
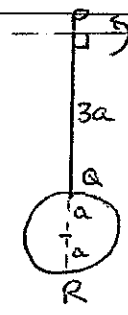
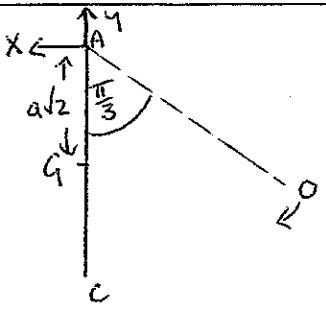
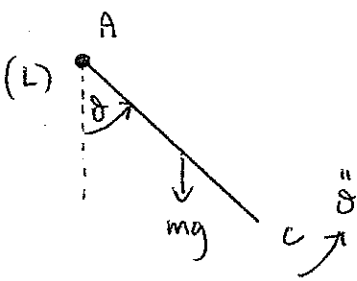


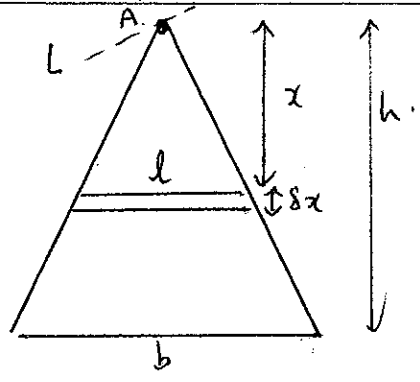
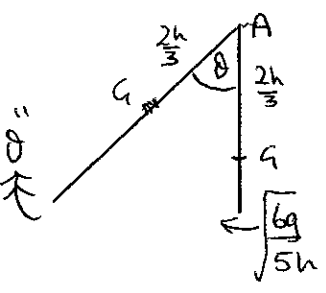
June 2005  
6681 Mechanics M5  
Mark Scheme

publications

Question Number	Scheme	Marks
1.	$ 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}  = 3 \Rightarrow F_1 = 6\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ $  \mathbf{i} + 8\mathbf{j} - 4\mathbf{k}   = 9 \Rightarrow F_2 = 2\mathbf{i} + 16\mathbf{j} - 8\mathbf{k}$ $WD = (6\mathbf{i} + 3\mathbf{j} + 6\mathbf{k} + 2\mathbf{i} + 16\mathbf{j} - 8\mathbf{k}) \cdot (3\mathbf{i} + 2\mathbf{j} - \mathbf{k} - \mathbf{i} - \mathbf{j} - \mathbf{k})$ $= (8\mathbf{i} + 19\mathbf{j} - 2\mathbf{k}) \cdot (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ $= \underline{39 \text{ J}}$	B1 B1 M1 A1 ↓ on F M1 A1 (6)
2.	$I.F. = e^{\int 2t dt} = e^{2t}$ $\Rightarrow e^{2t} \frac{d\mathbf{r}}{dt} + 2e^{2t} \mathbf{r} = 3e^{2t} \mathbf{j}$ $\Rightarrow \frac{d}{dt} (\mathbf{r} e^{2t}) = 3e^{2t} \mathbf{j}$ $e^{2t} \mathbf{r} = 3e^{2t} \mathbf{j} (+c)$ $t=0, \mathbf{r} = 2\mathbf{i} - \mathbf{j} \Rightarrow 2\mathbf{i} - \mathbf{j} = 3\mathbf{j} + c$ $2\mathbf{i} - 4\mathbf{j} = c$ $\Rightarrow \underline{\mathbf{r} = 3e^{-t} \mathbf{j} + (2\mathbf{i} - 4\mathbf{j}) e^{-2t}}$	B1 M1 M1 A1 M1 A1 A1 (7)
3.	<p>a) </p> <p><math>\underline{R} = \underline{F}_1 + \underline{F}_2</math>  <math>\underline{R} = (-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) \text{ N}</math></p> <p>b) <math>M(0): \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ -3 \\ 8 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}</math>  <math>\begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 7 \\ -3 \\ 8 \end{pmatrix} = \begin{pmatrix} 2y - 3z \\ -z - 2x \\ 3x + y \end{pmatrix}</math>  <math>\Rightarrow \begin{pmatrix} 6 \\ -6 \\ 12 \end{pmatrix} = \begin{pmatrix} 2y - 3z \\ -z - 2x \\ 3x + y \end{pmatrix}</math>                      Take <math>z = 0</math>, one solution is <math>x = 3, y = 3, z = 0</math>  <math>\therefore \underline{\mathbf{r}} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}</math> is an equation of the line of action of <math>\underline{R}</math> </p>	M1 A1 (2) M1 A2 ↓ on R A1 A1 M1 A1 (7) (9)

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<p>4 (a)</p> 	<p>DISC: <math>I_{diam} = \frac{1}{2} \left( \frac{1}{2} m a^2 \right) = \frac{1}{4} m a^2</math>  <math>I_L = \frac{1}{4} m a^2 + m (4a)^2 = \frac{65}{4} m a^2</math></p> <p>ROD: <math>I_L = \frac{4}{3} \left( \frac{3a}{2} \right)^2 = 3 m a^2</math></p> <p><math>I_{TOTAL} = \frac{65}{4} m a^2 + 3 m a^2 = \frac{77}{4} m a^2</math> *</p> <p>(b) CAM: <math>\frac{77 m a^2}{4} \omega = \left[ \frac{77}{4} m a^2 + \frac{1}{2} m (4a)^2 \right] \omega'</math>  <math>\Rightarrow \omega' = \frac{77}{109} \omega</math></p>	<p>M1 A1 M1 A1 B1 M1 A1 (7) M1 A1 A1 A1 (4) (11)</p>
<p>5(a)</p>  	<p>Energy: <math>\frac{1}{2} \cdot \frac{8 m a^2}{3} \dot{\theta}^2 = m g a \sqrt{2} \left( 1 - \cos \frac{\pi}{3} \right)</math>  <math>\Rightarrow \dot{\theta}^2 = \frac{3 g \sqrt{2}}{8 a}</math></p> <p>R↑: <math>4 - m g = m a \sqrt{2} \dot{\theta}^2 = m a \sqrt{2} \cdot \frac{3 g \sqrt{2}}{8 a}</math>  <math>\Rightarrow 4 = \frac{7 m g}{4}</math></p> <p>m(L): <math>-m g a \sqrt{2} \sin \theta = \frac{8 m a^2}{3} \ddot{\theta}</math></p> <p>For small <math>\theta</math>, <math>\sin \theta \approx \theta</math>  <math>\Rightarrow -\frac{3 g \sqrt{2}}{8 a} \theta \approx \ddot{\theta}</math></p> <p>Approx SHM  <math>\Rightarrow \text{period} = 2\pi \sqrt{\frac{8 a}{3 g \sqrt{2}}}</math></p>	<p>M1 A1 A1 M1 A1 M1 A1 (7) M1 A1 A1 M1 A1 (5) (12)</p>

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6(a)	<div style="text-align: center;"> </div> <p>Impulse - momentum:</p> $-kv \Delta t = (m + \Delta m)(v + \Delta v) + (-\Delta m)(v - u) - mv$ $-kv \Delta t = mv + m\Delta v + \Delta m v - \Delta m v + \Delta m u - mv$ $-kv = m \frac{\Delta v}{\Delta t} + u \frac{\Delta m}{\Delta t}$ <p>limit as <math>\Delta t \rightarrow 0</math> : <math>-kv = m \frac{dv}{dt} + u \frac{dm}{dt}</math></p> $\frac{dm}{dt} = -\lambda \Leftrightarrow m = M - \lambda t$ $-kv = (M - \lambda t) \frac{dv}{dt} - \lambda u$ $\frac{dv}{dt} = \frac{\lambda u - kv}{M - \lambda t} \quad *$	<p>M1 A3</p> <p>B1</p> <p>M1</p> <p>A1 (7)</p>
(b)	$\int_0^t \frac{dt}{M - \lambda t} = \int_0^v \frac{dv}{\lambda u - kv}$ $-\frac{1}{\lambda} \left[ \ln(M - \lambda t) \right]_0^t = -\frac{1}{k} \left[ \ln(\lambda u - kv) \right]_0^v$ $\frac{k}{\lambda} \left( \ln(M - \lambda t) - \ln M \right) = \ln(\lambda u - kv) - \ln \lambda u$ $\left( \frac{M - \lambda t}{M} \right)^{\frac{k}{\lambda}} = \frac{\lambda u - kv}{\lambda u}$ $v = \frac{\lambda u}{k} \left[ 1 - \left( 1 - \frac{\lambda t}{M} \right)^{\frac{k}{\lambda}} \right] \quad *$	<p>M1 A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1 (6)</p> <p>(13)</p>

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<p>7. (a)</p>	 $\frac{l}{x} = \frac{b}{h} \quad \left( = \frac{2}{\sqrt{3}} \right)$ $\Rightarrow l = \frac{bx}{h} = \frac{2x}{\sqrt{3}}$ $\rho = \frac{m}{\frac{1}{2}bh} = \frac{2m}{bh}$ $\delta m = \frac{bx}{h} \cdot \delta x \cdot \frac{2m}{bh} = \frac{2mx \delta x}{h^2}$ $\delta I_L \approx \frac{1}{3} \delta m \left( \frac{l}{2} \right)^2 + \delta m x^2$ $= \frac{1}{12} (l^2 + 12x^2) \delta m$ $= \frac{1}{12} \left( \frac{4x^2}{3} + 12x^2 \right) \frac{2mx \delta x}{h^2}$ $= \frac{80}{36} \frac{m}{h^2} x^3 \delta x$ $= \frac{20}{9} \frac{m}{h^2} x^3 \delta x$ $I_L = \frac{20}{9} \frac{m}{h^2} \int_0^h x^3 \delta x$ $= \frac{20}{9} \frac{m}{h^2} \cdot \frac{h^4}{4}$ $= \frac{5mh^2}{9}$	<p>M1 A1  M1  M1 M1 A1  A1  M1  A1 (9)</p>
<p>(b)</p>	 <p>Energy:</p> $\frac{1}{2} \frac{5mh^2}{9} \dot{\theta}^2 = mg \frac{2h}{3} (1 - \cos \theta)$ $\Rightarrow \frac{1}{2} \frac{5mh^2}{9} \frac{6g}{5h} = mg \frac{2h}{3} (1 - \cos \theta)$ $\cos \theta = \frac{1}{2}$ $\theta = \frac{\pi}{3} \quad (60^\circ)$	<p>M1 A1 A1  M1 A1 (5)</p>
<p>(c)</p>	<p>M(A): <math>-mg \cdot \frac{2h}{3} \sin \theta = \frac{5mh^2}{9} \ddot{\theta}</math> (max <math>L^r</math> acc<math>^n</math> when at rest)</p> $ \ddot{\theta} _{\max} = \frac{3\sqrt{3}g}{5h}$	<p>M1 A1 A1 on <math>\theta</math>  A1 (3)  (17)</p>