Paper Reference(s)
6681

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Tuesday 18 June 2002 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions. Pages 6, 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A small smooth ring of mass 0.5 kg moves along a smooth horizontal wire. The only forces acting on the ring are its weight, the normal reaction from the wire, and a constant force (5**i** + **j** – 3**k**) N. The ring is initially at rest at the point with position vector (**i** + **j** + **k**) m, relative to a fixed origin.

Find the speed of the ring as it passes through the point with position vector (3**i** + **k**) m.

(5)

2. Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body. $\mathbf{F}_1 = (2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ N, $\mathbf{F}_2 = (\mathbf{i} + \mathbf{j} - 4\mathbf{k})$ N and $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j} + r\mathbf{k})$ N, where p, q and r are constants. All three forces act through the point with position vector (3**i** – 2**j** + **k**) m, relative to a fixed origin. The three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are equivalent to a single force (5**i** – 4**j** + 2**k**) N, acting at the origin, together with a couple **G**.

(*a*) Find the values of p, q and r.

(3)

(*b*) Find **G**.

(3)

3. At time t seconds, the position vector of a particle P is **r** metres, relative to a fixed origin. The particle moves in such a way that

$$\frac{d^2\mathbf{r}}{dt^2} - 4\frac{d\mathbf{r}}{dt} = \mathbf{0}.$$

At $t = 0$, P is moving with velocity (8**i** – 6**j**) m s^{-1}.

Find the speed of P when $t = \frac{1}{2}\ln 2$.

(7)

4. A uniform plane lamina of mass m is in the shape of an equilateral triangle of side $2a$. Find, using integration, the moment of inertia of the lamina about one of its edges.

(9)

5. A rocket is launched vertically upwards from rest. Initially, the total mass of the rocket and its fuel is 1000 kg. The rocket burns fuel at a rate of 10 kg s^{-1}. The burnt fuel is ejected vertically downwards with a speed of 2000 m s^{-1} relative to the rocket, and burning stops after one minute. At time *t* seconds, $t \leq 60$, after the launch, the speed of the rocket is *v* m s^{-1}. Air resistance is assumed to be negligible.

(*a*) Show that

$$-9.8(100 - t) = (100 - t)\frac{dv}{dt} - 2000.$$

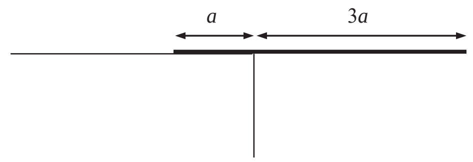
(8)

(*b*) Find the speed of the rocket when burning stops.

(6)

6.

Figure 1



A rough uniform rod, of mass *m* and length 4*a*, is rod is held on a rough horizontal table. The rod is perpendicular to the edge of the table and a length 3*a* projects horizontally over the edge, as shown in Fig. 1.

(*a*) Show that the moment of inertia of the rod about the edge of the table is $\frac{7}{3}ma^2$.

(2)

The rod is released from rest and rotates about the edge of the table. When the rod has turned through an angle θ, its angular speed is $\dot{\theta}$. Assuming that the rod has not started to slip,

(*b*) show that $\dot{\theta}^2 = \frac{6g\sin\theta}{7a}$,

(3)

(*c*) find the angular acceleration of the rod,

(2)

(*d*) find the normal reaction of the table on the rod.

(4)

The coefficient of friction between the rod and the edge of the table is μ.

(*e*) Show that the rod starts to slip when $\tan\theta = \frac{4}{13}\mu$

(6)

7. A uniform plane circular disc, of mass m and radius a, hangs in equilibrium from a point B on its circumference. The disc is free to rotate about a fixed smooth horizontal axis which is in the plane of the disc and tangential to the disc at B. A particle P, of mass m, is moving horizontally with speed u in a direction which is perpendicular to the plane of the disc. At time $t = 0$, P strikes the disc at its centre and adheres to the disc.

(*a*) Show that the angular speed of the disc immediately after it has been struck by P is $\dfrac{4u}{9a}$.

(6)

It is given that $u^2 = \frac{1}{10}ag$, and that air resistance is negligible.

(*b*) Find the angle through which the disc turns before it first comes to instantaneous rest.

(5)

The disc first returns to its initial position at time $t = T$.

(*c*) (i) Write down an equation of motion for the disc.

(ii) Hence find T in terms of a, g and m, using a suitable approximation which should be justified.

(6)

END

Paper Reference(s)

6681

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Thursday 19 June 2003 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

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Instructions to Candidates

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When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has six questions.

Advice to Candidates

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N13907A

1. *In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane and* **k** *is a unit vector vertically upwards.*

A small smooth ring of mass 0.1 kg is threaded onto a smooth horizontal wire which is parallel to $(\mathbf{i} + 2\mathbf{j})$. The only forces acting on the ring are its weight, the normal reaction from the wire and a constant force $(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ N. The ring starts from rest at the point A on the wire, whose position vector relative to a fixed origin is $(2\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$ m, and passes through the point B with speed 5 m s^{-1}. Find the position vector of B.

(6)

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2. With respect to a fixed origin O, the position vector, $\mathbf{r}$ metres, of a particle P at time t seconds satisfies

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \mathbf{r} = (\mathbf{i} - \mathbf{j})\mathrm{e}^{-2t}.$$

Given that P is at O when $t = 0$, find

(*a*) $\mathbf{r}$ in terms of t,

(7)

(b) a cartesian equation of the path of P.

(2)

3.

Figure 1

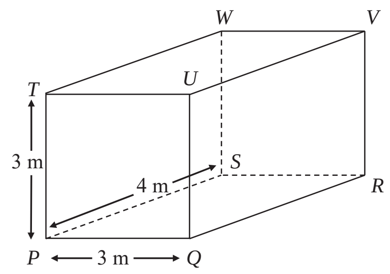

Figure 1 shows a box in the shape of a cuboid $PQRSTUVW$ where $\overrightarrow{PQ} = 3\mathbf{i}$ metres, $\overrightarrow{PS} = 4\mathbf{j}$ metres and $\overrightarrow{PT} = 3\mathbf{k}$ metres. A force $(4\mathbf{i} - 2\mathbf{j})$ N acts at Q, a force $(4\mathbf{i} + 2\mathbf{j})$ N acts at R, a force $(-2\mathbf{j} + \mathbf{k})$ N acts at T, and a force $(2\mathbf{j} + \mathbf{k})$ N acts at W. Given that these are the only forces acting on the box, find

(*a*) the resultant force acting on the box,

(2)

(*b*) the resultant vector moment about P of the four forces acting on the box.

(5)

When an additional force $\mathbf{F}$ acts on the box at a point X on the edge PS, the box is in equilibrium.

(*c*) Find $\mathbf{F}$.

(1)

(*d*) Find the length of PX.

(5)

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4. A rocket-driven car propels itself forwards in a straight line on a horizontal track by ejecting burnt fuel backwards at a constant rate λ kg s^{-1} and at a constant speed U m s^{-1} relative to the car. At time t seconds, the speed of the car is v m s^{-1} and the total resistance to the motion of the car has magnitude kv N, where k is a positive constant. When $t = 0$ the total mass of the car, including fuel, is M kg. Assuming that at time t seconds some fuel remains in the car,

(*a*) show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\lambda U - kv}{M - \lambda t},$$

(7)

(*b*) find the speed of the car at time t seconds, given that it starts from rest when $t = 0$ and that $\lambda = k = 10$.

(6)

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5. A uniform rod AB, of mass m and length $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis through A. The rod is hanging in equilibrium with B below A when it is hit by a particle of mass m moving horizontally with speed v in a vertical plane perpendicular to the axis. The particle strikes the rod at B and immediately adheres to it.

(*a*) Show that the angular speed of the rod immediately after the impact is $\dfrac{3v}{8a}$.

(5)

Given that the rod rotates through 120° before first coming to instantaneous rest,

(*b*) find v in terms of a and g.

(6)

(*c*) find, in terms of m and g, the magnitude of the vertical component of the force acting on the rod at A immediately after the impact.

(5)

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6. (*a*) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius a, about an axis through its centre O perpendicular to the plane of the disc is $\frac{1}{2}ma^2$.

(4)

The line AB is a diameter of the disc and P is the mid-point of OA. The disc is free to rotate about a fixed smooth horizontal axis L. The axis lies in the plane of the disc, passes through P and is perpendicular to OA. A particle of mass m is attached to the disc at A and a particle of mass $2m$ is attached to the disc at B.

(*b*) Show that the moment of inertia of the loaded disc about L is $\frac{21}{4}ma^2$.

(6)

At time $t = 0$, PB makes a small angle with the downward vertical through P and the loaded disc is released from rest. By obtaining an equation of motion for the disc and using a suitable approximation,

(*c*) find the time when the loaded disc first comes to instantaneous rest.

(8)

END

Paper Reference(s)

6681
Edexcel GCE
Mechanics M5
Advanced/Advanced Subsidiary
Friday 25 June 2004 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has six questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body. $\mathbf{F}_1 = (12\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$ N and acts at the point with position vector $(2\mathbf{i} - 3\mathbf{j})$ m, $\mathbf{F}_2 = (-3\mathbf{j} + 2\mathbf{k})$ N and acts at the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m. The force $\mathbf{F}_3$ acts at the point with position vector $(2\mathbf{i} - \mathbf{k})$ m.

Given that this set of forces is equivalent to a couple, find

(*a*) $\mathbf{F}_3$,
(2)

(*b*) the magnitude of the couple.
(5)

---

2. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are the only forces acting on a particle P of mass 2 kg. The particle is initially at rest at the point A with position vector $(-2\mathbf{i} - \mathbf{j} - 4\mathbf{k})$ m. Four seconds later, P is at the point B with position vector $(6\mathbf{i} - 5\mathbf{j} + 8\mathbf{k})$ m.

Given that $\mathbf{F}_1 = (12\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$ N, find

(*a*) $\mathbf{F}_2$,
(5)

(*b*) the work done on P as it moves from A to B.
(3)

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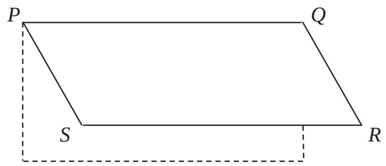
3. A uniform lamina of mass m is in the shape of a rectangle $PQRS$, where $PQ = 8a$ and $QR = 6a$.

(*a*) Find the moment of inertia of the lamina about the edge PQ.

(2)

Figure 1



The flap on a letterbox is modelled as such a lamina. The flap is free to rotate about an axis along its horizontal edge PQ, as shown in Fig. 1. The flap is released from rest in a horizontal position. It then swings down into a vertical position.

(*b*) Show that the angular speed of the flap as it reaches the vertical position is $\sqrt{\left(\frac{g}{2a}\right)}$.

(3)

(*c*) Find the magnitude of the vertical component of the resultant force of the axis PQ on the flap, as it reaches the vertical position.

(4)

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4. A uniform circular disc, of mass m and radius r, has a diameter AB. The point C on AB is such that $AC = \frac{1}{2}r$. The disc can rotate freely in a vertical plane about a horizontal axis through C, perpendicular to the plane of the disc. The disc makes small oscillations in a vertical plane about the position of equilibrium in which B is below A.

(*a*) Show that the motion is approximately simple harmonic.

(6)

(*b*) Show that the period of this approximate simple harmonic motion is $\pi\sqrt{\left(\frac{6r}{g}\right)}$.

(1)

The speed of B when it is vertically below A is $\sqrt{\left(\frac{gr}{54}\right)}$. The disc comes to rest when CB makes an angle α with the downward vertical.

(*c*) Find an approximate value of α.

(3)

---

5. A rocket is launched vertically upwards under gravity from rest at time $t = 0$. The rocket propels itself upward by ejecting burnt fuel vertically downwards at a constant speed u relative to the rocket. The initial mass of the rocket, including fuel, is M. At time t, before all the fuel has been used up, the mass of the rocket, including fuel, is $M(1 - kt)$ and the speed of the rocket is v.

(*a*) Show that $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{ku}{1-kt} - g$.

(7)

(*b*) Hence find the speed of the rocket when $t = \frac{1}{3k}$.

(3)

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6. A particle P of mass 2 kg moves in the x-y plane. At time t seconds its position vector is $\mathbf{r}$ metres. When $t = 0$, the position vector of P is $\mathbf{i}$ metres and the velocity of P is $(-\mathbf{i} + \mathbf{j})$ m s^{-1}.

The vector $\mathbf{r}$ satisfies the differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 2\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + 2\mathbf{r} = \mathbf{0}.$$

(*a*) Find $\mathbf{r}$ in terms of t.

(8)

(*b*) Show that the speed of P at time t is $\mathrm{e}^{-t}\sqrt{2}$ m s^{-1}.

(5)

(*c*) Find, in terms of e, the loss of kinetic energy of P in the interval $t = 0$ to $t = 1$.

(2)

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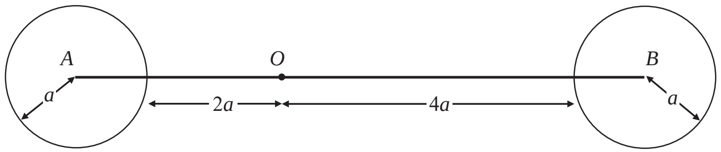
7. **Figure 2**



A body consists of two uniform circular discs, each of mass m and radius a, with a uniform rod. The centres of the discs are fixed to the ends A and B of the rod, which has mass $3m$ and length $8a$. The discs and the rod are coplanar, as shown in Fig. 2. The body is free to rotate in a vertical plane about a smooth fixed horizontal axis. The axis is perpendicular to the plane of the discs and passes through the point O of the rod, where $AO = 3a$.

(*a*) Show that the moment of inertia of the body about the axis is $54ma^2$.

(6)

The body is held at rest with AB horizontal and is then released. When the body has turned through an angle of 30°, the rod AB strikes a small fixed smooth peg P where $OP = 3a$. Given that the body rebounds from the peg with its angular speed halved by the impact,

(*b*) show that the magnitude of the impulse exerted on the body by the peg at the impact is $9m\sqrt{\left(\frac{5ga}{6}\right)}$.

(10)

END

Paper Reference(s)

6681/01
Edexcel GCE
Mechanics M5
Advanced/Advanced Subsidiary
Friday 24 June 2005 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac or Green)	Nil
Answer Book (AB16)	
Graph paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 7 questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N21145A

1. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are the only forces acting on a particle. $\mathbf{F}_1$ has magnitude 9 N and acts in the direction of $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. $\mathbf{F}_2$ has magnitude 18 N and acts in the direction of $\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$.

Find the total work done by the two forces in moving the particle from the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m to the point with position vector $(3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ m.

(Total 6 marks)

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2. At time t seconds the position vector of a particle P, relative to a fixed origin O, is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + 2\mathbf{r} = 3\mathrm{e}^{-t}\mathbf{j}.$$

Given that $\mathbf{r} = 2\mathbf{i} - \mathbf{j}$ when $t = 0$, find $\mathbf{r}$ in terms of t.

(Total 7 marks)

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3. A system of forces acting on a rigid body consists of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at a point A of the body, together with a couple of moment $\mathbf{G}$. $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ N and $\mathbf{F}_2 = (-2\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ N. The position vector of the point A is $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m and $\mathbf{G} = (7\mathbf{i} - 3\mathbf{j} + 8\mathbf{k})$ Nm.

Given that the system is equivalent to a single force $\mathbf{R}$,

(*a*) find $\mathbf{R}$,

(2)

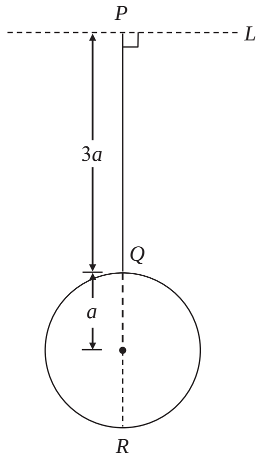
(*b*) find a vector equation for the line of action of $\mathbf{R}$.

(7)

(Total 9 marks)

4.

Figure 1



A thin uniform rod PQ has mass m and length $3a$. A thin uniform circular disc, of mass m and radius a, is attached to the rod at Q in such a way that the rod and the diameter QR of the disc are in a straight line with $PR = 5a$. The rod together with the disc form a composite body, as shown in Figure 1. The body is free to rotate about a fixed smooth horizontal axis L through P, perpendicular to PQ and in the plane of the disc.

(*a*) Show that the moment of inertia of the body about L is $\frac{77ma^2}{4}$.

(7)

When PR is vertical, the body has angular speed ω and the centre of the disc strikes a stationary particle of mass $\frac{1}{2}m$. Given that the particle adheres to the centre of the disc,

(*b*) find, in terms of ω, the angular speed of the body immediately after the impact.

(4)

(Total 11 marks)

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5. A uniform square lamina $ABCD$, of mass m and side $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through A and is perpendicular to the plane of the lamina. The moment of inertia of the lamina about L is $\frac{8ma^2}{3}$.

Given that the lamina is released from rest when the line AC makes an angle of $\frac{\pi}{3}$ with the downward vertical,

(*a*) find the magnitude of the vertical component of the force acting on the lamina at A when the line AC is vertical.

(7)

Given instead that the lamina now makes small oscillations about its position of stable equilibrium,

(*b*) find the period of these oscillations.

(5)

(Total 12 marks)

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6. A rocket-driven car moves along a straight horizontal road. The car has total initial mass M. It propels itself forwards by ejecting mass backwards at a constant rate λ per unit time at a constant speed U relative to the car. The car starts from rest at time $t = 0$. At time t the speed of the car is v. The total resistance to motion is modelled as having magnitude kv, where k is a constant.

Given that $t < \frac{M}{\lambda}$, show that

(*a*) $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\lambda U - kv}{M - \lambda t}$,

(7)

(*b*) $v = \frac{\lambda U}{k}\left\{1 - \left(1 - \frac{\lambda t}{M}\right)^{\frac{k}{\lambda}}\right\}$.

(6)

(Total 13 marks)

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7. A uniform lamina of mass m is in the shape of an equilateral triangle ABC of perpendicular height h. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis L through A and perpendicular to the lamina.

(*a*) Show, by integration, that the moment of inertia of the lamina about L is $\frac{5mh^2}{9}$.

(9)

The centre of mass of the lamina is G. The lamina is in equilibrium, with G below A, when it is given an angular speed $\sqrt{\left(\frac{6g}{5h}\right)}$.

(*b*) Find the angle between AG and the downward vertical, when the lamina first comes to rest.

(5)

(*c*) Find the greatest magnitude of the angular acceleration during the motion.

(3)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Friday 23 June 2006 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

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Information for Candidates

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Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N22333A

1. (*a*) Prove, using integration, that the moment of inertia of a uniform rod, of mass m and length $2a$, about an axis perpendicular to the rod through one end is $\frac{4}{3}ma^2$.
(3)

(*b*) Hence, or otherwise, find the moment of inertia of a uniform square lamina, of mass M and side $2a$, about an axis through one corner and perpendicular to the plane of the lamina.
(3)

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2. A particle of mass 0.5 kg is at rest at the point with position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$ m. The particle is then acted upon by two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$. These are the only two forces acting on the particle. Subsequently, the particle passes through the point with position vector $(4\mathbf{i} + 5\mathbf{j} - 5\mathbf{k})$ m with speed 12 m s^{-1}. Given that $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ N, find $\mathbf{F}_2$.
(9)

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3. A particle P moves in the x-y plane and has position vector $\mathbf{r}$ metres at time t seconds. It is given that $\mathbf{r}$ satisfies the differential equation

$$\frac{d^2\mathbf{r}}{dt^2} = 2\frac{d\mathbf{r}}{dt}.$$

When $t = 0$, P is at the point with position vector $3\mathbf{i}$ metres and is moving with velocity $\mathbf{j}$ m s^{-1}.

(*a*) Find $\mathbf{r}$ in terms of t.
(8)

(*b*) Describe the path of P, giving its cartesian equation.
(2)

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4. A force system consists of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a rigid body.

$\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j})$ N and acts at the point with position vector $(-\mathbf{i} + 4\mathbf{j})$ m.
$\mathbf{F}_2 = (-\mathbf{j} + \mathbf{k})$ N and acts at the point with position vector $(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ m.
$\mathbf{F}_3 = (3\mathbf{i} - \mathbf{j} + \mathbf{k})$ N and acts at the point with position vector $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m.
It is given that this system can be reduced to a single force $\mathbf{R}$.

(*a*) Find $\mathbf{R}$.
(2)

(*b*) Find a vector equation of the line of action of $\mathbf{R}$, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a parameter.
(10)

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5. A space-ship is moving in a straight line in deep space and needs to reduce its speed from U to V. This is done by ejecting fuel from the front of the space-ship at a constant speed k relative to the space-ship. When the speed of the space-ship is v, its mass is m.

(a) Show that, while the space-ship is ejecting fuel, $\frac{\mathrm{d}m}{\mathrm{d}v} = \frac{m}{k}$.

(6)

The initial mass of the space-ship is M.

(b) Find, in terms of U, V, k and M, the amount of fuel which needs to be used to reduce the speed of the space-ship from U to V.

(6)

6. A uniform circular disc, of mass m, radius a and centre O, is free to rotate in a vertical plane about a fixed smooth horizontal axis. The axis passes through the mid-point A of a radius of the disc.

(a) Find an equation of motion for the disc when the line AO makes an angle θ with the downward vertical through A.

(5)

(b) Hence find the period of small oscillations of the disc about its position of stable equilibrium.

(2)

When the line AO makes an angle θ with the downward vertical through A, the force acting on the disc at A is **F**.

(c) Find the magnitude of the component of **F** perpendicular to AO.

(5)

7. Particles P and Q have mass $3m$ and m respectively. Particle P is attached to one end of a light inextensible string and Q is attached to the other end. The string passes over a circular pulley which can freely rotate in a vertical plane about a fixed horizontal axis through its centre O. The pulley is modelled as a uniform circular disc of mass $2m$ and radius a. The pulley is sufficiently rough to prevent the string slipping. The system is at rest with the string taut. A third particle R of mass m falls freely under gravity from rest for a distance a before striking and adhering to Q. Immediately before R strikes Q, particles P and Q are at rest with the string taut.

(a) Show that, immediately after R strikes Q, the angular speed of the pulley is $\frac{1}{3}\sqrt{\left(\frac{g}{2a}\right)}$.

(5)

When R strikes Q, there is an impulse in the string attached to Q.

(b) Find the magnitude of this impulse.

(3)

Given that P does not hit the pulley,

(c) find the distance that P moves upwards before first coming to instantaneous rest.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Tuesday 26 June 2007 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N25909RA

1. A bead of mass 0.5 kg is threaded on a smooth straight wire. The only forces acting on the bead are a constant force $(4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k})$ N and the normal reaction of the wire. The bead starts from rest at the point A with position vector $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ m and moves to the point B with position vector $(4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ m.

Find the speed of the bead when it reaches B.

(4)

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2. At time t seconds, the position vector of a particle P is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the differential equation

$$\frac{d^2\mathbf{r}}{dt^2} + 3\frac{d\mathbf{r}}{dt} = \mathbf{0}.$$

When $t = 0$, the velocity of P is $(8\mathbf{i} - 12\mathbf{j})$ m s^{-1}.

Find the velocity of P when $t = \frac{2}{3}\ln 2$.

(7)

---

3. A uniform rod AB, of mass m and length $2a$, is free to rotate about a fixed smooth axis which passes through A and is perpendicular to the rod. The rod has angular speed ω when it strikes a particle P of mass m and adheres to it. Immediately before the rod strikes P, P is at rest and at a distance x from A. Immediately after the rod strikes P, the angular speed of the rod is $\frac{3}{4}\omega$.

Find x in terms of a.

(5)

4.

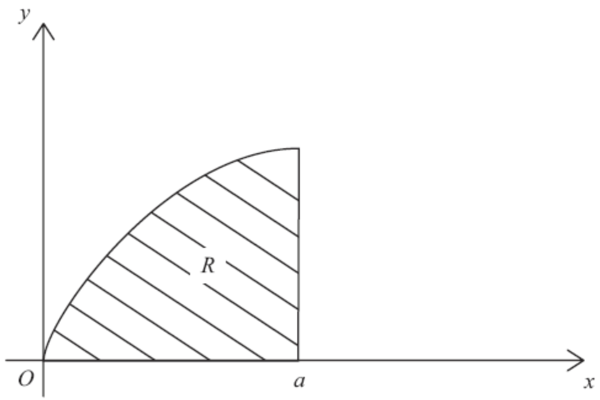

Figure 1

A region R is bounded by the curve $y^2 = 4ax$ ($y > 0$), the x-axis and the line $x = a$ ($a > 0$), as shown in Figure 1. A uniform solid S of mass M is formed by rotating R about the x-axis through 360°. Using integration, prove that the moment of inertia of S about the x-axis is $\frac{4}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.)

(7)

---

5. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a rigid body, where

$\mathbf{F}_1 = (3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$ N and

$\mathbf{F}_2 = (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N.

The force $\mathbf{F}_1$ acts at the point with position vector $(\mathbf{i} - 2\mathbf{j})$ m , and the force $\mathbf{F}_2$ acts at the point with position vector $(3\mathbf{i} - \mathbf{k})$ m. The two forces are equivalent to a single force $\mathbf{F}$ acting at the point with position vector $(\mathbf{i} - \mathbf{k})$ m, together with a couple $\mathbf{G}$.

(*a*) Find $\mathbf{F}$.

(1)

(*b*) Find the magnitude of $\mathbf{G}$.

(8)

6.

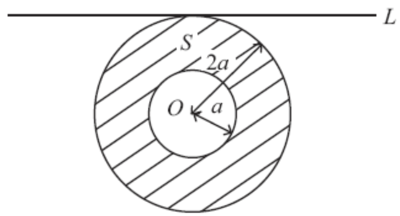

Figure 2

A lamina S is formed from a uniform disc, centre O and radius $2a$, by removing the disc of centre O and radius a, as shown in Figure 2. The mass of S is M.

(*a*) Show that the moment of inertia of S about an axis through O and perpendicular to its plane is $\frac{5}{2}Ma^2$.

(3)

The lamina is free to rotate about a fixed smooth horizontal axis L. The axis L lies in the plane of S and is a tangent to its outer circumference, as shown in Figure 2.

(*b*) Show that the moment of inertia of S about L is $\frac{21}{4}Ma^2$.

(4)

S is displaced through a small angle from its position of stable equilibrium and, at time $t = 0$, it is released from rest. Using the equation of motion of S, with a suitable approximation,

(*c*) find the time when S first passes through its position of stable equilibrium.

(6)

---

7. A motor boat of mass M is moving in a straight line, with its engine switched off, across a stretch of still water. The boat is moving with speed U when, at time $t = 0$, it develops a leak. The water comes in at a constant rate so that at time t, the mass of water in the boat is λt. At time t the speed of the boat is v and it experiences a total resistance to motion of magnitude $2\lambda v$.

(*a*) Show that $(M + \lambda t)\dfrac{\mathrm{d}v}{\mathrm{d}t} + 3\lambda v = 0$.

(6)

(*b*) Show that the time taken for the speed of the boat to reduce to $\frac{1}{2}U$ is $\dfrac{M}{\lambda}(2^{\frac{1}{3}} - 1)$.

(6)

The boat sinks when the mass of water inside the boat is M.

(*c*) Show that the boat does not sink before the speed of the boat is $\frac{1}{2}U$.

(2)

8.

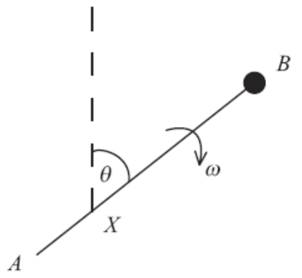

Figure 3

A uniform rod AB has mass $3m$ and length $2a$. It is free to rotate in a vertical plane about a smooth fixed horizontal axis through the point X on the rod, where $AX = \frac{1}{2}a$. A particle of mass m is attached to the rod at B. At time $t = 0$, the rod is vertical, with B above A, and is given an initial angular speed $\sqrt{\frac{g}{a}}$. When the rod makes an angle θ with the upward vertical, the angular speed of the rod is ω, as shown in Figure 3.

(*a*) By using the principle of the conservation of energy, show that

$$\omega^2 = \frac{g}{2a}(5 - 3\cos\theta).$$

(8)

(*b*) Find the angular acceleration of the rod when it makes an angle θ with the upward vertical.

(3)

When $\theta = \phi$, the resultant force of the axis on the rod is in a direction perpendicular to the rod.

(*c*) Find $\cos\phi$.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Friday 26 June 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

N29947RA

1. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A small bead of mass 0.5 kg is threaded on a smooth horizontal wire. The bead is initially at rest at the point with position vector $(\mathbf{i} - 6\mathbf{j})$ m. A constant horizontal force **P** N then acts on the bead causing it to move along the wire. The bead passes through the point with position vector $(7\mathbf{i} - 14\mathbf{j})$ m with speed $2\sqrt{7}$ m s^{-1}.

Given that **P** is parallel to $(6\mathbf{i} + \mathbf{j})$, find **P**.

(6)

---

2. The velocity **v** m s^{-1} of a particle P at time t seconds satisfies the vector differential equation

$$\frac{d\mathbf{v}}{dt} + 4\mathbf{v} = \mathbf{0}.$$

The position vector of P at time t seconds is **r** metres.

Given that at $t = 0$, $\mathbf{r} = (\mathbf{i} - \mathbf{j})$ and $\mathbf{v} = (-8\mathbf{i} + 4\mathbf{j})$, find **r** at time t seconds.

(7)

---

3. A system of forces consists of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on a rigid body.

$\mathbf{F}_1 = (-2\mathbf{i} + \mathbf{j} - \mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_1 = (\mathbf{i} - \mathbf{j} + \mathbf{k})$ m.

$\mathbf{F}_2 = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_2 = (4\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ m.

Given that the system is equivalent to a single force **R** N, acting at the point with position vector $(5\mathbf{i} + \mathbf{j} - \mathbf{k})$ m, together with a couple **G** N m, find

(*a*) **R**,

(2)

(*b*) the magnitude of **G**.

(9)

---

4. At time $t = 0$ a rocket is launched from rest vertically upwards. The rocket propels itself upwards by expelling burnt fuel vertically downwards with constant speed U m s^{-1} relative to the rocket. The initial mass of the rocket is M_0 kg. At time t seconds, where $t < 2$, its mass is $M_0(1 - \frac{1}{2}t)$ kg, and it is moving upwards with speed v m s^{-1}.

(*a*) Show that

$$\frac{dv}{dt} = \frac{U}{(2-t)} - 9.8.$$

(7)

(*b*) Hence show that $U > 19.6$.

(2)

(*c*) Find, in terms of U, the speed of the rocket one second after its launch.

(5)

5.

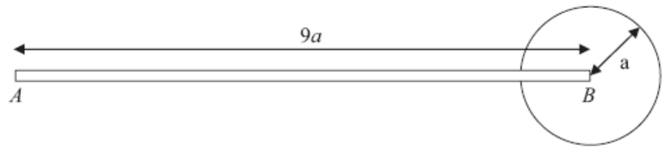

Figure 1

A pendulum P is modelled as a uniform rod AB, of length $9a$ and mass m, rigidly fixed to a uniform circular disc of radius a and mass $2m$. The end B of the rod is attached to the centre of the disc, and the rod lies in the plane of the disc, as shown in Figure 1. The pendulum is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through the end A and is perpendicular to the plane of the disc.

(*a*) Show that the moment of inertia of P about L is $190ma^2$.

(4)

The pendulum makes small oscillations about L.

(*b*) By writing down an equation of motion for P, find the approximate period of these small oscillations.

(7)

---

6. A uniform solid right circular cylinder has mass M, height h and radius a. Find, using integration, its moment of inertia about a diameter of one of its circular ends.

[*You may assume without proof that the moment of inertia of a uniform circular disc, of mass m and radius a, about a diameter is* $\frac{1}{4}ma^2$.]

(10)

---

7. A uniform square lamina *ABCD*, of mass $2m$ and side $3a\sqrt{2}$, is free to rotate in a vertical plane about a fixed smooth horizontal axis *L* which passes through *A* and is perpendicular to the plane of the lamina. The moment of inertia of the lamina about *L* is $24ma^2$.

The lamina is at rest with *C* vertically above *A*. At time $t = 0$ the lamina is slightly displaced. At time *t* the lamina has rotated through an angle θ.

(*a*) Show that

$$2a\left(\frac{d\theta}{dt}\right)^2 = g(1 - \cos\theta).$$

(4)

(*b*) Show that, at time *t*, the magnitude of the component of the force acting on the lamina at *A*, in a direction perpendicular to *AC*, is $\frac{1}{2}mg\sin\theta$.

(7)

When the lamina reaches the position with *C* vertically below *A*, it receives an impulse which acts at *C*, in the plane of the lamina and in a direction which is perpendicular to the line *AC*. As a result of this impulse the lamina is brought immediately to rest.

(*c*) Find the magnitude of the impulse.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6681

Edexcel GCE

Mechanics M5

Advanced Level

Tuesday 23 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of *g* is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

M34277A

1. At time $t = 0$, a particle P of mass 3 kg is at rest at the point A with position vector $(\mathbf{j} - 3\mathbf{k})$ m. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ then act on the particle P and it passes through the point B with position vector $(8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ m.

Given that $\mathbf{F}_1 = (4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ N and $\mathbf{F}_2 = (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})$ N and that $\mathbf{F}_1$ and $\mathbf{F}_2$ are the *only* two forces acting on P, find the velocity of P as it passes through B, giving your answer as a vector.

(7)

---

2. At time t seconds, the position vector of a particle P is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the vector differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = 4\mathbf{r} = \mathrm{e}^{2t}\mathbf{j}.$$

When $t = 0$, P has position vector $(\mathbf{i} + \mathbf{j})$ m and velocity $2\mathbf{i}$ m s^{-1}.

Find an expression for $\mathbf{r}$ in terms of t .

(11)

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3. A spaceship is moving in a straight line in deep space and needs to increase its speed. This is done by ejecting fuel backwards from the spaceship at a constant speed c relative to the spaceship. When the speed of the spaceship is v, its mass is m.

(*a*) Show that, while the spaceship is ejecting fuel,

$$\frac{\mathrm{d}v}{\mathrm{d}m} = -\frac{c}{m}.$$

(5)

The initial mass of the spaceship is m_0 and at time t the mass of the spaceship is given by $m = m_0(1 - kt)$, where k is a positive constant.

(*b*) Find the acceleration of the spaceship at time t .

(4)

4.

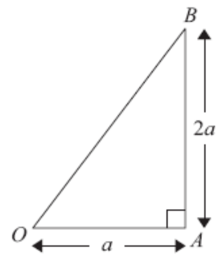

Figure 1

A uniform lamina of mass M is in the shape of a right-angled triangle OAB. The angle OAB is 90°, $OA = a$ and $AB = 2a$, as shown in Figure 1.

(*a*) Prove, using integration, that the moment of inertia of the lamina OAB about the edge OA is $\frac{2}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through one end and perpendicular to the rod is $\frac{4}{3}ml^2$.)

(6)

The lamina OAB is free to rotate about a fixed smooth horizontal axis along the edge OA and hangs at rest with B vertically below A. The lamina is then given a horizontal impulse of magnitude J. The impulse is applied to the lamina at the point B, in a direction which is perpendicular to the plane of the lamina. Given that the lamina first comes to instantaneous rest after rotating through an angle of 120°,

(*b*) find an expression for J, in terms of M, a and g .

(7)

---

5. Two forces $\mathbf{F}_1 = (2\mathbf{i} + \mathbf{j})$ N and $\mathbf{F}_2 = (-2\mathbf{j} - \mathbf{k})$ N act on a rigid body. The force $\mathbf{F}_1$ acts at the point with position vector $\mathbf{r}_1 = (3\mathbf{i} + \mathbf{j} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts at the point with position vector $\mathbf{r}_2 = (\mathbf{i} - 2\mathbf{j})$ m. A third force $\mathbf{F}_3$ acts on the body such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are in equilibrium.

(*a*) Find the magnitude of $\mathbf{F}_3$.

(4)

(*b*) Find a vector equation of the line of action of $\mathbf{F}_3$.

(8)

The force $\mathbf{F}_3$ is replaced by a fourth force $\mathbf{F}_4$, acting through the origin O, such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ are equivalent to a couple.

(*c*) Find the magnitude of this couple.

(4)

---

6. A pendulum consists of a uniform rod AB, of length $4a$ and mass $2m$, whose end A is rigidly attached to the centre O of a uniform square lamina $PQRS$, of mass $4m$ and side a. The rod AB is perpendicular to the plane of the lamina. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through B. The axis L is perpendicular to AB and parallel to the edge PQ of the square.

(*a*) Show that the moment of inertia of the pendulum about L is $75ma^2$.

(4)

The pendulum is released from rest when BA makes an angle α with the downward vertical through B, where $\tan \alpha = \frac{7}{24}$. When BA makes an angle θ with the downward vertical through B, the magnitude of the component, in the direction AB, of the force exerted by the axis L on the pendulum is X.

(*b*) Find an expression for X in terms of m, g and θ.

(9)

Using the approximation $\theta \approx \sin \theta$,

(*c*) find an estimate of the time for the pendulum to rotate through an angle α from its initial rest position.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6681/01
Edexcel GCE

Mechanics M5

Advanced

Friday 28 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

N35394A

1. At time $t = 0$, the position vector of a particle P is $-3\mathbf{j}$ m. At time t seconds, the position vector of P is $\mathbf{r}$ metres and the velocity of P is $\mathbf{v}$ m s^{-1}. Given that

$$\mathbf{v} - 2\mathbf{r} = 4e^{t}\mathbf{j} ,$$

find the time when P passes through the origin.

(7)

2.

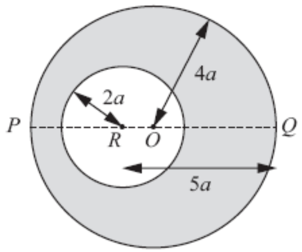

Figure 1

A uniform circular disc has mass $4m$, centre O and radius $4a$. The line POQ is a diameter of the disc. A circular hole of radius $2a$ is made in the disc with the centre of the hole at the point R on PQ where $QR = 5a$, as shown in Figure 1.

The resulting lamina is free to rotate about a fixed smooth horizontal axis L which passes through Q and is perpendicular to the plane of the lamina.

(*a*) Show that the moment of inertia of the lamina about L is $69ma^2$.

(7)

The lamina is hanging at rest with P vertically below Q when it is given an angular velocity Ω. Given that the lamina turns through an angle $\frac{2\pi}{3}$ before it first comes to instantaneous rest,

(*b*) find Ω in terms of g and a.

(6)

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3. A uniform lamina ABC of mass m is in the shape of an isosceles triangle with $AB = AC = 5a$ and $BC = 8a$.

(*a*) Show, using integration, that the moment of inertia of the lamina about an axis through A, parallel to BC, is $\frac{9}{2}ma^2$.

(6)

The foot of the perpendicular from A to BC is D. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis which passes through D and is perpendicular to the plane of the lamina. The lamina is released from rest when DA makes an angle α with the downward vertical. It is given that the moment of inertia of the lamina about an axis through A, perpendicular to BC and in the plane of the lamina, is $\frac{8}{3}ma^2$.

(*b*) Find the angular acceleration of the lamina when DA makes an angle θ with the downward vertical.

(8)

Given that α is small,

(*c*) find an approximate value for the period of oscillation of the lamina about the vertical.

(2)

---

4. Two forces $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ N and $\mathbf{F}_2 = (3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ N act on a rigid body.

The force $\mathbf{F}_1$ acts through the point with position vector $(2\mathbf{i} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts through the point with position vector $(\mathbf{j} + 2\mathbf{k})$ m.

(*a*) If the two forces are equivalent to a single force $\mathbf{R}$, find

(i) $\mathbf{R}$,

(2)

(ii) a vector equation of the line of action of $\mathbf{R}$, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$.

(6)

(*b*) If the two forces are equivalent to a single force acting through the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m together with a couple of moment $\mathbf{G}$, find the magnitude of $\mathbf{G}$.

(5)

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5. A raindrop falls vertically under gravity through a cloud. In a model of the motion the raindrop is assumed to be spherical at all times and the cloud is assumed to consist of stationary water particles. At time $t = 0$, the raindrop is at rest and has radius a. As the raindrop falls, water particles from the cloud condense onto it and the radius of the raindrop is assumed to increase at a constant rate λ. A time t the speed of the raindrop is v.

(*a*) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{3\lambda v}{(\lambda t + a)} = g.$$

(8)

(*b*) Find the speed of the raindrop when its radius is $3a$.

(7)

6. A uniform circular disc has mass m, centre O and radius $2a$. It is free to rotate about a fixed smooth horizontal axis L which lies in the same plane as the disc and which is tangential to the disc at the point A. The disc is hanging at rest in equilibrium with O vertically below A when it is struck at O by a particle of mass m. Immediately before the impact the particle is moving perpendicular to the plane of the disc with speed $3\sqrt{(ag)}$. The particle adheres to the disc at O.

(*a*) Find the angular speed of the disc immediately after the impact.

(5)

(*b*) Find the magnitude of the force exerted on the disc by the axis immediately after the impact.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Friday 24 June 2011 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P35416A

1. A particle moves from the point A with position vector $(3\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ m to the point B with position vector $(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$ m under the action of the force $(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})$ N. Find the work done by the force.

(4)

---

2. A particle P moves in the x-y plane so that its position vector $\mathbf{r}$ metres at time t seconds satisfies the differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} - 4\mathbf{r} = -3\mathrm{e}^t\mathbf{j}.$$

When $t = 0$, the particle is at the origin and is moving with velocity $(2\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find $\mathbf{r}$ in terms of t.

(10)

---

3. A rocket propels itself by its engine ejecting burnt fuel. Initially the rocket has total mass M, of which a mass kM, $k < 1$, is fuel. The rocket is at rest when its engine is started. The burnt fuel is ejected with constant speed c, relative to the rocket, in a direction opposite to that of the rocket's motion.

Assuming that there are no external forces, find the speed of the rocket when all its fuel has been burnt.

(7)

---

4. Two forces $\mathbf{F}_1 = (3\mathbf{j} + \mathbf{k})$ N and $\mathbf{F}_2 = (4\mathbf{i} + \mathbf{j} - \mathbf{k})$ N act on a rigid body.

The force $\mathbf{F}_1$ acts at the point with position vector $(2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ m and the force $\mathbf{F}_2$ acts at the point with position vector $(-3\mathbf{i} + 2\mathbf{k})$ m.

The two forces are equivalent to a single force $\mathbf{R}$ acting at the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m together with a couple of moment $\mathbf{G}$.

Find,

(*a*) $\mathbf{R}$,

(2)

(*b*) $\mathbf{G}$.

(4)

A third force $\mathbf{F}_3$ is now added to the system. The force $\mathbf{F}_3$ acts at the point with position vector $(2\mathbf{i} - \mathbf{k})$ m and the three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are equivalent to a couple.

(*c*) Find the magnitude of the couple.

(6)

---

5. A uniform rod PQ, of mass m and length $2a$, is made to rotate in a vertical plane with constant angular speed $\sqrt{\left(\frac{g}{a}\right)}$ about a fixed smooth horizontal axis through the end P of the rod.

Show that, when the rod is inclined at an angle θ to the downward vertical, the magnitude of the force exerted on the axis by the rod is $2mg\,|\cos(\frac{1}{2}\theta)|$.

(8)

---

6. A uniform rod AB of mass $4m$ is free to rotate in a vertical plane about a fixed smooth horizontal axis, L, through A. The rod is hanging vertically at rest when it is struck at its end B by a particle of mass m. The particle is moving with speed u, in a direction which is horizontal and perpendicular to L, and after striking the rod it rebounds in the opposite direction with speed v. The coefficient of restitution between the particle and the rod is 1.

Show that $u = 7v$.

(7)

---

7. Prove, using integration, that the moment of inertia of a uniform solid right circular cone, of mass M and base radius a, about its axis is $\frac{3}{10}Ma^2$.

[You may assume, without proof, that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre and perpendicular to its plane is $\frac{1}{2}mr^2$.]

(10)

---

8. A pendulum consists of a uniform rod PQ, of mass $3m$ and length $2a$, which is rigidly fixed at its end Q to the centre of a uniform circular disc of mass m and radius a. The rod is perpendicular to the plane of the disc. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through the end P of the rod and is perpendicular to the rod.

(*a*) Show that the moment of inertia of the pendulum about L is $\frac{33}{4}ma^2$.

(5)

The pendulum is released from rest in the position where PQ makes an angle α with the downward vertical. At time t, PQ makes an angle θ with the downward vertical.

(*b*) Show that the angular speed, $\dot{\theta}$, of the pendulum satisfies

$$\dot{\theta}^2 = \frac{40g(\cos\theta - \cos\alpha)}{33a}.$$

(4)

(*c*) Hence, or otherwise, find the angular acceleration of the pendulum.

(3)

Given that $\alpha = \frac{\pi}{20}$ and that PQ has length $\frac{8}{33}$ m,

(*d*) find, to 3 significant figures, an approximate value for the angular speed of the pendulum 0.2 s after it has been released from rest.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Monday 25 June 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle P moves in a plane such that its position vector $\mathbf{r}$ metres at time t seconds ($t > 0$) satisfies the differential equation

$$\frac{d\mathbf{r}}{dt} - \frac{2}{t}\mathbf{r} = 4\mathbf{i}.$$

When $t = 1$, the particle is at the point with position vector $(\mathbf{i} + \mathbf{j})$ m.

Find $\mathbf{r}$ in terms of t.

(9)

---

2. A rocket, with initial mass 1500 kg, including 600 kg of fuel, is launched vertically upwards form rest. The rocket burns fuel at a rate of 15 k g s^{-1} and the burnt fuel is ejected vertically downwards with a speed of 1000 m s^{-1} relative to the rocket. At time t seconds after launch ($t \leq 40$) the rocket has mass m kg and velocity v m s^{-1}.

(*a*) Show that

$$\frac{dv}{dt} + \frac{1000}{m}\frac{dm}{dt} = -9.8.$$

(5)

(*b*) Find v at time t, $0 \leq t \leq 40$.

(5)

---

3. A uniform rod PQ, of mass m and length $3a$, is free to rotate about a fixed smooth horizontal axis L, which passes through the end P of the rod and is perpendicular to the rod. The rod hangs at rest in equilibrium with Q vertically below P. One end of a light inextensible string of length $2a$ is attached to the rod at P and the other end is attached to a particle of mass $3m$. The particle is held with the string taut, and horizontal and perpendicular to L, and is then released. After colliding, the particle sticks to the rod forming a body B.

(*a*) Show that the moment of inertia of B about L is $15ma^2$.

(2)

(*b*) Show that B first comes to instantaneous rest after it has turned through an angle $\arccos \frac{9}{25}$.

(10)

---

4. A body consists of a uniform plane circular disc, of radius r and mass $2m$, with a particle of mass $3m$ attached to the circumference of the disc at the point P.

The line PQ is a diameter of the disc. The body is free to rotate in a vertical plane about a fixed smooth horizontal axis, L, which is perpendicular to the plane of the disc and passes through Q. The body is held with QP making an angle β with the downward vertical through Q, where $\sin \beta = 0.25$, and released from rest. Find the magnitude of the component, perpendicular to PQ, of the force acting on the body at Q at the instant when it is released.

[*You may assume that the moment of inertia of the body about L is* $15mr^2$.]

(6)

---

5. The points P and Q have position vectors $4\mathbf{i} - 6\mathbf{j} - 12\mathbf{k}$ and $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ respectively, relative to a fixed origin O.

Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, act along $\overrightarrow{OP}$, $\overrightarrow{QO}$ and $\overrightarrow{QP}$ respectively, and have magnitudes 7 N, 3 N and $3\sqrt{10}$ N respectively.

(*a*) Express $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ in vector form.

(3)

(*b*) Show that the resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is $(2\mathbf{i} - 10\mathbf{j} - 16\mathbf{k})$ N.

(2)

(*c*) Find a vector equation of the line of action of this resultant, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a parameter.

(5)

---

6. A uniform circular pulley, of mass $4m$ and radius r, is free to rotate about a fixed smooth horizontal axis which passes through the centre of the pulley and is perpendicular to the plane of the pulley. A light inextensible string passes over the pulley and has a particle of mass $2m$ attached to one end of a particle of mass $3m$ attached to the other end. The particles hang with the string vertical and taut on each side of the pulley. The rim of the pulley is sufficiently rough to prevent the string slipping. The system is released from rest.

(*a*) Find the angular acceleration of the pulley.

(8)

When the angular speed of the pulley is Ω, the string breaks and a constant braking couple of magnitude G is applied to the pulley which brings it to rest.

(*b*) Find an expression for the angle turned through by the pulley from the instant when the string breaks to the instant when the pulley first comes to rest.

(4)

---

7. (a) A uniform lamina of mass m is in the shape of a triangle ABC. The perpendicular distance of C from the line AB is h. Prove, using integration, that the moment of inertia of the lamina about AB is $\frac{1}{6}mh^2$.

(7)

(b) Deduce the radius of gyration of a uniform square lamina of side $2a$, about a diagonal.

(3)

The points X and Y are the mid-points of the sides RQ and RS respectively of a square $PQRS$ of side $2a$. A uniform lamina of mass M is in the shape of $PQXYS$.

(c) Show that the moment of inertia of this lamina about XY is $\frac{79}{84}Ma^2$.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6681/01R

Edexcel GCE

Mechanics M5 (R)

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P42961A

1. A particle moves in a plane in such a way that its position vector **r** metres at time t seconds satisfies the differential equation

$$\frac{d^2\mathbf{r}}{dt^2} - 2\frac{d\mathbf{r}}{dt} = \mathbf{0}$$

When $t = 0$, the particle is at the origin and is moving with velocity $(4\mathbf{i} + 2\mathbf{j})$ m s^{-1}.

Find **r** in terms of t.

(7)

---

2. Three forces $\mathbf{F}_1 = (3\mathbf{i} - \mathbf{j} + \mathbf{k})$ N, $\mathbf{F}_2 = (2\mathbf{i} - \mathbf{k})$ N, and $\mathbf{F}_3$ act on a rigid body.

The force $\mathbf{F}_1$ acts through the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m, the force $\mathbf{F}_2$ acts through the point with position vector $(\mathbf{i} - 2\mathbf{j})$ m and the force $\mathbf{F}_3$ acts through the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m.

Given that the system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ reduces to a couple **G**,

(*a*) find **G**.

(6)

The line of action of $\mathbf{F}_3$ is changed so that the system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ now reduces to a couple $(6\mathbf{i} + 8\mathbf{j} + 2\mathbf{k})$ N m.

(*b*) Find an equation of the new line of action of $\mathbf{F}_3$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where **a** and **b** are constant vectors.

(5)

---

3. A spacecraft is moving in a straight line in deep space. The spacecraft moves by ejecting burnt fuel backwards at a constant speed of 2000 m s^{-1} relative to the spacecraft. The burnt fuel is ejected at a constant rate of c kg s^{-1}. At time t seconds the total mass of the spacecraft, including fuel, is m kg and the speed of the spacecraft is v m s^{-1}.

(*a*) Show that, while the spacecraft is ejecting burnt fuel,

$$m\frac{dv}{dt} = 2000c$$

(7)

At time $t = 0$, the mass of the spacecraft is M_0 kg and the speed of the spacecraft is 2000 m s^{-1}. When $t = 50$, the spacecraft is still ejecting burnt fuel and its speed is 6000 m s^{-1}.

(*b*) Find c in terms of M_0.

(7)

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4. Show, using integration, that the moment of inertia of a uniform solid right circular cone of mass M, height h and base radius a, about an axis through the vertex, parallel to the base, is

$$\frac{3M}{20}(a^2 + 4h^2)$$

[*You may assume without proof that the moment of inertia of a uniform circular disc, of radius r and mass m, about a diameter is* $\frac{1}{4}mr^2$.]

(13)

5.

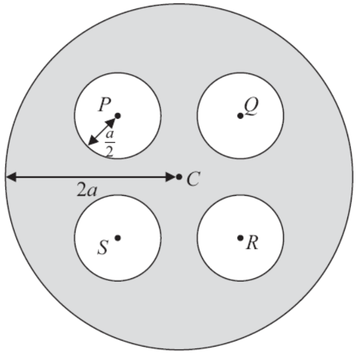

Figure 1

A uniform circular lamina has radius $2a$ and centre C. The points P, Q, R and S on the lamina are the vertices of a square with centre C and $CP = a$. Four circular discs, each of radius $\frac{a}{2}$, with centres P, Q, R and S, are removed from the lamina. The remaining lamina forms a template T, as shown in Figure 1.

The radius of gyration of T about an axis through C, perpendicular to T, is k.

(*a*) Show that $k^2 = \dfrac{55a^2}{24}$

(7)

The template T is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to T and passes through a point on its outer rim.

(*b*) Write down an equation of rotational motion for T and deduce that the period of small oscillations of T about its stable equilibrium position is

$$2\pi\sqrt{\left(\frac{151a}{48g}\right)}$$

(8)

6. A uniform circular disc, of radius r and mass m, is free to rotate in a vertical plane about a fixed smooth horizontal axis L which is perpendicular to the plane of the disc and passes through a point which is $\frac{1}{4}r$ from the centre of the disc. The disc is held at rest with its centre vertically above the axis. The disc is then slightly disturbed from its rest position. You may assume without proof that the moment of inertia of the disc about L is $\dfrac{9mr^2}{16}$.

(*a*) Show that the angular speed of the disc when it has turned through $\frac{\pi}{2}$ is $\sqrt{\left(\frac{8g}{9r}\right)}$.

(4)

(*b*) Find the magnitude of the force exerted on the disc by the axis when the disc has turned through $\frac{\pi}{2}$.

(11)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6681/01
Edexcel GCE
Mechanics M5
Advanced/Advanced Subsidiary
Monday 24 June 2013 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
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Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P41823A

1. Solve the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} - 2\mathbf{r} = \mathbf{0}$$

given that when $t = 0$, $\mathbf{r}.\mathbf{j} = 0$ and $\mathbf{r} \times \mathbf{j} = \mathbf{i} + \mathbf{k}$.

(7)

---

2. A uniform square lamina S has side $2a$. The radius of gyration of S about an axis through a vertex, perpendicular to S, is k.

(*a*) Show that $k^2 = \dfrac{8a^2}{3}$.

(4)

The lamina S is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to S and passes through a vertex.

(*b*) By writing down an equation of rotational motion for S, find the period of small oscillations of S about its position of stable equilibrium.

(5)

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3. A raindrop falls vertically under gravity through a stationary cloud. At time $t = 0$, the raindrop is at rest and has mass m_0. As the raindrop falls, water condenses onto it from the cloud so that the mass of the raindrop increases at a constant rate c. At time t, the mass of the raindrop is m and the speed of the raindrop is v. The resistance to the motion of the raindrop has magnitude mkv, where k is a constant. Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} + v\left(k + \frac{c}{m_0 + ct}\right) = g$$

(7)

---

4. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act through the points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively.

$\mathbf{r}_1 = (-2\mathbf{i} + 3\mathbf{j})$ m, $\quad \mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ N

$\mathbf{r}_2 = (3\mathbf{i} + 2\mathbf{k})$ m, $\quad \mathbf{F}_2 = (-2\mathbf{i} + \mathbf{j} - \mathbf{k})$ N

Given that the system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is in equilibrium,

(*a*) find $\mathbf{F}_3$,

(2)

(*b*) find a vector equation of the line of action of $\mathbf{F}_3$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

(5)

The force $\mathbf{F}_3$ is replaced by a force $\mathbf{F}_4$ acting through the point with position vector $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ m. The system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ is equivalent to a single force $(3\mathbf{i} + \mathbf{j} + \mathbf{k})$ N acting through the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m together with a couple.

(*c*) Find the magnitude of this couple.

(8)

5.

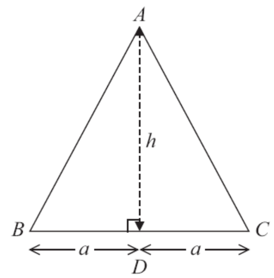

Figure 1

A uniform triangular lamina *ABC*, of mass *M*, has *AB* = *AC* and *BC* = 2*a*. The mid-point of *BC* is *D* and *AD* = *h*, as shown in Figure 1.

Show, using integration, that the moment of inertia of the lamina about an axis through *A*, perpendicular to the plane of the lamina, is

$$\frac{M}{6}(a^2 + 3h^2)$$

[*You may assume without proof that the moment of inertia of a uniform rod, of length 2l and mass m, about an axis through its midpoint and perpendicular to the rod, is* $\frac{1}{3}ml^2$.]

(10)

6.

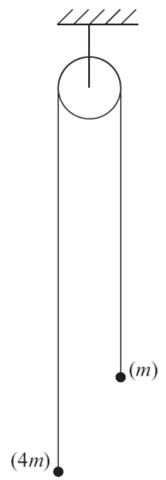

Figure 2

A light inextensible string has a particle of mass m attached to one end and a particle of mass $4m$ attached to the other end. The string passes over a rough pulley which is modelled as a uniform circular disc of radius a and mass $2m$, as shown in Figure 2.

The pulley can rotate in a vertical plane about a fixed horizontal axis which passes through the centre of the pulley and is perpendicular to the plane of the pulley. As the pulley rotates, a frictional couple of constant magnitude $2mga$ acts on it.

The system is held with the string vertical and taut on each side of the pulley and released from rest. Given that the string does not slip on the pulley, find the initial angular acceleration of the pulley.

(10)

7. A uniform circular disc, of radius r and mass m, is free to rotate in a vertical plane about a fixed smooth horizontal axis. This axis is perpendicular to the plane of the disc and passes through a point A on the circumference of the disc. The disc is held with AB horizontal, where AB is a diameter of the disc, and released from rest.

(*a*) Find the magnitude of

(i) the horizontal component,

(ii) the vertical component

of the force exerted on the disc by the axis immediately after the disc is released.

(11)

When AB is vertical the disc is instantaneously brought to rest by a horizontal impulse which acts in the plane of the disc and is applied to the disc at B.

(*b*) Find the magnitude of the impulse.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6681/01R

Edexcel GCE

Mechanics M5 (R)

Advanced/Advanced Subsidiary

Wednesday 18 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681R), your surname, initials and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A bead P of mass 0.2 kg is threaded on a smooth straight horizontal wire. The bead is at rest at the point A with position vector $(4\mathbf{i} - \mathbf{j})$ m. A force $(0.2\mathbf{i} + 0.3\mathbf{j})$ N acts on P and moves it to the point B with position vector $(13\mathbf{i} + 5\mathbf{j})$ m.

Find the speed of P at B.

(5)

---

2. A uniform equilateral triangular lamina ABC has mass m and sides of length $\sqrt{3}a$. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis L, which passes through A and is perpendicular to the lamina. The midpoint of BC is D. The lamina is held with AD making an angle of 60° with the upward vertical through A and released from rest. The moment of inertia of the lamina about the axis L is $\frac{5ma^2}{4}$.

Find the speed of D when AD is vertical.

(8)

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3. A uniform rectangular lamina $ABCD$, where $AB = a$ and $BC = 2a$, has mass $2m$. The lamina is free to rotate about its edge AB, which is fixed and vertical. The lamina is at rest when it is struck at C by a particle P of mass m. The particle P is moving horizontally with speed U in a direction which is perpendicular to the lamina. The coefficient of restitution between P and the lamina is 0.5.

Find the angular speed of the lamina immediately after the impact.

(8)

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4. A uniform solid sphere has mass M and radius a. Prove, using integration, that the moment of inertia of the sphere about a diameter is $\frac{2Ma^2}{5}$.

[*You may assume without proof that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre and perpendicular to its plane is* $\frac{1}{2}mr^2$.]

(8)

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5. A particle moves in a plane so that its position vector **r** metres at time t seconds satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}+(\tan t)\mathbf{r}=(\cos^2 t)\mathbf{i}-(3\cos t)\mathbf{j}, \qquad 0 \le t < \frac{\pi}{2}$$

When $t = 0$, the particle is at the point with position vector $4\mathbf{j}$ m.

Find **r** in terms of t.

(8)

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6. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body at the points with position vectors, $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively, where

$\mathbf{F}_1 = (2\mathbf{i} - \mathbf{j} + \mathbf{k})$ N $\qquad \mathbf{F}_2 = (3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ N $\qquad \mathbf{F}_3 = (-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ N

$\mathbf{r}_1 = (\mathbf{i} - \mathbf{k})$ m $\qquad \mathbf{r}_2 = (2\mathbf{i} - \mathbf{j} + \mathbf{k})$ m $\qquad \mathbf{r}_3 = (\mathbf{i} + \mathbf{j} - \mathbf{k})$ m

The system of the three forces is equivalent to a single force **R** acting at the point with position vector $(3\mathbf{i} - \mathbf{j} + \mathbf{k})$ m, together with a couple of moment **G**.

(*a*) Find **R**.

(2)

(*b*) Find **G**.

(9)

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7. A raindrop absorbs water as it falls vertically under gravity through a cloud. In a model of the motion the cloud is assumed to consist of stationary water particles. At time t, the mass of the raindrop is m and the speed of the raindrop is v. At time $t = 0$, the raindrop is at rest. The rate of increase of the mass of the raindrop with respect to time is modelled as being mkv, where k is a positive constant.

(*a*) Ignoring air resistance, show from first principles, that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - kv^2$$

(5)

(*b*) Find the time taken for the raindrop to reach a speed of $\frac{1}{2}\sqrt{\left(\frac{g}{k}\right)}$.

(4)

8.

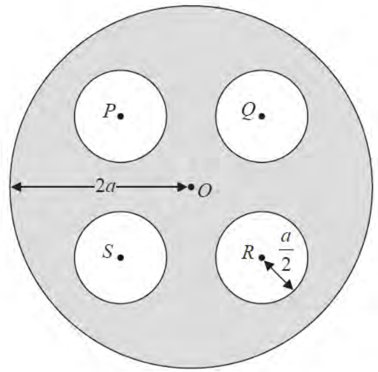

Figure 1

A uniform circular disc of radius $2a$ has centre O. The points P, Q, R and S on the disc are the vertices of a square with centre O and $OP = a$. Four circular holes, each of radius $\frac{a}{2}$, and with centres P, Q, R and S, are drilled in the disc to produce the lamina L, shown shaded in Figure 1. The mass of L is M.

(*a*) Show that the moment of inertia of L about an axis through O, and perpendicular to the plane of L, is $\frac{55Ma^2}{24}$.

(8)

The lamina L is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to L and which passes through a point A on the circumference of L. At time t, AO makes an angle θ with the downward vertical through A.

(*b*) Show that $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{48g}{151a}\sin\theta$.

(4)

(*c*) Hence find the period of small oscillations of L about its position of stable equilibrium.

(2)

The magnitude of the component, in a direction perpendicular to AO, of the force exerted on L by the axis is X.

(*d*) Find X in terms of M, g and θ.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Wednesday 18 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43161A

1. A small bead is threaded on a smooth, straight horizontal wire which passes through the point $A(-3, 1)$ and the point $B(2, 5)$ in the x-y plane. The bead moves under the action of a horizontal force **F** of magnitude 8.5 N whose line of action is parallel to the line with equation $15x - 8y + 4 = 0$. The unit on both the x and y axes has length one metre. Find the work done by **F** as it moves the bead from A to B.

(8)

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2. A particle P moves in a plane so that its position vector, $\mathbf{r}$ metres at time t seconds, satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \mathbf{r} = t\mathbf{i} + \mathrm{e}^{-t}\mathbf{j}$$

When $t = 0$ the particle is at the point with position vector $(\mathbf{i} + \mathbf{j})$ m.

Find $\mathbf{r}$ in terms of t.

(9)

---

3. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body at the points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively.

$\mathbf{F}_1 = (2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ N and $\mathbf{r}_1 = (\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ m,
$\mathbf{F}_2 = (\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})$ N and $\mathbf{r}_2 = (3\mathbf{i} - \mathbf{j} - \mathbf{k})$ m,
$\mathbf{F}_3 = (-3\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ N and $\mathbf{r}_3 = (\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ m.

Show that the system is equivalent to a couple and find the magnitude of the vector moment of this couple.

(9)

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4. A spacecraft is travelling in a straight line in deep space where all external forces can be assumed to be negligible. The spacecraft decelerates by ejecting fuel at a constant speed k relative to the spacecraft, in the direction of motion of the spacecraft. At time t, the spacecraft has speed v and mass m.

(*a*) Show, from first principles, that while the spacecraft is ejecting fuel,

$$\frac{\mathrm{d}v}{\mathrm{d}m} - \frac{k}{m} = 0$$

(5)

At time $t = 0$, the spacecraft has speed U and mass M.

(*b*) Find the mass of the spacecraft when it comes to rest.

(6)

Given that $m = Me^{-\alpha t^2}$, where α is a positive constant, and that the spacecraft comes to rest at time $t = T$,

(*c*) find, in terms of U and T only, the distance travelled by the spacecraft in decelerating from speed U to rest.

(6)

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5. A uniform rod AB, of mass m and length $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis L. The axis L is perpendicular to the rod and passes through the point P of the rod, where $AP = \frac{2}{3}a$.

(*a*) Find the moment of inertia of the rod about L.

(3)

The rod is held at rest with B vertically above P and is slightly displaced.

(*b*) Find the angular speed of the rod when PB makes an angle θ with the upward vertical.

(4)

(*c*) Find the magnitude of the angular acceleration of the rod when PB makes an angle θ with the upward vertical.

(3)

(*d*) Find, in terms of g and a only, the angular speed of the rod when the force acting on the rod at P is perpendicular to the rod.

(5)

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6. (*a*) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius a, about an axis through the centre of the disc and perpendicular to the plane of the disc is $\frac{1}{2}ma^2$.

(5)

[*You may assume without proof that the moment of inertia of a uniform hoop of mass m and radius r about an axis through its centre and perpendicular to its plane is* mr^2.]

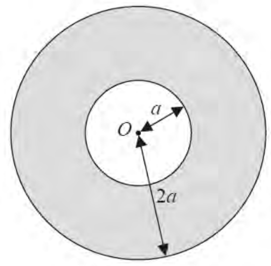

Figure 1

A uniform plane shape S of mass M is formed by removing a uniform circular disc with centre O and radius a from a uniform circular disc with centre O and radius $2a$, as shown in Figure 1. The shape S is free to rotate about a fixed smooth axis L, which passes through O and lies in the plane of the shape.

(*b*) Show that the moment of inertia of S about L is $\frac{5}{4}Ma^2$.

(4)

The shape S is at rest in a horizontal plane and is free to rotate about the axis L. A particle of mass M falls vertically and strikes S at the point A, where $OA = \frac{3}{2}a$ and OA is perpendicular to L. The particle adheres to S at A. Immediately before the particle strikes S the speed of the particle is u.

(*c*) Find, in terms of M and u, the loss in kinetic energy due to the impact.

(8)

TOTAL FOR PAPER: 75 MARKS

END