EDEXCEL MECHANICS M5 (6681) – JUNE 2004 **MARK SCHEME**

Question Number	Scheme	Marks
1. (*a*)	$\begin{pmatrix}12\\-4\\6\end{pmatrix} + \begin{pmatrix}0\\-3\\2\end{pmatrix} + \mathbf{F}_3 = \mathbf{0} \Rightarrow \mathbf{F}_3 = \begin{pmatrix}-12\\7\\-8\end{pmatrix}$N	M1 A1 (2)
(*b*)	$\mathbf{G} = \begin{pmatrix}2\\-3\\0\end{pmatrix} \times \begin{pmatrix}12\\-4\\6\end{pmatrix} + \begin{pmatrix}1\\1\\1\end{pmatrix} \times \begin{pmatrix}0\\-3\\2\end{pmatrix} + \begin{pmatrix}2\\0\\-1\end{pmatrix} \times \begin{pmatrix}-12\\7\\-8\end{pmatrix}$	M1
	$= \begin{pmatrix}-18\\-12\\28\end{pmatrix} + \begin{pmatrix}5\\-2\\-3\end{pmatrix} + \begin{pmatrix}7\\28\\14\end{pmatrix}$	A2, 1,0 f.t
	$= \begin{pmatrix}-6\\14\\39\end{pmatrix} \Rightarrow \lvert\mathbf{G}\rvert = \sqrt{(6^2 + 14^2 + 39^2)} \approx 41.9$ Nm	M1 A1 (5)
		(7 marks)
2. (*a*)	$\mathbf{AB} = 8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$	
	$8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k} = \frac{1}{2} \times \mathbf{a} \times 4^2 \Rightarrow \mathbf{a} = \mathbf{i} - \frac{1}{2}\mathbf{j} + \frac{3}{2}\mathbf{k}$	M1 A1 ft
	$(12\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) + \mathbf{F}_2 = 2(\mathbf{i} - \frac{1}{2}\mathbf{j} + \frac{3}{2}\mathbf{k})$	M1
	$\Rightarrow \mathbf{F}_2 = (-10\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$N	A1 (5)
(*b*)	Work done $= (\mathbf{F}_1 + \mathbf{F}_2).\mathbf{AB}$	
	$= (2\mathbf{i} - \mathbf{j} + 3\mathrm{k}).(8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k})$	M1
	$= 16 + 4 + 36 = 56$ J	M1 A1 (3)
		(8 marks)

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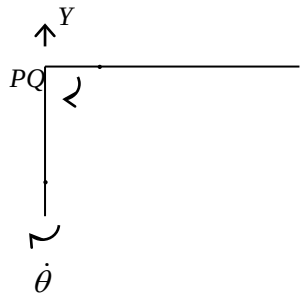
Question Number	Scheme	Marks
3. (*a*)	$I_{PQ} = \frac{4}{3}m(3a)^2 = 12\,ma^2$	M1 A1 (2)
(*b*)	 Energy: $\frac{1}{2} \times 12\,ma^2 \times \dot\theta^2 = mg \times 3a$	M1 A1 ft
	$\Rightarrow \dot\theta = \sqrt{\left(\frac{g}{2a}\right)}$ (*)	A1 (3)
(c)	$R(\uparrow)$: $Y - mg = m \times 3a\dot\theta^2$	M1 A1
	$Y = mg + m \times 3a \times \frac{g}{2a} = \frac{5}{2}\,mg$	M1 A1 (4)
		(9 marks)
4. (*a*)	$I_C = \frac{1}{2}mr^2 + m(\frac{1}{2}r)^2 = \frac{3}{4}mr^2$	M1 A1
	M(*C*): $\frac{3}{4}mr^2\ddot\theta = -mg\frac{r}{2}\sin\theta$	M1 A1 ft
	$\sin\theta \approx \theta \Rightarrow \ddot\theta \approx -\frac{2g}{3r}\theta$ approx. SHM	M1 A1 (6)
(*b*)	Period $= 2\pi\sqrt{\frac{3r}{2g}} = \pi\sqrt{\frac{6r}{g}}$ (*)	A1 (1)
(*c*)	$\dot\theta_{\max} = \omega\alpha \Rightarrow \frac{2}{3r}\sqrt{\frac{gr}{54}} = \sqrt{\frac{2g}{3r}}\,\alpha$	M1 A1
	$\Rightarrow \alpha = \frac{1}{9}^{c}$	A1 (3)
		(10 marks)
Alt	$mg \times \frac{1}{2}r(1 - \cos\alpha) = \frac{1}{2}(\frac{3}{4}mr^2)\left(\frac{gr}{54}\right)\left(\frac{2}{3r}\right)^2$	M1 A1
	$\cos\alpha = \frac{161}{162}$, $\alpha = 6.4°$ (AWRT) or 0.11^{c} (AWRT)	A1

(ft = follow through mark)

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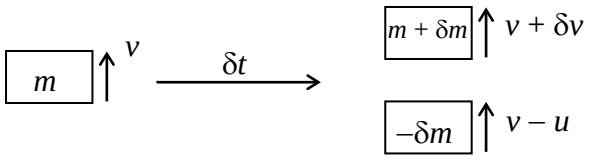
Question Number	Scheme	Marks
5. (*a*)		
	$(m + \delta m)(v + \delta m) + (-\delta m)(v - u) - mv = -mg\delta t$	M1 A2 (–1ee)
	$mv + m\delta v + v\delta m - v\delta m + u\delta m - mv = -mg\delta t$	
	$m\frac{dv}{dt} + u\frac{dm}{dt} = -mg$	A1
	$m = M(1 - kt) \Rightarrow \frac{dm}{dt} = -kM$	B1
	$M(1 - kt)\frac{dv}{dt} + u(-kM) = -M(1 - kt)g$	M1
	$\frac{dv}{dt} = \frac{ku}{1-kt} - g$ (*)	A1 (7)
(*b*)	$v = \int_0^{\frac{1}{3}k} \frac{ku}{1-kt} - g\,dt$	M1
	$= \left[-u\ln(1-kt) - gt \right]_0^{\frac{1}{3}k}$	A1
	$u\ln\left(\frac{3}{2}\right) - \frac{g}{3k}$	A1 (3)
		(10 marks)

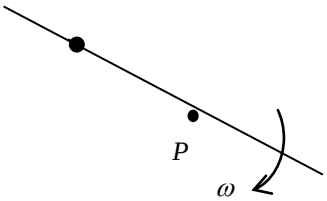
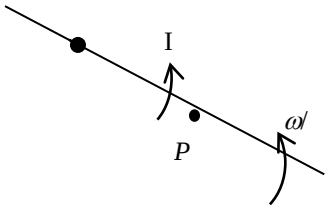
(ft = follow through mark)

EDEXCEL MECHANICS M5 (6681) – JUNE 2004 **MARK SCHEME**

Question Number	Scheme	Marks
6. (*a*)	Aux equ. $m^2 + 2m + 2 = 0$	
	$\Rightarrow \quad m = -1 \pm \mathbf{i}$	M1 A1
	G. soln.: $\mathbf{r} = e^{-t}(\mathbf{A}\cos t + \mathbf{B}\sin t)$	A1 ft
	$t = 0, \mathbf{r} = \mathbf{i} \Rightarrow \mathbf{A} = \mathbf{i}$	M1 A1
	$\dot{\mathbf{r}} = -e^{-t}(\mathbf{A}\cos t + \mathbf{B}\sin t) + e^{-t}(-\mathbf{A}\sin t + \mathbf{B}\cos t)$	M1
	$t = 0, \dot{\mathbf{r}} = (-\mathbf{i} + \mathbf{j}) \quad \Rightarrow \quad -\mathbf{i} + \mathbf{j} = -\mathbf{i} + \mathbf{B} \Rightarrow \mathbf{B} = \mathbf{j}$	M1
	$\therefore \mathbf{r} = e^{-t}(\cos t\,\mathbf{i} + \sin t\,\mathbf{j})$	A1 (8)
(*b*)	$\dot{\mathbf{r}} = -e^{-t}(\cos t\,\mathbf{i} + \sin t\,\mathbf{j}) + e^{-t}(-\sin t\,\mathbf{i} + \cos t\,\mathbf{j})$	M1
	$= e^{-t}\{-(\cos t + \sin t)\mathbf{i} + (\cos t - \sin t)\mathbf{j}\}$	
	Speed $= e^{-t}\sqrt{\{(\cos t + \sin t)^2 + (\cos t - \sin t)^2\}}$	M1 A1
	$= e^{-t}\sqrt{\{1 + 2\cos t \sin t + 1 - 2\cos t \sin t\}}$	M1
	$= e^{-t}\sqrt{2}$ (*)	A1 (5)
(c)	Loss of KE $= \frac{1}{2} \times 2 \times 2(1 - e^{-2})$	M1
	$= 2\left(1 - \frac{1}{e^2}\right) \quad (\approx 1.73$ (AWRT))	A1 (2)
		(15 marks)

(cso = correct solution only)

EDEXCEL MECHANICS M5 (6681) – JUNE 2004 **MARK SCHEME**

Question Number	Scheme	Marks
7. (*a*)	$I = \frac{1}{2}ma^2 + m(3a)^2 + \frac{1}{2}ma^2 + m(5a)^2$,	M1 A1 A1
	$\frac{1}{3}(3m)(4a)^2 + 3ma^2$	M1 A1
	$= 54\,ma^2$ (*)	A1 (6)
	$\frac{1}{2} \times 54ma^2 \times \omega^2 = mg \times \frac{5a}{2} + 3mg \times \frac{a}{2} - mg \times \frac{3a}{2}$	M1 A2, 1, 0
	$\Rightarrow \omega = \sqrt{\frac{5g}{54a}} = \frac{1}{3}\sqrt{\frac{5g}{6a}}$	M1 A1
	$3a \times I = 54ma^2\left[\frac{1}{2}\sqrt{\frac{5g}{54a}} - \left(-\sqrt{\frac{5g}{54a}}\right)\right]$	M1 A2, 1, 0 ft
	$I = \frac{54ma^2}{3a} \times \frac{3}{2}\sqrt{\frac{5g}{54a}}$	
	$= \frac{54ma^2}{3a} \times \frac{3}{2} \times \frac{1}{3}\sqrt{\frac{5g}{6a}}$	
	$= 9m\sqrt{\frac{5ga}{6}}$	M1 A1 (10)
		(16 marks)

(cso = correct solution only; ft = follow through mark)