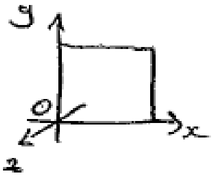
June 2006
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1. (a)	 $I = \int_0^{2a} \frac{m}{2a} x^2\,dx$	M1
	$= \frac{m}{2a}\left[\frac{x^3}{3}\right]_0^{2a}$	A1
	$= \frac{4}{3}ma^2$ *	A1 (3)
(b)	 $I_x = I_y = \frac{4}{3}ma^2$ (stretching rule)	M1
	$I_z = I_x + I_y = \frac{8}{3}ma^2$ ($\perp^{r}$ axes)	M1 A1 (3) (6)
2.	$\mathbf{d} = \begin{pmatrix}4\\5\\-5\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	B1
	$\mathbf{F}\cdot(2\mathbf{i}+2\mathbf{j}-\mathbf{k}) = \frac{1}{2}\times\frac{1}{2}\times 12^2 = 36$	M1 A2
	but $\mathbf{F} = \lambda(2\mathbf{i}+2\mathbf{j}-\mathbf{k})$ (particle starts from rest)	M1
	$\Rightarrow \lambda(2\mathbf{i}+2\mathbf{j}-\mathbf{k})\cdot(2\mathbf{i}+2\mathbf{j}-\mathbf{k}) = 36$	M1
	$\Rightarrow 9\lambda = 36$	
	$\Rightarrow \lambda = 4$	A1
	$\mathbf{F}_2 = 4\begin{pmatrix}2\\2\\-1\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix} = 7\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	M1 A1 (9)
3. (a)	$m^2 - 2m = m(m-2) = 0$	M1
	$\Rightarrow m = 0$ or $m = 2$	A1
	$\Rightarrow \mathbf{r} = \mathbf{A} + \mathbf{B}e^{2t}$	A1
	$t=0,\ \mathbf{r} = 3\mathbf{i} \Rightarrow \mathbf{A} + \mathbf{B} = 3\mathbf{i}$	M1 A1
	$\dot{\mathbf{r}} = 2\mathbf{B}e^{2t}$	M1
	$t=0,\ \dot{\mathbf{r}} = \mathbf{j} \Rightarrow \mathbf{B} = \frac{1}{2}\mathbf{j}$	A1
	$\Rightarrow \mathbf{r} = (3\mathbf{i} - \frac{1}{2}\mathbf{j}) + \frac{1}{2}\mathbf{j}e^{2t} = 3\mathbf{i} + \frac{1}{2}\mathbf{j}(e^{2t} - 1)$	A1 (8)
(b)	Particle moves in a straight line	B1
	Equation of line is $x = 3$	B1 (2) (10)

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4. (a)	$\underline{R} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \begin{pmatrix}0\\-1\\1\end{pmatrix} + \begin{pmatrix}3\\-1\\1\end{pmatrix} = \begin{pmatrix}4\\0\\2\end{pmatrix} = \underline{(4\underline{i} + 2\underline{k})\text{ N}}$	M1 A1 (2)
(b)	$\begin{pmatrix}-1\\4\\6\end{pmatrix} \times \begin{pmatrix}1\\2\\0\end{pmatrix} + \begin{pmatrix}2\\1\\1\end{pmatrix} \times \begin{pmatrix}0\\-1\\1\end{pmatrix} + \begin{pmatrix}1\\-1\\2\end{pmatrix} \times \begin{pmatrix}3\\-1\\1\end{pmatrix}$	M1
	$= \begin{pmatrix}0\\0\\-6\end{pmatrix} + \begin{pmatrix}2\\-2\\-2\end{pmatrix} + \begin{pmatrix}1\\5\\2\end{pmatrix}$	A1 A1 A1
	$= \begin{pmatrix}3\\3\\-6\end{pmatrix}$	A1
	$\begin{pmatrix}x\\y\\z\end{pmatrix} \times \begin{pmatrix}4\\0\\2\end{pmatrix} = \begin{pmatrix}3\\3\\-6\end{pmatrix}$	M1
	$\begin{pmatrix}2y\\4z-2x\\-4y\end{pmatrix} = \begin{pmatrix}3\\3\\-6\end{pmatrix}$	A1 ft.
	e.g. $x = -3/2,\ y = 3/2,\ z = 0$	B1
	$\underline{\underline{r} = \begin{pmatrix}-3/2\\3/2\\0\end{pmatrix} + \lambda \begin{pmatrix}2\\0\\1\end{pmatrix}}$	M1 A1 (10) (12)
5. (a)	$mv \approx (m+\delta m)(v+\delta v) + (-\delta m)(K + v + \delta v)$	M1 A3
	$\cancel{mv} \approx \cancel{mv} + m\,\delta v + \cancel{v\,\delta m} - K\,\delta m - \cancel{v\,\delta m}$	M1
	$K\,\delta m \approx m\,\delta v$	
	In the limit, as $\delta t \to 0$,	
	$\underline{\dfrac{dm}{dv} = \dfrac{m}{K}}$ *	A1 (6)
(b)	$\displaystyle\int_M^{m_1} \frac{dm}{m} = \int_U^V \frac{dv}{K}$	M1
	$\ln m_1 - \ln M = \frac{1}{K}(V-U)$	A1
	$\ln \frac{m_1}{M} = \frac{1}{K}(V-U)$	M1
	$m_1 = M e^{\left(\frac{V-U}{K}\right)}$	A1
	Amount of fuel $= M - m_1 = \underline{M\left(1 - e^{\frac{V-U}{K}}\right)}$	M1 A1 (6) (12)

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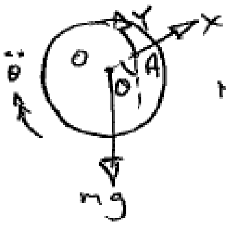
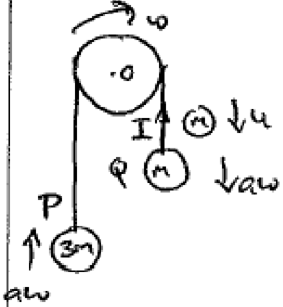
6(a)		
	$I_A = \frac{1}{2}ma^2 + m(\frac{1}{2}a)^2 = \frac{3}{4}ma^2$	M1 A1
	M(A), $-mg\frac{a}{2}\sin\theta = \frac{3}{4}ma^2\ddot{\theta}$	M1
	$-\frac{2g}{3a}\sin\theta = \ddot{\theta}$	A2 (5)
(b)	For small θ, $-\frac{2g}{3a}\theta = \ddot{\theta}$	M1
	$T = 2\pi\sqrt{\frac{3a}{2g}}$	A1 (2)
(c)	R(↖), $Y - mg\sin\theta = m\frac{a}{2}\ddot{\theta}$	M1 A2
	$\Rightarrow Y = mg\sin\theta + \frac{ma}{2}\left(-\frac{2g}{3a}\sin\theta\right)$	M1
	$= \frac{2mg\sin\theta}{3}$	A1 (5) (12)
7. (a)		
	$u = \sqrt{2ag}$	B1
	CAM about O:	
	$m\sqrt{2ag}\,a = 2ma^2\omega + 3ma^2\omega + \frac{1}{2}2ma^2\omega$	M1 A2
	$\frac{\sqrt{2ag}}{6a} = \omega$	
	$\frac{1}{3}\sqrt{\frac{g}{2a}} = \omega$ ✱	A1 (5)
(b)	For Q: $-I = 2ma\omega - mu$	M1 A1
	$\Rightarrow I = 6ma\omega - 2ma\omega = 4ma\omega$	
	$= \frac{4ma}{3}\sqrt{\frac{g}{2a}} = \frac{m}{3}\sqrt{8ag}$	A1 (3)
(c)	PE Gain of P = KE loss of P + KE loss of Q + KE loss of pulley + PE loss of Q	
	$3mgd = \frac{1}{2}3ma^2\omega^2 + \frac{1}{2}2ma^2\omega^2 + \frac{1}{2}ma^2\omega^2 + 2mgd$	M1 A3
	$\cancel{m}gd = 3\cancel{m}a^2\omega^2$	
	$gd = 3a^2 \cdot \frac{1}{9}\frac{g}{2a} = \frac{a}{6}$	M1 A1
		(6)
		(14)