EDEXCEL MECHANICS M5 (6681) – JUNE 2003 PROVISIONAL MARK SCHEME

Question Number	Scheme	Marks
1.	Work done by force = $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ **. AB**	M1
	Attempt at equating work done to KE	M1
	$\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} \lambda \\ 2\lambda \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} x-2 \\ y+2 \\ z+3 \end{pmatrix} = \frac{1}{2}(0.1)5^2$	A1
	Solving for λ (λ = 0.25) or forming sufficient equations in x, y	M1
	[e.g $x + 2y = -0.75$, $y + 2 = 2(x - 2)$]	
	Method to find **OB**	M1
	[**OB** = $\begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ or solving for x, y]	
	OB = $2\frac{1}{4}$**i** – $1\frac{1}{2}$**j** – 3**k** any form	A1
		(6 marks)
Alt.	Non-vector approach:	
	$\lvert F \rvert \cos\theta = ma$ applied; [$a = 10\sqrt{5}$]	M1
	Method to find "s": $5^2 = 2(10\sqrt{5})s$ [$s = \dfrac{\sqrt{5}}{4}$]	M1 A1
	Finding λ	M1
	Method to find **OB**	M1 A1

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2. (*a*)	*Integrating factor approach:*	
	I F = $e^{\int 1 dt} = e^{t}$	B1
	Multiplying through $\Rightarrow$ $\frac{d}{dt}(\mathbf{r}e^{t}) = (\mathbf{i} - \mathbf{j})\,e^{-t}$	M1A1
	Integrating $\Rightarrow$ $\mathbf{r}\,e^{t} = -(\mathbf{i} - \mathbf{j})\,e^{-t}\,(+\,\mathbf{c})$	M1 A1 ft
	Using $\mathbf{r} = \mathbf{0}$, $t = 0$ to find $\mathbf{c}$ $[\mathbf{c} = \mathbf{i} - \mathbf{j}]$	M1
	$\Rightarrow$ $\mathbf{r} = -(\mathbf{i} - \mathbf{j})\,e^{-2t} + (\mathbf{i} - \mathbf{j})\,e^{-t}$	A1 (7)
(*b*)	Writing $\mathbf{r} = f(t)\mathbf{i} + g(t)\mathbf{j}$ or $x = f(t)$, $y = g(t)$ and attempt to eliminate t	M1
	$y = -x$	A1 (2)
		(9 marks)
Alt. (*a*)	AE $m + 1 = 0 \Rightarrow \mathbf{r} = \mathbf{A}e^{-t}$ [Form of PI: $\mathbf{r} = \mathbf{B}e^{-2t}$]	B1
	Equation for PI: $-2\,e^{-2t}\,\mathbf{B} + \mathbf{B}e^{-2t} = (\mathbf{i} - \mathbf{j})e^{-2}$	M1 A1
	$\mathbf{B} = -(\mathbf{i} - \mathbf{j})$	A1 ft
	General Solution: $\mathbf{r} = \mathbf{A}e^{-t} + (-\mathbf{i} + \mathbf{j})\,e^{-2t}$	M1
	Using $\mathbf{r} = \mathbf{0}$, $t = 0$ to find $\mathbf{A}$	M1
	$\mathbf{r} = (\mathbf{i} - \mathbf{j})e^{-t} + (-\mathbf{i} + \mathbf{j})e^{-2t}$	A1 (7)

(ft = follow through mark)

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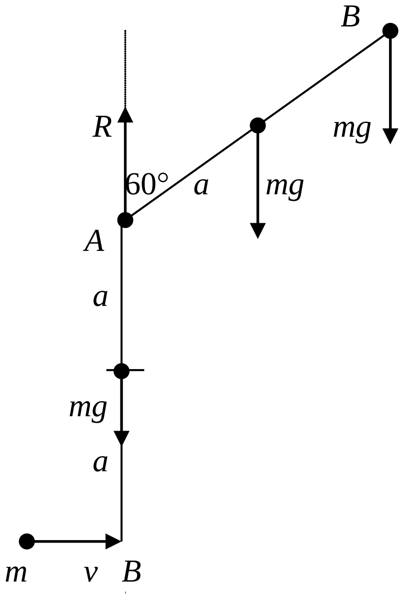
3. (*a*)	**R** = $\begin{pmatrix}4\\-2\\0\end{pmatrix}+\begin{pmatrix}4\\2\\0\end{pmatrix}+\begin{pmatrix}0\\-2\\1\end{pmatrix}+\begin{pmatrix}0\\2\\1\end{pmatrix}=\begin{pmatrix}8\\0\\2\end{pmatrix}$ or $8\mathbf{i}+2\mathbf{k}$	M1 A1 (2)
(*b*)	Finding one of $\begin{pmatrix}3\\0\\0\end{pmatrix}\times\begin{pmatrix}4\\-2\\0\end{pmatrix}$, $\begin{pmatrix}3\\4\\0\end{pmatrix}\times\begin{pmatrix}4\\2\\0\end{pmatrix}$, $\begin{pmatrix}0\\0\\3\end{pmatrix}\times\begin{pmatrix}0\\-2\\1\end{pmatrix}$, $\begin{pmatrix}0\\4\\3\end{pmatrix}\times\begin{pmatrix}0\\2\\1\end{pmatrix}$	M1
	$=\begin{pmatrix}0\\0\\-6\end{pmatrix},\begin{pmatrix}0\\0\\-10\end{pmatrix},\begin{pmatrix}6\\0\\0\end{pmatrix},\begin{pmatrix}-2\\0\\0\end{pmatrix}$	A2, 1, 0
	[A1 one correct, A2 at least three correct]	
	Resultant $=\begin{pmatrix}4\\0\\-16\end{pmatrix}$ any form	M1 A1 (5)
(*c*)	**F = –** 8**i –** 2**k**	B1ft (1)
(*d*)	For equilibrium $\mathbf{r}\times\begin{pmatrix}-8\\0\\-2\end{pmatrix}=-\begin{pmatrix}4\\0\\-16\end{pmatrix}$ or equivalent	M1
	$\mathbf{PX}=\begin{pmatrix}0\\\lambda\\0\end{pmatrix}\Rightarrow\mathbf{r}\times\begin{pmatrix}-8\\0\\-2\end{pmatrix}=\begin{pmatrix}-2\lambda\\0\\8\lambda\end{pmatrix}$	M1 A1 ft
	Finding λ ; $PX = 2$.	M1; A1 (5)
		(13 marks)

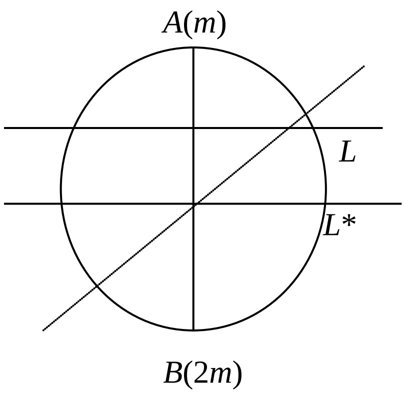
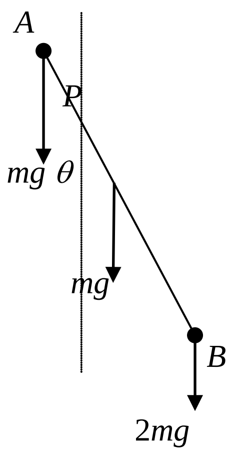
(ft = follow through mark)

Question Number	Scheme	Marks
4. (*a*)	$(m+\delta m)(v+\delta v)+(-\delta m)(v-U)-mv=-kv\delta t$	M1A1A1
	$\Rightarrow \quad m\dfrac{\mathrm{d}v}{\mathrm{d}t}+U\dfrac{\mathrm{d}m}{\mathrm{d}t}=-kv$	A1
	$m=M-\lambda t$	B1
	$\Rightarrow (M-\lambda t)\dfrac{\mathrm{d}v}{\mathrm{d}t}=\lambda U-kv$	M1
	$\Rightarrow \quad \dfrac{\mathrm{d}v}{\mathrm{d}t}=\dfrac{\lambda U-kv}{M-\lambda t}$ (*) (no incorrect working seen)	A1 cso (7)
(*b*)	Separating variables: $\displaystyle\int\frac{\mathrm{d}v}{U-v}=\int\frac{10}{M-10t}\mathrm{d}t$ or equivalent	M1
	Integrating: $\ln(U-v)=\ln(M-10t)$ $(+c)$	M1 A1
	Using limits correctly: $[\ ]_v^0=[\ ]_t^0$ applied or $t=0$, $v=0$ to find "c"	M1
	$\left[c=\ln\left(\dfrac{U}{M}\right)\right]$	
	Complete method to find v $\left[\ln\left(\dfrac{U}{U-v}\right)=\ln\left(\dfrac{M}{M-10t}\right)\right]$	M1
	$v=\dfrac{10Ut}{M}$	A1 (6)
		(13 marks)

(cso = correct solution only)

Question Number	Scheme	Marks
5. (*a*)	 $I_A = \{\frac{4}{3}ma^2 + m(2a)^2\}$	M1 A1
	$mv\,(2a) = I_A\,\omega = \dfrac{16ma^2}{3}\omega$	M1 A1ft
	$\omega = \dfrac{3v}{8a}$ * no wrong working seen	A1 cso (5)
	Gain in PE = $mg\,3a\,(1+\cos 60^\circ)$	M1 A1
	Attempt at $\frac{1}{2}I\ \omega^2$ = gain in PE	M1
	$\dfrac{1}{2}\left(\dfrac{16ma^2}{3}\right)_c\left(\dfrac{3v}{8a}\right)^2 = mg\,3a\,(1+\cos 60^\circ)$	A1 ft
	Finding v $v = \sqrt{12ga}$	M1 A1 (6)
(*c*)	Acceleration of C of G = $(\frac{3}{2}a\,\omega^2)$	B1
	$R - 2mg = mr\,\omega^2$; $= 2m(\frac{3}{2}a\,\omega^2)$	M1 A1
	Substitution of ω and v and finding $R = \ldots$	M1
	$R = \dfrac{113}{16}mg$	A1 (5)
		(16 marks)

(cso = correct solution only; ft = follow through mark)

Question Number	Scheme	Marks
6 (*a*)	$(\delta I) = (\rho)2\pi r\,\delta r \times r^2$	M1
	Using $(\rho) = \dfrac{m}{\pi a^2}$	M1
	Completion: $I = \dfrac{2m}{a^2}\left[\dfrac{r^4}{4}\right]_0^a = \frac{1}{2}ma^2$ (*)	M1 A1 (4)
	 Disc: Use of $\perp^r$ axis theorem to find I_{L*}	M1
	$I_{L*} = \frac{1}{2}(\frac{1}{2}ma^2) = \frac{1}{4}ma^2$	A1
	Use of parallel axis theorem	
	$I_L = \frac{1}{4}ma^2 + m\left(\dfrac{a}{2}\right)^2 = \frac{1}{2}ma^2$	M1 A1
	For loaded disc: $I = \frac{1}{2}ma^2 + m\left(\dfrac{a}{2}\right)^2 + 2m\left(\dfrac{3a}{2}\right)^2 = \dfrac{21}{4}ma^2$ (*)	M1 A1 cso (6)
(*c*)	$I\ddot{\theta} = \left\{mg\left(\dfrac{a}{2}\right)\sin\theta - mg\left(\dfrac{a}{2}\right)\sin\theta - 2mg\left(\dfrac{3a}{2}\right)\sin\theta\right\}$	M1 A1 A1
	 [A1 for signs, A1 "terms"]	
	$\left[\dfrac{21}{4}ma^2\ddot{\theta} = -3mga\sin\theta\right]$	
	For small angles $\theta \approx \sin\theta \Rightarrow$	
	$\dfrac{21}{4}ma^2\ddot{\theta} = -3mga\theta$	M1
	$\ddot{\theta} = -\dfrac{4g}{7a}\theta$	A1 ft
	$\Rightarrow$ SHM with $\omega^2 = \dfrac{4g}{7a}$	M1
	Time = $\dfrac{\pi}{\omega}$; $= \pi\sqrt{\dfrac{7a}{4g}}$ or $\dfrac{\pi}{2}\sqrt{\dfrac{7a}{g}}$	M1; A1 (8)
		(18 marks)

(cao = correct answer only; ft = follow through mark; (*) indicates final line is given on the paper)