$ $a\omega^2 = \frac{8}{15}g(\cos\theta - \frac{24}{25}) = \frac{8}{375}g(25\cos\theta - 24)$ $X - 6mg\cos\theta = 2m2a\omega^2 + 4m4a\omega^2 = 20ma\omega^2$ $X = 6mg\cos\theta + 20m\frac{8}{375}g(25\cos\theta - 24)$ $= \frac{50mg\cos\theta}{3} - \frac{256mg}{25}$	M1 A2 A1 M1 A2 D M1 A1 (9)
(c)	$-2mg2a\sin\theta - 4mg4a\sin\theta = 75ma^2\ddot{\theta}$ $\ddot{\theta} = -\frac{4g}{15a}\sin\theta$ $\approx -\frac{4g}{15a}\theta, \text{ SHM}$ $\text{Time} = \frac{1}{4}2\pi\sqrt{\frac{15a}{4g}}$ $= \frac{\pi}{4}\sqrt{\frac{15a}{g}}$	M1 A1 A1 M1 M1 A1 (6) [19]