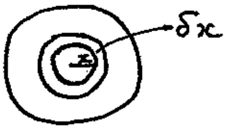
edexcel

January 2006
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1.	$\underset{\sim}{AB} = k(2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k})$; $AB = 6 \Rightarrow k\sqrt{(2^2+1^2+2^2)} = 6$	M1
	$\Rightarrow k = 2 \Rightarrow \underset{\sim}{AB} = 4\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k}$	A1
	Work done $= (7\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}) \cdot (4\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k})$	M1
	$= \underline{\underline{12\,J}}$	A1 (4)
2.	 $\delta m = \dfrac{m}{\pi a^2} \cdot 2\pi x\, \delta x$	M1 A1
	$\delta I = \dfrac{m}{\pi a^2} \cdot 2\pi x^3\, \delta x$	M1
	$I = \dfrac{2m}{a^2} \int_0^a x^3\, dx = \underline{\tfrac{1}{2} m a^2}$	M1 A1 (5)
3.	Either CF: $\dot{\underset{\sim}{r}} + 2\underset{\sim}{r} = \underset{\sim}{0}$	OR $IF = e^{2t}$	B1
	$\Rightarrow \underset{\sim}{r} = \underset{\sim}{A}e^{-2t}$ M1 A1	$\dfrac{d}{dt}(\underset{\sim}{r}e^{2t}) = 4\underset{\sim}{i}e^{2t}$	M1
	PI $\underset{\sim}{r} = 2\underset{\sim}{i}$ B1	$\underset{\sim}{r}e^{2t} = 2\underset{\sim}{i}e^{2t} + \underset{\sim}{A}$	A1
	GS $\underset{\sim}{r} = \underset{\sim}{A}e^{-2t} + 2\underset{\sim}{i}$ A1	$\underset{\sim}{r} = 2\underset{\sim}{i} + \underset{\sim}{A}e^{-2t}$	A1
	$t = 0$ $\underset{\sim}{r} = 2\underset{\sim}{j} \Rightarrow \underset{\sim}{A} = 2\underset{\sim}{j} - 2\underset{\sim}{i}$	M1
	$\underset{\sim}{r} = \underline{(2\underset{\sim}{j} - 2\underset{\sim}{i})e^{-2t} + 2\underset{\sim}{i}}$ or $\underline{2\underset{\sim}{i}(1-e^{-2t}) + 2\underset{\sim}{j}e^{-2t}}$	A1 (6)
4.	 (a) $M(A)$: $\tfrac{4}{3}ma^2\ddot{\theta} = -mga\sin\theta$	M1 A1
	Small $\theta \Rightarrow \sin\theta \simeq \theta \rightarrow \ddot{\theta} = -\dfrac{3g}{4a}\theta$	M1
	$\therefore$ Approx. SHM	A1 (4)
	(b) Time $= \tfrac{1}{4}$ period $= \tfrac{1}{4} \cdot 2\pi\sqrt{\dfrac{4a}{3g}} = \underline{\pi\sqrt{\dfrac{a}{3g}}}$	M1 A1 (2) (6)

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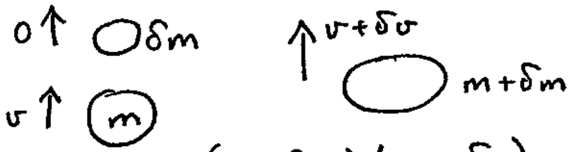
5.	MI of disc abt diameter $= \frac{1}{4}ma^2$	M1
	$\therefore$ MI of disc about axis $= \frac{1}{4}ma^2 + ma^2 = \frac{5}{4}ma^2$	M1 A1
	CAM: $m.a.\sqrt{kga} = \left(\frac{5}{4}ma^2 + ma^2\right)\omega$	M1 A1
	$\Rightarrow \omega = \frac{4}{9}\sqrt{\frac{kg}{a}}$	A1
	Energy: $\frac{1}{2}\cdot\frac{9ma^2}{4}\cdot\frac{16kg}{81a} = 2mga$	M1 A1 A1
	$\Rightarrow \underline{k = 9}$	A1 (10)
6.	(a) $\underline{SQ} = \underline{q} - \underline{s} = 4\underline{j} - 3\underline{k} \Rightarrow \underline{F}_1 = 16\underline{j} - 12\underline{k}$	M1 A1
	$\underline{SR} = -3\underline{i} - 2\underline{j} \Rightarrow \underline{F}_2 = -6\underline{i} - 4\underline{j}$	M1 A1
	Net couple alone $\Rightarrow \Sigma \underline{F}_i = \underline{0}$	
	$\Rightarrow \underline{F}_3 = \underline{6\underline{i} - 12\underline{j} + 12\underline{k}}$	M1 A1 (6)
	(b) M(S) $\underline{G} = \underline{SP} \times \underline{F}_3$	M1
	$= (-7\underline{i} + 4\underline{j} - 2\underline{k}) \times (6\underline{i} - 12\underline{j} + 12\underline{k})$	A1
	(or complete expressⁿ if about another pt.)	
	$\underline{G} = 24\underline{i} + 72\underline{j} + 60\underline{k}$	M1 A1
	$= 12(2\underline{i} + 6\underline{j} + 5\underline{k})$	
	$\lvert\underline{G}\rvert = \underline{12\sqrt{65}}$	M1 A1 (6)

7.		
	(a) $(m+\delta m)(v+\delta v) - mv = -mg\,\delta t$	M1 A2,1,0
	$\cancel{mv} + m\delta v + v\delta m + \cancel{\delta m\,\delta v} - \cancel{mv} = -mg\,\delta t$	
	$\underline{m\frac{dv}{dt} + v\frac{dm}{dt} = -mg}$ (*)	M1 A1 (5)
	(b) $m = Me^{kt} \Rightarrow \frac{dm}{dt} = kMe^{kt}$	B1
	Hence $\cancel{M}e^{kt}\frac{dv}{dt} + \cancel{k}v.\,k\cancel{M}e^{kt} = -\cancel{M}e^{kt}g$	M1
	$\Rightarrow \underline{\frac{d}{dt}(ve^{kt}) = -ge^{kt}}$ (*)	A1 (3)
	(c) $ve^{kt} = -g\int e^{kt}\,dt$	M1
	$= -\frac{g}{k}e^{kt}\ (+C)$	A1
	$t = 0,\ v = \frac{g}{2k} \Rightarrow C = \frac{3g}{2k}$	B1
	At highest pt $v = 0$: $-\frac{g}{k}e^{kt} + \frac{3g}{2k} = 0$	M1
	$\Rightarrow e^{kt} = \frac{3}{2}$	A1
	Hence $m = Me^{kt} = \underline{\frac{3M}{2}}$	M1 A1 (7)

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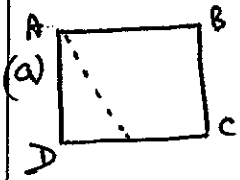
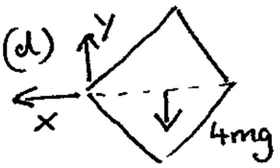
8.	(a)  $I_{AB} = \underset{(BC,\ AD)}{2 \times \frac{4}{3}ma^2} + \underset{(CD)}{m(2a)^2} = \frac{20}{3}ma^2$	M1 A1 A1
	By $\perp$r axis : $I_A = 2 \times \frac{20}{3}ma^2 = \underline{\frac{40}{3}ma^2}$ (*)	M1 A1 (5)
	[or $I_A = \underset{(AB,\ AD)}{2 \times \frac{4}{3}ma^2}, + \underset{(BC,\ CD)}{2\left(\frac{1}{3}ma^2 + 5ma^2\right)}$	[M1, M1 A1 A1
	$= \underline{\frac{40}{3}ma^2}$	A1]
	(b) M(A) $\frac{40ma^2}{3}\ddot{\theta} = 4mg.a\sqrt{2}$	M1 A1
	$\ddot{\theta} = \underline{\frac{3g\sqrt{2}}{10a}}$	A1 (3)
	(c) $\frac{1}{2}.\frac{40ma^2}{3}\dot{\theta}^2 = 4mg.a\sqrt{2}$	M1 A1
	$\dot{\theta} = \sqrt{\frac{3g\sqrt{2}}{5a}}$	A1 (3)
	(d)  R($\leftarrow$) $X = 4ma\sqrt{2}\,\dot{\theta}^2$	M1
	$= 4ma\sqrt{2}.\frac{3g\sqrt{2}}{5a} = \frac{24mg}{5}$	A1
	R($\uparrow$) $4mg - Y = 4ma\sqrt{2}\,\ddot{\theta}$	M1
	$Y = 4mg - 4ma\sqrt{2}.\frac{3g\sqrt{2}}{10a}$	
	$Y = \frac{8mg}{5}$	A1
	$R = \sqrt{(X^2 + Y^2)} = \frac{8mg}{5}\sqrt{(1^2 + 3^2)}$	M1
	$= \underline{\frac{8\sqrt{10}}{5}mg}$	A1