Edexcel Maths M5

Past Paper Pack

2009-2013

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	8	1	/	0	1	Signature	

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Tuesday 23 June 2009 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
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Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. You must write your answer to each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. At time $t = 0$, a particle P of mass 3 kg is at rest at the point A with position vector $(\mathbf{j} - 3\mathbf{k})$ m. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ then act on the particle P and it passes through the point B with position vector $(8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ m.

Given that $\mathbf{F}_1 = (4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ N and $\mathbf{F}_2 = (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})$ N and that $\mathbf{F}_1$ and $\mathbf{F}_2$ are the *only* two forces acting on P, find the velocity of P as it passes through B, giving your answer as a vector.

(7)

Leave blank

2. At time t seconds, the position vector of a particle P is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the vector differential equation

$$\frac{d^2\mathbf{r}}{dt^2} + 4\mathbf{r} = e^{2t}\mathbf{j}.$$

When $t = 0$, P has position vector $(\mathbf{i} + \mathbf{j})$ m and velocity $2\mathbf{i}$ m s^{-1}.

Find an expression for $\mathbf{r}$ in terms of t.

(11)

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Question 2 continued

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3. A spaceship is moving in a straight line in deep space and needs to increase its speed. This is done by ejecting fuel backwards from the spaceship at a constant speed c relative to the spaceship. When the speed of the spaceship is v, its mass is m.

(a) Show that, while the spaceship is ejecting fuel,

$$\frac{dv}{dm} = -\frac{c}{m}.$$

(5)

The initial mass of the spaceship is m_0 and at time t the mass of the spaceship is given by $m = m_0(1 - kt)$, where k is a positive constant.

(b) Find the acceleration of the spaceship at time t.

(4)

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Question 3 continued

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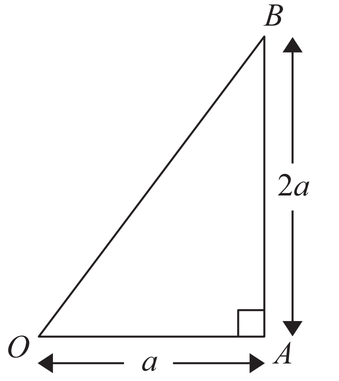

Figure 1

A uniform lamina of mass M is in the shape of a right-angled triangle OAB. The angle OAB is 90°, $OA = a$ and $AB = 2a$, as shown in Figure 1.

(a) Prove, using integration, that the moment of inertia of the lamina OAB about the edge OA is $\frac{2}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through one end and perpendicular to the rod is $\frac{4}{3}ml^2$.)

(6)

The lamina OAB is free to rotate about a fixed smooth horizontal axis along the edge OA and hangs at rest with B vertically below A. The lamina is then given a horizontal impulse of magnitude J. The impulse is applied to the lamina at the point B, in a direction which is perpendicular to the plane of the lamina. Given that the lamina first comes to instantaneous rest after rotating through an angle of 120°,

(b) find an expression for J, in terms of M, a and g.

(7)

Leave blank

Question 4 continued

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5. Two forces $\mathbf{F}_1 = (2\mathbf{i} + \mathbf{j})$ N and $\mathbf{F}_2 = (-2\mathbf{j} - \mathbf{k})$ N act on a rigid body. The force $\mathbf{F}_1$ acts at the point with position vector $\mathbf{r}_1 = (3\mathbf{i} + \mathbf{j} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts at the point with position vector $\mathbf{r}_2 = (\mathbf{i} - 2\mathbf{j})$ m. A third force $\mathbf{F}_3$ acts on the body such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are in equilibrium.

(a) Find the magnitude of $\mathbf{F}_3$.

(4)

(b) Find a vector equation of the line of action of $\mathbf{F}_3$.

(8)

The force $\mathbf{F}_3$ is replaced by a fourth force $\mathbf{F}_4$, acting through the origin *O*, such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ are equivalent to a couple.

(c) Find the magnitude of this couple.

(4)

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Question 5 continued

N 3 4 2 7 7 R A 0 1 7 2 4

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6. A pendulum consists of a uniform rod AB, of length $4a$ and mass $2m$, whose end A is rigidly attached to the centre O of a uniform square lamina $PQRS$, of mass $4m$ and side a. The rod AB is perpendicular to the plane of the lamina. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through B. The axis L is perpendicular to AB and parallel to the edge PQ of the square.

(a) Show that the moment of inertia of the pendulum about L is $75ma^2$.

(4)

The pendulum is released from rest when BA makes an angle α with the downward vertical through B, where $\tan\alpha = \frac{7}{24}$. When BA makes an angle θ with the downward vertical through B, the magnitude of the component, in the direction AB, of the force exerted by the axis L on the pendulum is X.

(b) Find an expression for X in terms of m, g and θ.

(9)

Using the approximation $\theta \approx \sin\theta$,

(c) find an estimate of the time for the pendulum to rotate through an angle α from its initial rest position.

(6)

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Question 6 continued

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 8 1 / 0 1	Signature	

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Monday 28 June 2010 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
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Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

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1. At time $t = 0$, the position vector of a particle P is $-3\mathbf{j}$ m. At time t seconds, the position vector of P is $\mathbf{r}$ metres and the velocity of P is $\mathbf{v}$ m s^{-1}. Given that

$$\mathbf{v} - 2\mathbf{r} = 4e^{t}\,\mathbf{j},$$

find the time when P passes through the origin.

(7)

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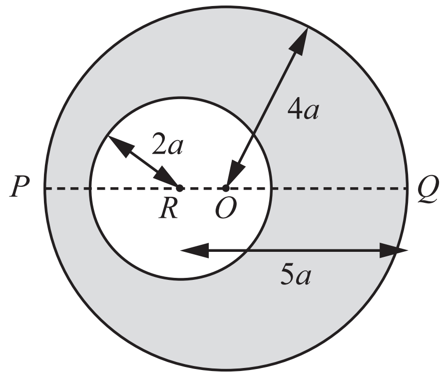

Figure 1

A uniform circular disc has mass $4m$, centre O and radius $4a$. The line POQ is a diameter of the disc. A circular hole of radius $2a$ is made in the disc with the centre of the hole at the point R on PQ where $QR = 5a$, as shown in Figure 1.

The resulting lamina is free to rotate about a fixed smooth horizontal axis L which passes through Q and is perpendicular to the plane of the lamina.

(a) Show that the moment of inertia of the lamina about L is $69ma^2$.

(7)

The lamina is hanging at rest with P vertically below Q when it is given an angular velocity Ω. Given that the lamina turns through an angle $\frac{2\pi}{3}$ before it first comes to instantaneous rest,

(b) find Ω in terms of g and a.

(6)

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Question 2 continued

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3. A uniform lamina ABC of mass m is in the shape of an isosceles triangle with $AB = AC = 5a$ and $BC = 8a$.

(a) Show, using integration, that the moment of inertia of the lamina about an axis through A, parallel to BC, is $\frac{9}{2}ma^2$.

(6)

The foot of the perpendicular from A to BC is D. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis which passes through D and is perpendicular to the plane of the lamina. The lamina is released from rest when DA makes an angle α with the downward vertical. It is given that the moment of inertia of the lamina about an axis through A, perpendicular to BC and in the plane of the lamina, is $\frac{8}{3}ma^2$.

(b) Find the angular acceleration of the lamina when DA makes an angle θ with the downward vertical.

(8)

Given that α is small,

(c) find an approximate value for the period of oscillation of the lamina about the vertical.

(2)

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Question 3 continued

Turn over

N 3 5 3 9 4 A 0 9 2 4

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4. Two forces $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ N and $\mathbf{F}_2 = (3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ N act on a rigid body. The force $\mathbf{F}_1$ acts through the point with position vector $(2\mathbf{i} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts through the point with position vector $(\mathbf{j} + 2\mathbf{k})$ m.

(a) If the two forces are equivalent to a single force $\mathbf{R}$, find

(i) $\mathbf{R}$,

(2)

(ii) a vector equation of the line of action of $\mathbf{R}$, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$.

(6)

(b) If the two forces are equivalent to a single force acting through the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m together with a couple of moment $\mathbf{G}$, find the magnitude of $\mathbf{G}$.

(5)

N 3 5 3 9 4 A 0 1 2 2 4

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Question 4 continued

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5. A raindrop falls vertically under gravity through a cloud. In a model of the motion the raindrop is assumed to be spherical at all times and the cloud is assumed to consist of stationary water particles. At time $t = 0$, the raindrop is at rest and has radius a. As the raindrop falls, water particles from the cloud condense onto it and the radius of the raindrop is assumed to increase at a constant rate λ. A time t the speed of the raindrop is v.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{3\lambda v}{(\lambda t + a)} = g\,.$$

(8)

(b) Find the speed of the raindrop when its radius is $3a$.

(7)

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Question 5 continued

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6. A uniform circular disc has mass m, centre O and radius $2a$. It is free to rotate about a fixed smooth horizontal axis L which lies in the same plane as the disc and which is tangential to the disc at the point A. The disc is hanging at rest in equilibrium with O vertically below A when it is struck at O by a particle of mass m. Immediately before the impact the particle is moving perpendicular to the plane of the disc with speed $3\sqrt{(ag)}$. The particle adheres to the disc at O.

(a) Find the angular speed of the disc immediately after the impact.

(5)

(b) Find the magnitude of the force exerted on the disc by the axis immediately after the impact.

(6)

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Question 6 continued

June 2011

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	8	1	/	0	1	Signature	

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Friday 24 June 2011 – Afternoon

Time: 1 hour 30 minutes

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Team Leader's use only

Question Number	Leave Blank
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Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

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1. A particle moves from the point A with position vector $(3\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ m to the point B with position vector $(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$ m under the action of the force $(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})$ N. Find the work done by the force.

(4)

Leave blank

2. A particle P moves in the x-y plane so that its position vector $\mathbf{r}$ metres at time t seconds satisfies the differential equation

$$\frac{d^2\mathbf{r}}{dt^2} - 4\mathbf{r} = -3e^t\mathbf{j}$$

When $t = 0$, the particle is at the origin and is moving with velocity $(2\mathbf{i} + \mathbf{j})\ \mathrm{m\,s^{-1}}$.

Find $\mathbf{r}$ in terms of t.

(10)

Leave blank

Question 2 continued

Leave blank

3. A rocket propels itself by its engine ejecting burnt fuel. Initially the rocket has total mass M, of which a mass kM, $k < 1$, is fuel. The rocket is at rest when its engine is started. The burnt fuel is ejected with constant speed c, relative to the rocket, in a direction opposite to that of the rocket's motion. Assuming that there are no external forces, find the speed of the rocket when all its fuel has been burnt.

(7)

Leave blank

4. Two forces $\mathbf{F}_1 = (3\mathbf{j} + \mathbf{k})$ N and $\mathbf{F}_2 = (4\mathbf{i} + \mathbf{j} - \mathbf{k})$ N act on a rigid body.
The force $\mathbf{F}_1$ acts at the point with position vector $(2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ m and the force $\mathbf{F}_2$ acts at the point with position vector $(-3\mathbf{i} + 2\mathbf{k})$ m.
The two forces are equivalent to a single force $\mathbf{R}$ acting at the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m together with a couple of moment $\mathbf{G}$.

Find,

(a) $\mathbf{R}$,

(2)

(b) $\mathbf{G}$.

(4)

A third force $\mathbf{F}_3$ is now added to the system. The force $\mathbf{F}_3$ acts at the point with position vector $(2\mathbf{i} - \mathbf{k})$ m and the three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are equivalent to a couple.

(c) Find the magnitude of the couple.

(6)

Leave blank

5. A uniform rod PQ, of mass m and length $2a$, is made to rotate in a vertical plane with constant angular speed $\sqrt{\left(\frac{g}{a}\right)}$ about a fixed smooth horizontal axis through the end P of the rod.

Show that, when the rod is inclined at an angle θ to the downward vertical, the magnitude of the force exerted on the axis by the rod is $2mg\left|\cos(\frac{1}{2}\theta)\right|$.

(8)

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6. A uniform rod AB of mass $4m$ is free to rotate in a vertical plane about a fixed smooth horizontal axis, L, through A. The rod is hanging vertically at rest when it is struck at its end B by a particle of mass m. The particle is moving with speed u, in a direction which is horizontal and perpendicular to L, and after striking the rod it rebounds in the opposite direction with speed v. The coefficient of restitution between the particle and the rod is 1.

Show that $u = 7v$.

(7)

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7. Prove, using integration, that the moment of inertia of a uniform solid right circular cone, of mass M and base radius a, about its axis is $\frac{3}{10}Ma^2$.

[You may assume, without proof, that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre and perpendicular to its plane is $\frac{1}{2}mr^2$.]

(10)

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Question 7 continued

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8. A pendulum consists of a uniform rod PQ, of mass $3m$ and length $2a$, which is rigidly fixed at its end Q to the centre of a uniform circular disc of mass m and radius a. The rod is perpendicular to the plane of the disc. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through the end P of the rod and is perpendicular to the rod.

(a) Show that the moment of inertia of the pendulum about L is $\frac{33}{4}ma^2$.
(5)

The pendulum is released from rest in the position where PQ makes an angle α with the downward vertical. At time t, PQ makes an angle θ with the downward vertical.

(b) Show that the angular speed, $\dot{\theta}$, of the pendulum satisfies

$$\dot{\theta}^2 = \frac{40g(\cos\theta - \cos\alpha)}{33a}$$

(4)

(c) Hence, or otherwise, find the angular acceleration of the pendulum.
(3)

Given that $\alpha = \frac{\pi}{20}$ and that PQ has length $\frac{8}{33}$ m,

(d) find, to 3 significant figures, an approximate value for the angular speed of the pendulum 0.2 s after it has been released from rest.
(5)

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Question 8 continued

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Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 8 1 / 0 1	Signature	

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Monday 25 June 2012 – Afternoon

Time: 1 hour 30 minutes

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Team Leader's use only

Question Number	Leave Blank
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Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

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1. A particle P moves in a plane such that its position vector $\mathbf{r}$ metres at time t seconds ($t > 0$) satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} - \frac{2}{t}\mathbf{r} = 4\mathbf{i}$$

When $t = 1$, the particle is at the point with position vector $(\mathbf{i} + \mathbf{j})$ m.

Find $\mathbf{r}$ in terms of t.

(9)

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2. A rocket, with initial mass 1500 kg, including 600 kg of fuel, is launched vertically upwards from rest. The rocket burns fuel at a rate of 15 kg s^{-1} and the burnt fuel is ejected vertically downwards with a speed of 1000 m s^{-1} relative to the rocket. At time t seconds after launch ($t \leqslant 40$) the rocket has mass m kg and velocity v m s^{-1}.

(a) Show that

$$\frac{dv}{dt} + \frac{1000}{m}\frac{dm}{dt} = -9.8$$

(5)

(b) Find v at time t, $0 \leqslant t \leqslant 40$

(5)

Leave blank

Question 2 continued

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3. A uniform rod PQ, of mass m and length $3a$, is free to rotate about a fixed smooth horizontal axis L, which passes through the end P of the rod and is perpendicular to the rod. The rod hangs at rest in equilibrium with Q vertically below P. One end of a light inextensible string of length $2a$ is attached to the rod at P and the other end is attached to a particle of mass $3m$. The particle is held with the string taut, and horizontal and perpendicular to L, and is then released. After colliding, the particle sticks to the rod forming a body B.

(a) Show that the moment of inertia of B about L is $15ma^2$.

(2)

(b) Show that B first comes to instantaneous rest after it has turned through an angle $\arccos\left(\frac{9}{25}\right)$.

(10)

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Question 3 continued

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4. A body consists of a uniform plane circular disc, of radius r and mass $2m$, with a particle of mass $3m$ attached to the circumference of the disc at the point P.
The line PQ is a diameter of the disc. The body is free to rotate in a vertical plane about a fixed smooth horizontal axis, L, which is perpendicular to the plane of the disc and passes through Q. The body is held with QP making an angle β with the downward vertical through Q, where $\sin \beta = 0.25$, and released from rest. Find the magnitude of the component, perpendicular to PQ, of the force acting on the body at Q at the instant when it is released.

[*You may assume that the moment of inertia of the body about L is* $15mr^2$.]

(6)

Leave blank

5. The points P and Q have position vectors $4\mathbf{i} - 6\mathbf{j} - 12\mathbf{k}$ and $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ respectively, relative to a fixed origin O.

Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, act along $\overrightarrow{OP}$, $\overrightarrow{QO}$ and $\overrightarrow{QP}$ respectively, and have magnitudes 7 N, 3 N and $3\sqrt{10}$ N respectively.

(a) Express $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ in vector form.

(3)

(b) Show that the resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is $(2\mathbf{i} - 10\mathbf{j} - 16\mathbf{k})$ N.

(2)

(c) Find a vector equation of the line of action of this resultant, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a parameter.

(5)

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Question 5 continued

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6. A uniform circular pulley, of mass $4m$ and radius r, is free to rotate about a fixed smooth horizontal axis which passes through the centre of the pulley and is perpendicular to the plane of the pulley. A light inextensible string passes over the pulley and has a particle of mass $2m$ attached to one end and a particle of mass $3m$ attached to the other end. The particles hang with the string vertical and taut on each side of the pulley. The rim of the pulley is sufficiently rough to prevent the string slipping. The system is released from rest.

(a) Find the angular acceleration of the pulley.

(8)

When the angular speed of the pulley is Ω, the string breaks and a constant braking couple of magnitude G is applied to the pulley which brings it to rest.

(b) Find an expression for the angle turned through by the pulley from the instant when the string breaks to the instant when the pulley first comes to rest.

(4)

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Question 6 continued

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7. (a) A uniform lamina of mass m is in the shape of a triangle ABC. The perpendicular distance of C from the line AB is h. Prove, using integration, that the moment of inertia of the lamina about AB is $\frac{1}{6}mh^2$.

(7)

(b) Deduce the radius of gyration of a uniform square lamina of side $2a$, about a diagonal.

(3)

The points X and Y are the mid-points of the sides RQ and RS respectively of a square $PQRS$ of side $2a$. A uniform lamina of mass M is in the shape of $PQXYS$.

(c) Show that the moment of inertia of this lamina about XY is $\frac{79}{84}Ma^2$.

(6)

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Question 7 continued

Centre No.					
Candidate No.					

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Paper Reference(s)

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Mechanics M5

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

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Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. A particle moves in a plane in such a way that its position vector **r** metres at time t seconds satisfies the differential equation

$$\frac{d^2\mathbf{r}}{dt^2} - 2\frac{d\mathbf{r}}{dt} = \mathbf{0}$$

When $t = 0$, the particle is at the origin and is moving with velocity $(4\mathbf{i} + 2\mathbf{j})\text{ m s}^{-1}$.

Find **r** in terms of t.

(7)

Leave blank

2. Three forces $\mathbf{F}_1 = (3\mathbf{i} - \mathbf{j} + \mathbf{k})$ N, $\mathbf{F}_2 = (2\mathbf{i} - \mathbf{k})$ N, and $\mathbf{F}_3$ act on a rigid body.

The force $\mathbf{F}_1$ acts through the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m, the force $\mathbf{F}_2$ acts through the point with position vector $(\mathbf{i} - 2\mathbf{j})$ m and the force $\mathbf{F}_3$ acts through the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m.

Given that the system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ reduces to a couple $\mathbf{G}$,

(a) find $\mathbf{G}$.

(6)

The line of action of $\mathbf{F}_3$ is changed so that the system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ now reduces to a couple $(6\mathbf{i} + 8\mathbf{j} + 2\mathbf{k})$ N m.

(b) Find an equation of the new line of action of $\mathbf{F}_3$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(5)

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Question 2 continued

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3. A spacecraft is moving in a straight line in deep space. The spacecraft moves by ejecting burnt fuel backwards at a constant speed of 2000 m s^{-1} relative to the spacecraft. The burnt fuel is ejected at a constant rate of c kg s^{-1}. At time t seconds the total mass of the spacecraft, including fuel, is m kg and the speed of the spacecraft is v m s^{-1}.

(a) Show that, while the spacecraft is ejecting burnt fuel,

$$m\frac{dv}{dt} = 2000c$$

(7)

At time $t = 0$, the mass of the spacecraft is M_0 kg and the speed of the spacecraft is 2000 m s^{-1}. When $t = 50$, the spacecraft is still ejecting burnt fuel and its speed is 6000 m s^{-1}.

(b) Find c in terms of M_0.

(7)

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Question 3 continued

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4. Show, using integration, that the moment of inertia of a uniform solid right circular cone of mass M, height h and base radius a, about an axis through the vertex, parallel to the base, is

$$\frac{3M}{20}(a^2 + 4h^2)$$

[*You may assume without proof that the moment of inertia of a uniform circular disc, of radius r and mass m, about a diameter is* $\frac{1}{4}mr^2$.]

(13)

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Question 4 continued

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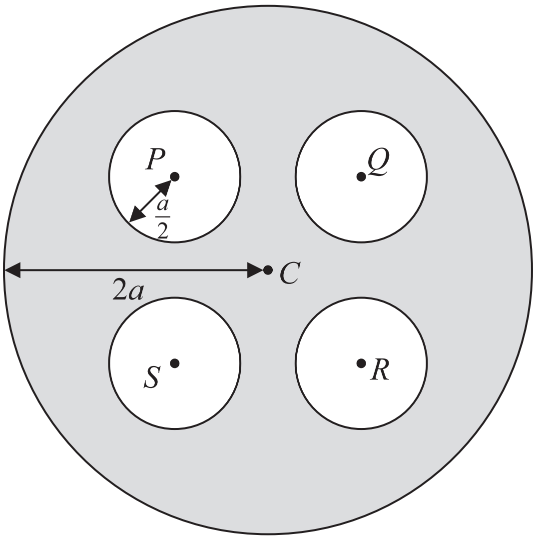

Figure 1

A uniform circular lamina has radius $2a$ and centre C. The points P, Q, R and S on the lamina are the vertices of a square with centre C and $CP = a$. Four circular discs, each of radius $\frac{a}{2}$, with centres P, Q, R and S, are removed from the lamina. The remaining lamina forms a template T, as shown in Figure 1.

The radius of gyration of T about an axis through C, perpendicular to T, is k.

(a) Show that $k^2 = \dfrac{55a^2}{24}$

(7)

The template T is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to T and passes through a point on its outer rim.

(b) Write down an equation of rotational motion for T and deduce that the period of small oscillations of T about its stable equilibrium position is

$$2\pi\sqrt{\left(\frac{151a}{48g}\right)}$$

(8)

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Question 5 continued

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6. A uniform circular disc, of radius r and mass m, is free to rotate in a vertical plane about a fixed smooth horizontal axis L which is perpendicular to the plane of the disc and passes through a point which is $\frac{1}{4}r$ from the centre of the disc. The disc is held at rest with its centre vertically above the axis. The disc is then slightly disturbed from its rest position. You may assume without proof that the moment of inertia of the disc about L is $\frac{9mr^2}{16}$.

(a) Show that the angular speed of the disc when it has turned through $\frac{\pi}{2}$ is $\sqrt{\left(\frac{8g}{9r}\right)}$.

(4)

(b) Find the magnitude of the force exerted on the disc by the axis when the disc has turned through $\frac{\pi}{2}$.

(11)

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Question 6 continued

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 8 1 / 0 1	Signature	

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
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Materials required for examination	Items included with question papers
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PEARSON

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1. Solve the differential equation

$$\frac{d\mathbf{r}}{dt} - 2\mathbf{r} = \mathbf{0}$$

given that when $t = 0$, $\mathbf{r}.\mathbf{j} = 0$ and $\mathbf{r} \times \mathbf{j} = \mathbf{i} + \mathbf{k}$.

(7)

Leave blank

2. A uniform square lamina S has side $2a$. The radius of gyration of S about an axis through a vertex, perpendicular to S, is k.

(a) Show that $k^2 = \dfrac{8a^2}{3}$.

(4)

The lamina S is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to S and passes through a vertex.

(b) By writing down an equation of rotational motion for S, find the period of small oscillations of S about its position of stable equilibrium.

(5)

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3. A raindrop falls vertically under gravity through a stationary cloud. At time $t = 0$, the raindrop is at rest and has mass m_0. As the raindrop falls, water condenses onto it from the cloud so that the mass of the raindrop increases at a constant rate c. At time t, the mass of the raindrop is m and the speed of the raindrop is v. The resistance to the motion of the raindrop has magnitude mkv, where k is a constant. Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} + v\left(k + \frac{c}{m_0 + ct}\right) = g$$

(7)

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4. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act through the points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively.

$\mathbf{r}_1 = (-2\mathbf{i} + 3\mathbf{j})$ m, $\quad$ $\mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ N

$\mathbf{r}_2 = (3\mathbf{i} + 2\mathbf{k})$ m, $\quad$ $\mathbf{F}_2 = (-2\mathbf{i} + \mathbf{j} - \mathbf{k})$ N

Given that the system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is in equilibrium,

(a) find $\mathbf{F}_3$,

(2)

(b) find a vector equation of the line of action of $\mathbf{F}_3$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

(5)

The force $\mathbf{F}_3$ is replaced by a force $\mathbf{F}_4$ acting through the point with position vector $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ m. The system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ is equivalent to a single force $(3\mathbf{i} + \mathbf{j} + \mathbf{k})$ N acting through the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m together with a couple.

(c) Find the magnitude of this couple.

(8)

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Question 4 continued

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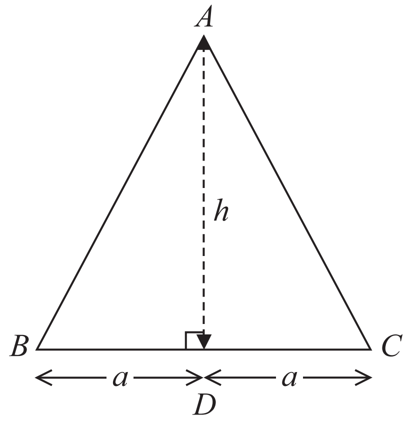

Figure 1

A uniform triangular lamina ABC, of mass M, has $AB = AC$ and $BC = 2a$. The mid-point of BC is D and $AD = h$, as shown in Figure 1.

Show, using integration, that the moment of inertia of the lamina about an axis through A, perpendicular to the plane of the lamina, is

$$\frac{M}{6}(a^2 + 3h^2)$$

[*You may assume without proof that the moment of inertia of a uniform rod, of length 2l and mass m, about an axis through its midpoint and perpendicular to the rod, is* $\frac{1}{3}ml^2$.]

(10)

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Question 5 continued

6.

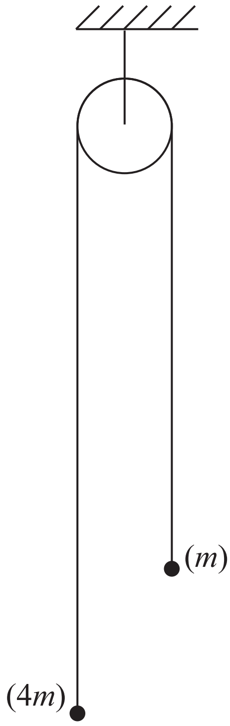

Figure 2

A light inextensible string has a particle of mass m attached to one end and a particle of mass $4m$ attached to the other end. The string passes over a rough pulley which is modelled as a uniform circular disc of radius a and mass $2m$, as shown in Figure 2.

The pulley can rotate in a vertical plane about a fixed horizontal axis which passes through the centre of the pulley and is perpendicular to the plane of the pulley. As the pulley rotates, a frictional couple of constant magnitude $2mga$ acts on it.

The system is held with the string vertical and taut on each side of the pulley and released from rest. Given that the string does not slip on the pulley, find the initial angular acceleration of the pulley.

(10)

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Question 6 continued

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7. A uniform circular disc, of radius r and mass m, is free to rotate in a vertical plane about a fixed smooth horizontal axis. This axis is perpendicular to the plane of the disc and passes through a point A on the circumference of the disc. The disc is held with AB horizontal, where AB is a diameter of the disc, and released from rest.

(a) Find the magnitude of

(i) the horizontal component,

(ii) the vertical component

of the force exerted on the disc by the axis immediately after the disc is released.

(11)

When AB is vertical the disc is instantaneously brought to rest by a horizontal impulse which acts in the plane of the disc and is applied to the disc at B.

(b) Find the magnitude of the impulse.

(6)

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Question 7 continued