

Mark Scheme (Final)

Summer 2007

GCE

GCE Mathematics (6681/01)

June 2007
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1.	$\mathbf{d} = \mathbf{AB} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ $\frac{1}{2} \cdot 0.5v^2 = \mathbf{F} \cdot \mathbf{d} = (4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} - 5\mathbf{k})$ $v = 6 \text{ m s}^{-1}$	B1 M1 A1 A1 (4)
2.	$\frac{d\mathbf{v}}{dt} + 3\mathbf{v} = \mathbf{0}$ $\text{IF} = e^{3t} \Rightarrow \frac{d(\mathbf{v}e^{3t})}{dt} = \mathbf{0}$ $\Rightarrow \mathbf{v}e^{3t} = \mathbf{A}$ $t = 0, \mathbf{v} = 8\mathbf{i} - 12\mathbf{j} \Rightarrow \mathbf{v} = (8\mathbf{i} - 12\mathbf{j})e^{-3t}$ $t = \frac{2}{3} \ln 2 \Rightarrow \mathbf{v} = (8\mathbf{i} - 12\mathbf{j})e^{-2 \ln 2} = (2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$	B1 M1 A1 M1 A1 DM1 A1 (7)
3.	$\frac{4}{3} ma^2 \omega = \left(\frac{4}{3} ma^2 + mx^2 \right) \frac{3}{4} \omega$ $\Rightarrow x = \frac{2}{3} a$	M1A1A1 DM1A1 (5)
4.	$V = \pi \int_0^a 4ax \, dx$ $= 2\pi a^3$ $\delta m = \frac{M}{2\pi a^3} \cdot \pi 4ax \delta x \quad \left(= \frac{2M}{a^2} x \delta x \right)$ $\delta I = \frac{1}{2} \frac{2M}{a^2} x \delta x \cdot y^2 = \frac{4M}{a} x^2 \delta x$ $I = \frac{4M}{a} \int_0^a x^2 \, dx = \frac{4}{3} Ma^2$	M1 A1 M1 M1A1 DM1A1 (7)

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5. (a)	$\mathbf{F} = \Sigma \mathbf{F}_i = (8\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) \text{ N}$	B1 (1)
	<p data-bbox="209 315 1334 421">(b) $\Sigma \mathbf{r}_i \times \mathbf{F}_i = (\mathbf{i} - 2\mathbf{j}) \times (3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}) + (3\mathbf{i} - \mathbf{k}) \times (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ $= (12\mathbf{i} + 6\mathbf{j} + 10\mathbf{k}) + (-\mathbf{i} - 11\mathbf{j} - 3\mathbf{k}) = (11\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$</p> <p data-bbox="209 421 1334 504">$\mathbf{G} + (\mathbf{i} - \mathbf{k}) \times (8\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) = (11\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$</p> <p data-bbox="209 504 1334 586">$(3\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}), \mathbf{G} = (8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$</p> <p data-bbox="209 586 1334 745">$\mathbf{G} = \sqrt{8^2 + (-1)^2 + 4^2} = 9 \text{ Nm.}$</p>	M1 A2, 1, 0 M1 B1, A1 M1A1 (8) (9)
6. (a)	$I_O = \frac{M}{3\pi a^2} \left(\frac{\pi}{2} (2a)^2 (2a)^2 - \frac{\pi}{2} (a)^2 (a)^2 \right)$ $= \frac{5Ma^2}{2} *$	M1 A1 A1 (3)
(b)	$I_{\text{diameter}} = \frac{1}{2} \frac{5Ma^2}{2} \quad (\text{perp. axes})$ $I_L = \frac{5Ma^2}{4} + M(2a)^2 \quad (\text{parallel axes})$ $= \frac{21Ma^2}{4}$	M1 A1 M1 A1 (4)
(c)	$M(L), -Mg2a \sin \theta = \frac{21Ma^2}{4} \ddot{\theta}$ $\sin \theta \approx \theta \Rightarrow \ddot{\theta} = -\frac{8g}{21a} \theta, \quad \text{so SHM}$ $\text{Time} = \frac{1}{4} 2\pi \sqrt{\frac{21a}{8g}}$ $= \frac{\pi}{2} \sqrt{\frac{21a}{8g}}$ <p style="text-align: center;">□</p>	M1 A1 DM1 A1 DM1 A1 (6) (13)

7.(a)	$(m + \delta m)(v + \delta v) - mv = -2\lambda v \delta t$ $m \frac{dv}{dt} + v \frac{dm}{dt} = -2\lambda v$ $\frac{dm}{dt} = \lambda; \quad m = M + \lambda t$ $(M + \lambda t) \frac{dv}{dt} + 3\lambda v = 0 \quad *$	M1 A1 B1 ; B1 D M1 A1 (6)
(b)	$-\int \frac{dv}{3\lambda v} = \int \frac{dt}{(M + \lambda t)}$ $-\frac{1}{3\lambda} [\ln v]_v^u = \frac{1}{\lambda} [\ln(M + \lambda t)]_0^T$ $\frac{1}{3} \ln 2 = \ln \frac{(M + \lambda T)}{M}$ $T = \frac{M}{\lambda} (2^{\frac{1}{3}} - 1) *$	M1 DM1 A1 DM1 DM1 A1 (6)
(c)	<p>Sinks at $T_s = \frac{M}{\lambda}$</p> <p>Reaches speed $\frac{1}{2}U$ at $T = \frac{M}{\lambda} (2^{\frac{1}{3}} - 1)$</p> <p>Since $(2^{\frac{1}{3}} - 1) < 1$, $T < T_s$</p> <p>i.e. Reaches speed $\frac{1}{2}U$ before it sinks</p>	M1 A1c.s.o. (2) (14)

<p>8.(a)</p>	<p>MI of rod + particle</p> $= \frac{1}{12} 3m(2a)^2 + 3m\left(\frac{1}{2}a\right)^2 + m\left(\frac{3}{2}a\right)^2$ $= 4ma^2$ $\frac{1}{2} 4ma^2(\omega^2 - \frac{g}{a}) = 3mg \frac{a}{2}(1 - \cos \theta) + mg \frac{3a}{2}(1 - \cos \theta)$ $\omega^2 = \frac{g}{2a}(5 - 3\cos \theta)$	<p>M1, M1</p> <p>A1</p> <p>M1 A ft A1</p> <p>DM1 A1 (8)</p>
<p>(b)</p>	$4ma^2 \omega^2 = 3mg \frac{1}{2} a \sin \theta + mg \frac{3}{2} a \sin \theta$ $\Rightarrow \omega^2 = \frac{3g \sin \theta}{4a}$	<p>M1 A1 ft</p> <p>A1 (3)</p>
<p>(c)</p>	$F + 4mg \cos \theta = 3m \frac{1}{2} a \omega^2 + m \frac{3}{2} a \omega^2$ $= 3ma \frac{g}{2a} (5 - 3\cos \theta)$ $\Rightarrow F = \frac{mg}{2} (15 - 17 \cos \theta)$ <p>When $\theta = \phi$, $F = 0$</p> $\Rightarrow \cos \phi = \frac{15}{17}$	<p>M1 A1</p> <p>DM1</p> <p>DM1</p> <p>A1 (5)</p> <p>16</p>