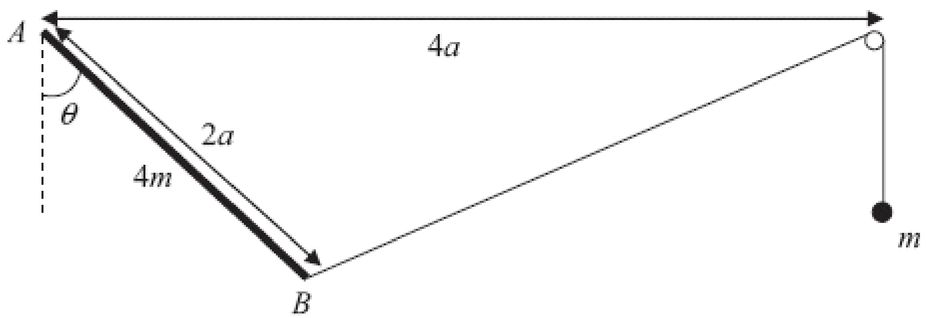
1.



The end A of a uniform rod AB, of length $2a$ and mass $4m$, is smoothly hinged to a fixed point. The end B is attached to one end of a light inextensible string which passes over a small smooth pulley, fixed at the same level as A. The distance from A to the pulley is $4a$. The other end of the string carries a particle of mass m which hangs freely, vertically below the pulley, with the string taut. The angle between the rod and the downward vertical is θ, where $0 < \theta < \frac{\pi}{2}$, as shown in the diagram above.

(a) Show that the potential energy of the system is

$$2mga\left(\sqrt{(5 - 4\sin\theta)} - 2\cos\theta\right) + \text{constant.}$$

(5)

(b) Hence, or otherwise, show that any value of θ which corresponds to a position of equilibrium of the system satisfies the equation

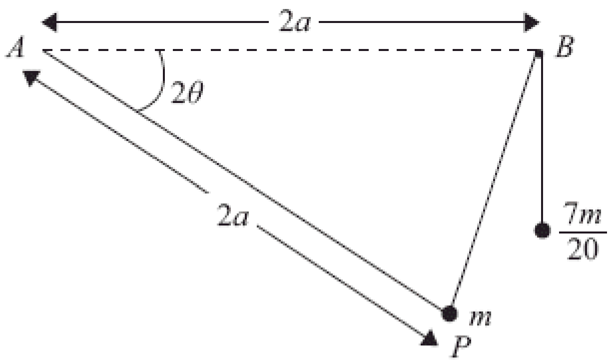
$$4\sin^3\theta - 6\sin^2\theta + 1 = 0.$$

(5)

(c) Given that $\theta = \frac{\pi}{6}$ corresponds to a position of equilibrium, determine its stability.

(5)
(Total 15 marks)

2.



A light inextensible string of length $2a$ has one end attached to a fixed point A. The other end of the string is attached to a particle P of mass m. A second light inextensible string of length L, where $L > \frac{12a}{5}$, has one of its ends attached to P and passes over a small smooth peg fixed at a point B. The line AB is horizontal and $AB = 2a$. The other end of the second string is attached to a particle of mass $\frac{7}{20}m$, which hangs vertically below B, as shown in the diagram above.

(a) Show that the potential energy of the system, when the angle $PAB = 2\theta$, is

$$\frac{1}{5}mga(7\sin\theta - 10\sin 2\theta) + \text{constant}.$$

(4)

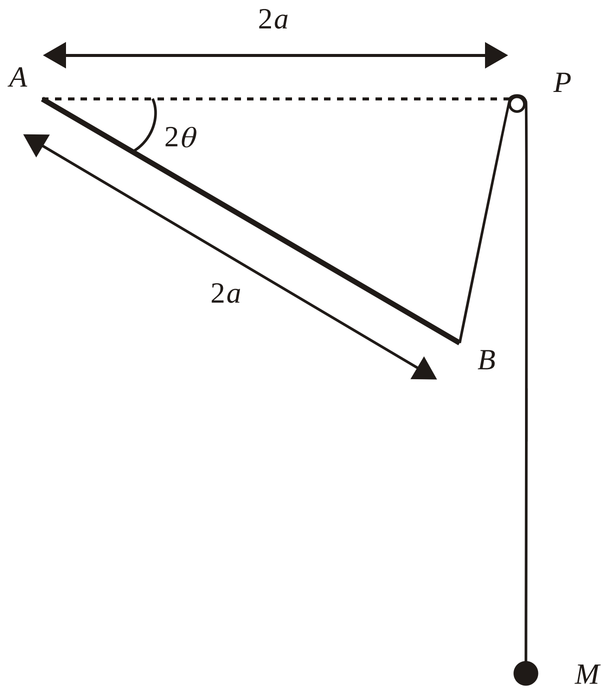
(b) Show that there is only one value of $\cos\theta$ for which the system is in equilibrium and find this value.

(8)

(c) Determine the stability of the position of equilibrium.

(4)
(Total 16 marks)

3.



A uniform rod AB, of length $2a$ and mass kM where k is a constant, is free to rotate in a vertical plane about the fixed point A. One end of a light inextensible string of length $6a$ is attached to the end B of the rod and passes over a small smooth pulley which is fixed at the point P. The line AP is horizontal and of length $2a$. The other end of the string is attached to a particle of mass M which hangs vertically below the point P, as shown in the diagram above. The angle PAB is 2θ, where $0^{\circ} \leq \theta \leq 180°$.

(a) Show that the potential energy of the system is

$$Mga(4\sin\theta - k\sin 2\theta) + \text{constant}.$$

(5)

The system has a position of equilibrium when $\cos\theta = \frac{3}{4}$.

(b) Find the value of k.

(5)

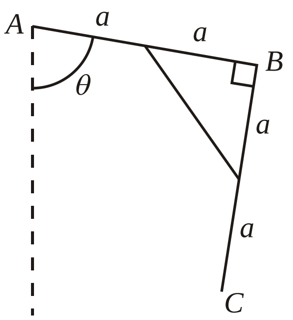
(c) Hence find the value of $\cos\theta$ at the other position of equilibrium.

(3)

(d) Determine the stability of each of the two positions of equilibrium.

(5)
(Total 18 marks)

4.



A framework consists of two uniform rods *AB* and *BC*, each of mass *m* and length 2*a*, joined at *B*. The mid-points of the rods are joined by a light rod of length $a\sqrt{2}$, so that angle *ABC* is a right angle. The framework is free to rotate in a vertical plane about a fixed smooth horizontal axis. This axis passes through the point *A* and is perpendicular to the plane of the framework. The angle between the rod *AB* and the downward vertical is denoted by θ, as shown in the diagram above.

(a) Show that the potential energy of the framework is

$$-mga(3\cos\theta + \sin\theta) + \text{constant}.$$

(4)

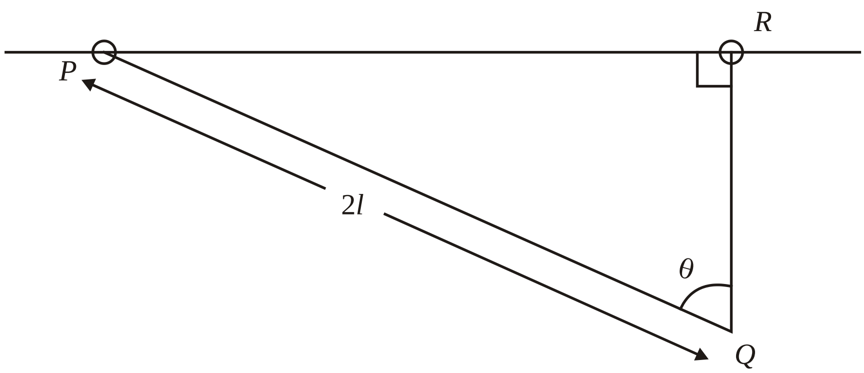
(b) Find the value of θ when the framework is in equilibrium, with *B* below the level of *A*.

(4)

(c) Determine the stability of this position of equilibrium.

(4)
(Total 12 marks)

5.



A uniform rod PQ has mass m and length $2l$. A small smooth light ring is fixed to the end P of the rod. This ring is threaded on to a fixed horizontal smooth straight wire. A second small smooth light ring R is threaded on to the wire and is attached by a light elastic string, of natural length l and modulus of elasticity kmg, to the end Q of the rod, where k is a constant.

(a) Show that, when the rod PQ makes an angle θ with the vertical, where $0 < \theta \leq \frac{\pi}{3}$, and Q is vertically below R, as shown in the figure above, the potential energy of the system is

$$mgl[2k\cos^2\theta - (2k + 1)\cos\theta] + \text{constant}$$

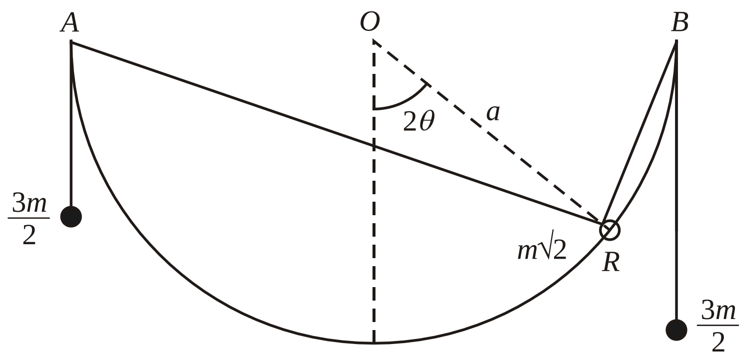
(7)

Given that there is a position of equilibrium with $\theta > 0$,

(b) show that $k > \frac{1}{2}$

(5)
(Total 12 marks)

6.



A smooth wire with ends A and B is in the shape of a semicircle of radius a. The mid-point of AB is O. The wire is fixed in a vertical plane and hangs below AB which is horizontal. A small ring R of mass $m\sqrt{2}$ is threaded on the wire and is attached to two light inextensible strings. The strings each have length $2a$ and pass over smooth pegs at A and B. The other end of each string is attached to a particle of mass $\frac{3m}{2}$. The particles hang vertically under gravity, as shown in the figure above.

(a) Show that, when the radius OR makes an angle 2θ with the vertical, the potential energy, V, of the system is given by

$$V = \sqrt{2}mga\,(3\cos\theta - \cos 2\theta) + \text{constant}.$$

(7)

(b) Find the values of θ for which the system is in equilibrium.

(6)

(c) Determine the stability of the position of equilibrium for which $\theta > 0$.

(4)
(Total 17 marks)

7. A non-uniform rod BC has mass m and length $3l$. The centre of mass of the rod is at distance l from B. The rod can turn freely about a fixed smooth horizontal axis through B. One end of a light elastic string, of natural length l and modulus of elasticity $\frac{mg}{6}$, is attached to C. The other end of the string is attached to a point P which is at a height $3l$ vertically above B.

(a) Show that, while the string is stretched, the potential energy of the system is

$$mgl(\cos^2\theta - \cos\theta) + \text{constant},$$

where θ is the angle between the string and the downward vertical and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

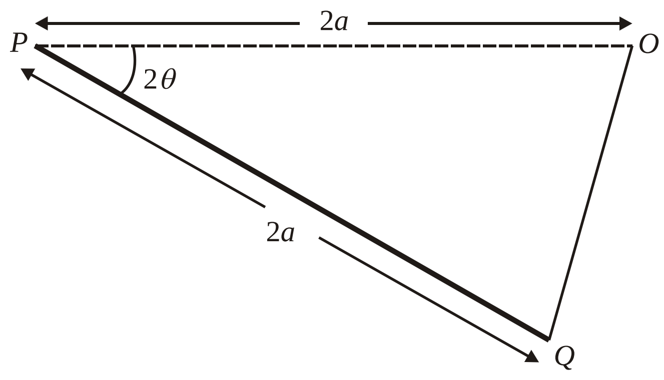
(6)

(b) Find the values of θ for which the system is in equilibrium with the string stretched.

(6)

(Total 12 marks)

8.



A uniform rod PQ, of length $2a$ and mass m, is free to rotate in a vertical plane about a fixed smooth horizontal axis through the end P. The end Q is attached to one end of a light elastic string, of natural length a and modulus of elasticity $\frac{mg}{2\sqrt{3}}$. The other end of the string is attached to a fixed point O, where OP is horizontal and $OP = 2a$, as shown in the diagram above. $\angle OPQ$ is denoted by 2θ.

(a) Show that, when the string is taut, the potential energy of the system is

$$-\frac{mga}{\sqrt{3}}(2\cos 2\theta + \sqrt{3}\sin 2\theta + 2\sin\theta) + \text{constant.}$$

(7)

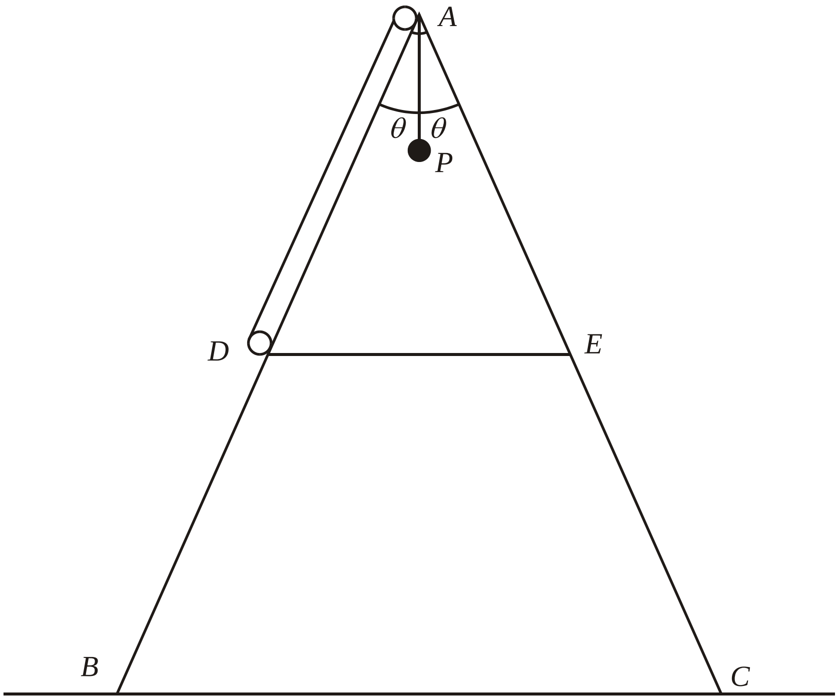
(b) Verify that there is a position of equilibrium at $\theta = \frac{\pi}{6}$.

(4)

(c) Determine whether this is a position of stable equilibrium.

(4)
(Total 15 marks)

9.



Two uniform rods AB and AC, each of mass $2m$ and length $2L$, are freely jointed at A. The mid-points of the rods are D and E respectively. A light inextensible string of length s is fixed to E and passes round small, smooth light pulleys at D and A. A particle P of mass m is attached to the other end of the string and hangs vertically. The points A, B and C lie in the same vertical plane with B and C on a smooth horizontal surface. The angles PAB and PAC are each equal to θ ($\theta > 0$), as shown in the diagram above.

(a) Find the length of AP in terms of s, L and θ.

(2)

(b) Show that the potential energy V of the system is given by

$$V = 2mgL\,(3\cos\theta + \sin\theta) + \text{constant}.$$

(4)

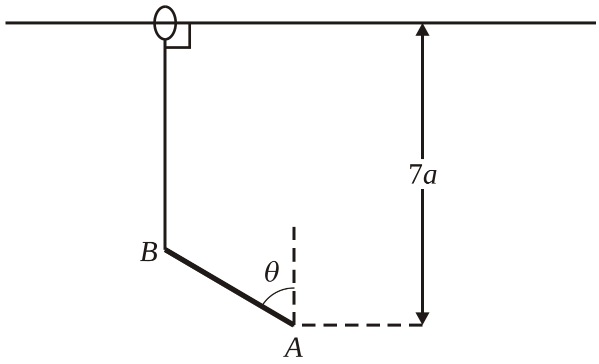
(c) Hence find the value of θ for which the system is in equilibrium.

(4)

(d) Determine whether this position of equilibrium is stable or unstable.

(4)
(Total 14 marks)

10.



A uniform rod *AB*, of length $2a$ and mass $8m$, is free to rotate in a vertical plane about a fixed smooth horizontal axis through *A*. One end of a light elastic string, of natural length a and modulus of elasticity $\frac{4}{5}mg$, is fixed to *B*. The other end of the string is attached to a small ring which is free to slide on a smooth straight horizontal wire which is fixed in the same vertical plane as *AB* at a height $7a$ vertically above *A*. The rod *AB* makes an angle θ with the upward vertical at *A*, as shown in the diagram above.

(a) Show that the potential energy V of the system is given by

$$V = \frac{8}{5}mg\,a\,(\cos^2\theta - \cos\theta) + \text{constant}.$$

(6)

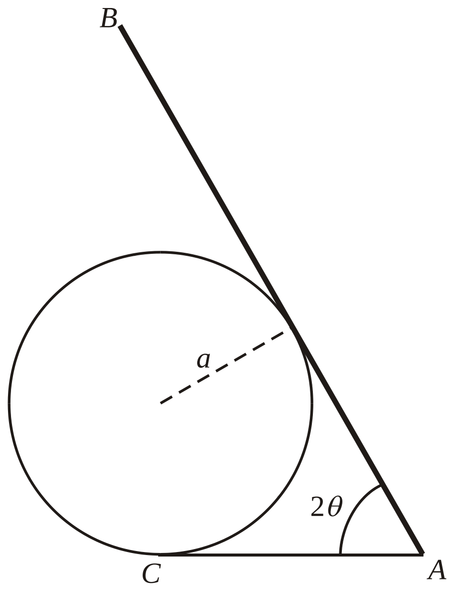
(b) Hence find the values of θ, $0 \leq \theta \leq \pi$, for which the system is in equilibrium.

(5)

(c) Determine the nature of these positions of equilibrium.

(4)
(Total 15 marks)

11.



The diagram above shows a uniform rod AB, of mass m and length $4a$, resting on a smooth fixed sphere of radius a. A light elastic string, of natural length a and modulus of elasticity $\frac{3}{4}mg$, has one end attached to the lowest point C of the sphere and the other end attached to A. The points A, B and C lie in a vertical plane with $\angle BAC = 2\theta$, where $\theta < \frac{\pi}{4}$. Given that AC is always horizontal,

(a) show that the potential energy of the system is

$$\frac{mga}{8}(16 \sin 2\theta + 3 \cot^2\theta - 6 \cot \theta) + \text{constant},$$

(7)

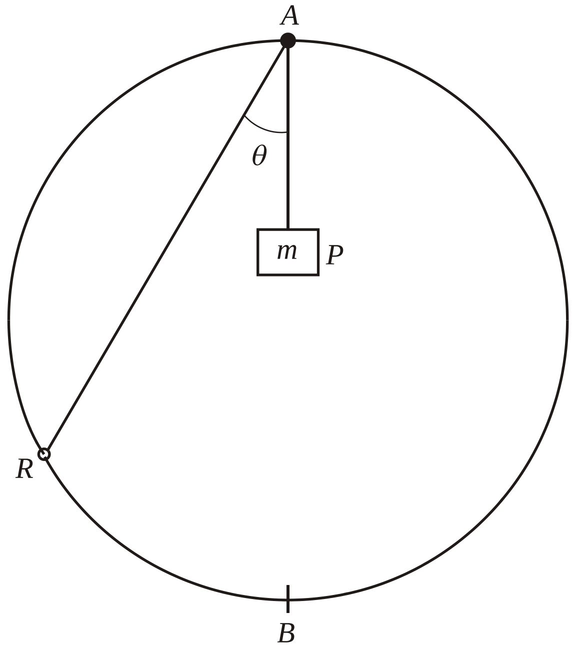
(b) show that there is a value of θ for which the system is in equilibrium such that $0.535 < \theta < 0.545$.

(6)

(c) Determine whether this position of equilibrium is stable or unstable.

(3)
(Total 16 marks)

12.



A smooth wire *AB*, in the shape of a circle of radius *r*, is fixed in a vertical plane with *AB* vertical. A small smooth ring *R* of mass *m* is threaded on the wire and is connected by a light inextensible string to a particle *P* of mass *m*. The length of the string is greater than the diameter of the circle. The string passes over a small smooth pulley which is fixed at the highest point *A* of the wire and angle $R\hat{A}P = \theta$, as shown in the diagram above.

(a) Show that the potential energy of the system is given by

$$2mgr(\cos\theta - \cos^2\theta) + \text{constant}.$$

(6)

(b) Hence determine the values of θ, $\theta \geq 0$, for which the system is in equilibrium.

(6)

(c) Determine the stability of each position of equilibrium.

(5)
(Total 17 marks)

1. (a) $\sqrt{4a^2 + 16a^2 - 16a^2 \sin\theta}$ M1 A1

Let length of string be L.

$V = -4mga\cos\theta - mg(L - \sqrt{4a^2 + 16a^2 - 16a^2 \sin\theta}$ M1 A1

$= -4mga\cos\theta - mgL + 2mga\sqrt{5 - 4\sin\theta}$

$= 2mga\left\{\sqrt{5 - 4\sin\theta} - 2\cos\theta\right\} + \text{constant}$ * * A1 5

(b) $V'(\theta) = 2mga\left\{\dfrac{-2\cos\theta}{\sqrt{5 - 4\sin\theta}} + 2\sin\theta\right\}$ M1 A1

For equilibrium, $V'(\theta) = 0$

$\left\{\dfrac{-2\cos\theta}{\sqrt{5 - 4\sin\theta}} + 2\sin\theta\right\} = 0$ M1

$\dfrac{\cos^2\theta}{5 - 4\sin\theta} = \sin^2\theta$

$1 - \sin^2\theta = \sin^2\theta(5 - 4\sin\theta)$ DM1

$4\sin^3\theta - 6\sin^2\theta + 1 = 0$ * * A1 5

(c) $V''(\theta) = 2mga\left(\dfrac{\left\{\sqrt{5 - 4\sin\theta}.2\sin\theta - \dfrac{-2\cos\theta.(-4\cos\theta)}{2\sqrt{5 - 4\sin\theta}}\right\}}{(5 - 4\sin\theta)} + 2\cos\theta\right)$ M1 A1 A1

$V''\left(\dfrac{\pi}{6}\right) = 2mga\left\{\dfrac{\sqrt{3} - \dfrac{8\text{x}\frac{3}{4}}{2\sqrt{3}}}{3} + \sqrt{3}\right\} = 2mga\sqrt{3} > 0$ so stable DM1 A1 5

[15]

2. (a) $V = -mg2a\sin 2\theta - \frac{7}{20}mg\left(L - 4a\sin\theta\right)$ M1 B1 A1

$= \frac{1}{5}\ mga(7\sin\theta - 10\sin 2\theta) - \frac{7}{20}mgL$ A1 4

(b) $\dfrac{dV}{d\theta} = \frac{1}{5}\ mga(7\cos\theta - 20\cos 2\theta)$ M1 A1

$\frac{1}{5}\ mga(7\cos\theta - 20\cos 2\theta) = 0$ DM1

$7\cos\theta - 20(2\cos^2\theta - 1) = 0$ DM1

$40\cos^2\theta - 7\cos\theta - 20 = 0$ A1

$(5\cos\theta - 4)(8\cos\theta + 5) = 0$ DM1 A1

$\cos\theta = \frac{4}{5}$ or $(\cos\theta = -\frac{5}{8} \Rightarrow 2\theta > 180°$ DM1 8

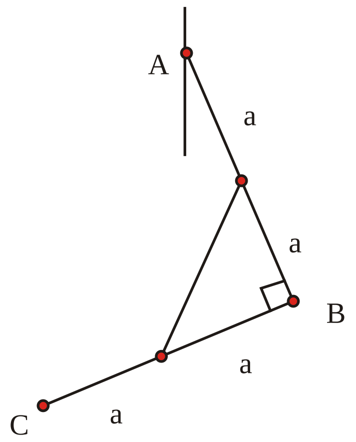
	(c)	$\frac{d^2V}{d\theta^2} = \frac{1}{5}\ mga(-7\sin\theta + 40\sin 2\theta)$	M1 A1	
		$= \frac{1}{5}\ mga(-7\sin\theta + 80\sin\theta\cos\theta)$		
		When $\cos\theta = \frac{4}{5}$,		
		$\frac{d^2V}{d\theta^2} = \frac{1}{5} mga(\frac{-21}{5} + 80 \text{x} \frac{3}{5} \text{x} \frac{4}{5}) = \frac{171}{25} mga$	M1	
		> 0 therefore stable	A1 cso	4

[16]

3.	(a)	PE of rod = $-kMga \sin 2\theta$	B1	
		$BP = 2 \times 2a \sin\theta = 4a \sin\theta$	M1	
		PE of mass = $-Mg(6a - 4a \sin\theta)$	A1	
		$V = -Mg(6a - 4a \sin\theta) - kMga \sin 2\theta$	M1	
		$= Mga(4 \sin\theta - k \sin 2\theta)$ + constant *	A1	5
	(b)	$\frac{dV}{d\theta} = Mga(4 \cos\theta - 2k \cos 2\theta)$	M1A1	
		so, $4 \times \frac{3}{4} - 2k\left(2\left(\frac{3}{4}\right)^2 - 1\right) = 0$	M1M1	
		$\Rightarrow k = 12$	A1	5
	(c)	$4 \cos\theta - 24(2 \cos^2\theta - 1) = 0$	M1	
		$12 \cos^2\theta - \cos\theta - 6 = 0$	DM1	
		$(4 \cos\theta - 3)(3 \cos\theta + 2) = 0$		
		$\cos\theta = -\frac{2}{3}$	A1	3
	(d)	$\frac{d^2V}{d\theta^2} = (Mga)(-4 \sin\theta + 4k \sin 2\theta)$	M1A1	
		when $\cos\theta = \frac{3}{4}, \frac{d^2V}{d\theta^2} = (Mga) \times 44.97.. \Rightarrow$ stable	M1A1	
		when $\cos\theta = \frac{-2}{3}, \frac{d^2V}{d\theta^2} = (Mga) \times -50.68.. \Rightarrow$ unstable	A1	5

[18]

4.



(a) $V = -mga\cos\theta - mg(2a\cos\theta + a\sin\theta)$ M1A1A1

$= -mga(3\cos\theta + \sin\theta)$ (+const)* A1 4

M1 Expression for the potential energy of the two rods.
Condone trig errors.
Condone sign errors. BC term in two parts

A1 correct expression for AB

A1 correct expression for BC

A1 Answer **as given**.

(b) $\frac{dV}{d\theta} = -mga(-3\sin\theta + \cos\theta)$ M1A1

$= 0 \Rightarrow \tan\theta = \frac{1}{3}$ M1

$\Rightarrow \theta = 0.32(1)^c$ or 18.4° accept awrt A1 4

M1 Attempt to differentiate V. Condone errors in signs and in constants.

A1 Derivative correct

M1 Set derivative = 0 and rearrange to a single trig function in θ

A1 Solve for θ

or M1A1 find the position of the centre of mass
M1A1 form and solve trig equation for θ

(c) $\frac{d^2V}{d\theta^2} = -mga(-3\cos\theta - \sin\theta)$ M1A1

$= mga\,(3\cos\theta + \sin\theta)$

Hence, when $\theta = 0.32^e$, $\frac{d^2V}{d\theta^2} > 0$ M1

i.e. stable A1 4

M1 Differentiate to obtain the second derivative

A1 Derivative correct

M1 Determine the sign of the second derivative

A1 Correct conclusion. cso

Or: M1 Find the value of $\frac{dV}{d\theta}$ on both sides of the minimum point

A1 signs correct

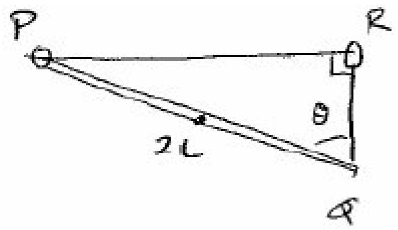
M1 Use the results to determine the nature of the turning point

A1 Correct conclusion, cso.

These 4 marks are dependent on the use of derivatives.

[12]

5. (a)



PE of rod $= -mgl\cos\theta$ B1

EPE of string $\frac{Kmg}{2l}(2l\cos - l)^2$ M1 A1

Total PE of system, $V = -mgl\cos\theta + \frac{kmgl}{2}(2\cos\theta - 1)^2 + c$ M1

$= -mgl\cos\theta + \frac{kmgl}{2}(4\cos^2\theta - 4\cos\theta + 1) + c$ M1 A1

$= mgl(-\cos\theta + 2k\cos^2\theta - 2k\cos\theta) + c'$

$= -mgl\cos\theta\,[2k\cos^2\theta - (2K + 1)\cos\theta] + c'$ A1 7

(b) $\frac{dv}{d\theta} = mgL(-4\text{K}\cos\theta\sin\theta + (2\text{K}+1)\sin\theta)$ M1 A1

At equilibrium, $mgl\sin\theta(-4\text{K}\cos\theta + (2\text{K}+1)) = 0$ M1

$\Rightarrow \sin\theta = 0 \text{ or } \cos\theta = \frac{2\text{K}+1}{-4\text{K}}$

$\Rightarrow \theta = 0\ (\theta > 0)\ \frac{2\text{K}+1}{-4\text{K}} < 1$ M1

$2\text{K}+1 < 4\text{K}$

$1 < 2\text{K}$

$\frac{1}{2} < \text{K}$ A1 5

[12]

6. (a) PE of R = $-\sqrt{2}mga\cos 2\theta$ (+c) (1) B1

PE of LH mass = $-\frac{3}{2}mg(2a - 2a\sin(45 + \theta))$ (+c) (2) M1 A1

PE of RH mass = $-\frac{3}{2}mg(2a - 2a\sin(45 - \theta))$ (+c) (3) A1

V = (1) + (2) + (3) (in terms of θ etc.) M1

$= -\sqrt{2}mga\cos 2\theta - \frac{3}{2}mg[4a - a\sqrt{2}(\cos\theta + \sin\theta + \cos\theta - \sin\theta)]$ M1

$= -\sqrt{2}mga\cos 2\theta - \frac{3}{2}mga(-2\sqrt{2}\cos\theta + 4)$

$= \underline{-\sqrt{2}mga(3\cos\theta + 4\cos 2\theta) + \text{constant}}$ A1 7

(b) $\frac{dV}{d\theta} = \sqrt{2}mga(-3\sin\theta + 2\sin 2\theta)$ M1 A1

$\frac{dV}{d\theta} = 0 \Rightarrow 2\sin 2\theta - 3\sin\theta = 0$ M1

$\Rightarrow \sin\theta(4\cos\theta - 3) = 0$ M1

$\Rightarrow \underline{\theta = 0, \text{ or } \theta = \pm\arccos\frac{3}{4} (= \pm 0.723)}$ A1, A1 6

(c) $\frac{d^2V}{d\theta^2} = \sqrt{2}\, mga(-3\cos\theta + 4\cos 2\theta)$ M1 A1

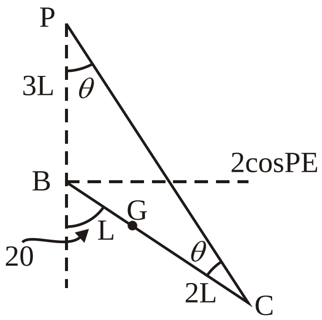
$\cos\theta = \frac{3}{4}$: $\frac{d^2V}{d\theta^2} = \sqrt{2}mga\left(-3.\frac{3}{4} + 4\left(2.\frac{9}{16} - 1\right)\right)$ M1

$\sqrt{2}mga\left(-\frac{9}{4} + \frac{1}{2}\right)$ A1

<u><0 \ Unstable</u> A1 4

[17]

7. (a)



GPE = $-mgL\cos 2\theta$ B1

EPE = $\frac{mg}{6} \cdot \frac{(6L\cos\theta - L)^2}{2L}$ M1

$= \frac{mg}{12L}(36L^2\cos^2\theta - 12L^2\cos\theta + L^2)$ M1

$= mgL(3\cos^2\theta - \cos\theta) + C$

$V = -mgL(2\cos^2\theta - 1) + mgL(3\cos^2\theta - \cos\theta) + C$ M1 M1

$= \underline{mgL(\cos^2\theta - \cos\theta) + C'}$ (*) A1 6

(b) $\frac{dV}{d\theta} = mgl(-2\cos\theta\sin\theta + \sin\theta) = 0$ M1 A1; M1

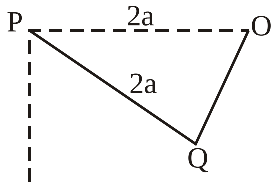
$\sin\theta(-2\cos\theta + 1) = 0$

$\sin\theta = 0$ or $\cos\theta = \frac{1}{2}$ M1

$\underline{\theta = 0}$ or $\underline{\theta = \pm\frac{\pi}{3}}$ A1 A1 6

[12]

8. (a)



$OQ = 4a \sin \theta$	B1	
$V = (-)\, mga \sin 2\theta;\ + \dfrac{mg}{2\sqrt{3}2a}(4a \sin \theta - a)^2 + C$	B1; M1 A1	
$= -mga \sin 2\theta + \dfrac{mga^2}{4a\sqrt{3}}(16 \sin^2 \theta - 8 \sin\theta + 1) + C$	M1	
$= -mga \sin 2\theta + \dfrac{mga}{4\sqrt{3}}(8(1 - \cos 2\theta) - 8 \sin \theta) + C$	M1	
i.e. $V = -\dfrac{mga}{\sqrt{3}}(2 \cos 2\theta + \sqrt{3} \sin 2\theta + 2 \sin \theta) + C$ (*)	A1 c.s.o	7

(b) $V'(\theta) = -\dfrac{mga}{\sqrt{3}}(-4 \sin 2\theta + 2\sqrt{3} \cos 2\theta + 2 \cos\theta)$	M1 A1	
$V'\left(\dfrac{\pi}{6}\right) = -\dfrac{mga}{\sqrt{3}}\left(-2\sqrt{3} + 2\sqrt{3}\dfrac{1}{2} + 2\dfrac{\sqrt{3}}{2}\right) = 0$	M1 A1	4

(c) $V''(\theta) = \dfrac{mga}{\sqrt{3}}(+8 \cos 2\theta + 4\sqrt{3} \sin 2\theta + 2 \sin \theta)$	M1 A1	
Hence, $V''\left(\dfrac{\pi}{6}\right) = \dfrac{11mga}{\sqrt{3}} > 0 \therefore$ stable	M1 A1 c.s.o	4

[15]

9. (a) $AP = s - AD - DE$	M1	
$= s - L - 2L\sin\theta$	A1	2

(b) $V(\theta) = 2 \times 2mg \times L\cos\theta + \ldots$	B1	
$= \ldots + mg(2L \cos\theta - AP)$	M1	
$= 4mgL \cos\theta + mg(2L \cos\theta + 2L \sin \theta)\ (+C)$	M1	
$= 2mgL(3 \cos \theta + \sin\theta)$ + constant (*) cso	A1	4

(c)	$V'(\theta) = 2mgL\ (-3\sin\theta + \cos\ \theta)$	M1	
	$= 0$	M1	
	$\tan\ \theta = \dfrac{1}{3}$	A1	
	$\theta \approx 18°$ awrt 18°, 0.32^c	A1	4
(d)	$V''(\theta) = 2mgL(-3\cos\ \theta) - \sin\ \theta)$	M1 A1	
	$\left(V''\left(\arctan\dfrac{1}{3}\right) = -2\sqrt{10}mgL\right)$		
	$V''(\theta) < 0$, for any acute θ	M1	
	Equilibrium is <u>unstable</u> ft any acute θ	A1 ft	4

[14]

10. M1

Þ $q = 0.32(1)^c$ or 18.4° accept awrt A1 4

M1 Attempt to differentiate V. Condone errors in signs and in constants.

A1 Derivative correct

M1 Set derivative = 0 and rearrange to a single trig function in *q*

A1 Solve for *q*

or M1A1 find the position of the centre of mass
M1A1 form and solve trig equation for *q*

(c) $\dfrac{d^2V}{d\theta^2}\ \dfrac{d^2V}{d\theta^2} > 0\ \dfrac{dV}{d\theta}$ **22.** (a)Extension of

string $= 7a - 2a\cos\ \theta - a$	B1	
$= 2a(3 - \cos\ \theta)$		
PE $= 8mga\cos\ \theta, + \dfrac{4mg}{5} \times \dfrac{4a^2}{2a}\ (3 - \cos\ \theta)^2$	B1, M1 A1	
$= 8mga\cos\ \theta + \dfrac{8mga}{5}\ (9 - 6\cos\ \theta - \cos^2\ \theta)$	M1	
$= \dfrac{8mga}{5}\ (\cos^2\ \theta - \cos\ \theta) + C$ (*)	A1	6

(b)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = \frac{8mga}{5}(-2\cos\theta\sin\theta + \sin\theta)$		M1 A1	
	$= 0, \Rightarrow \sin\theta = 0$ <u>or</u> $\cos\theta = \frac{1}{2}$		M1,	
	$\Rightarrow \theta = 0$ or π, <u>or</u> $\theta = \frac{\pi}{3}$		A1, A1	5

(c)	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = \frac{8mga}{5}(\cos\theta + 2\sin^2\theta - 2\cos^2\theta)$				M1 A1	
	$\theta = 0$	$V'' < 0$	$(= -\frac{8mga}{5})$	unstable		
	$\theta = \pi$	$V'' < 0$	$(= -3 \times \frac{8mga}{5})$	unstable	A1 ft	
	$\theta = \frac{\pi}{3}$	$V'' > 0$	$(= \frac{3}{2}.\frac{8mga}{5})$	stable	A1	4

[15]

11. (a)

P.E. of rod $= mg \times 2a\sin 2\theta$	B1	
$AC = a\cot\theta$	B1	
EPE in String $= \frac{1}{2} \times \frac{3}{4} \times \frac{mg}{a}(a\cot\theta - a)^2$	M1 A1	
Total P.E $V = mg.2a\sin 2\theta + \frac{3}{8}\frac{mg}{a}(a\cot\theta - a)^2$	M1	
$= \frac{mga}{8}[16\sin 2\theta + 3\cot^2\theta - 6\cot\theta + 3]$	M1	
i.e. $V = \frac{mga}{8}[16\sin 2\theta + 3\cot^2\theta - 6\cot\theta] + \text{const}$ (*)	A1 cso	7

(b)

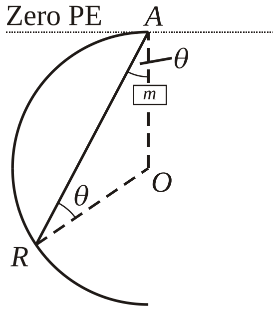
$\frac{\mathrm{d}v}{\mathrm{d}\theta} = \frac{mga}{8}[32\cos 2\theta - 6\cot\theta\,\mathrm{cosec}^2\theta + 6\,\mathrm{cosec}^2\theta]$	M1 A2, 1, 0	
$\left.\frac{\mathrm{d}v}{\mathrm{d}\theta}\right\|_{\theta=0.535} = \frac{mga}{8}(-0.5^{0.1\ldots}_{\ldots\ldots})$	M1	
$\left.\frac{\mathrm{d}v}{\mathrm{d}\theta}\right\|_{\theta=0.545} = \frac{mga}{8}(0.2^{99\ldots}_{\ldots\ldots})$	A1	
Change of sign $\therefore \frac{\mathrm{d}v}{\mathrm{d}\theta} = 0$ in range, so $\exists$ find a position of equilibrium	A1	6

(c) $\left.\frac{dv}{d\theta}\right|_{0.535} < 0, \left.\frac{dv}{d\theta}\right|_{\theta=0.545} > 0$ M1

So turning point is *minimum*, $\therefore$ equilibrium is *stable* A1, A1 3

[16]

12. (a)



$AR = 2r \cos\theta$ B1

For P: $-mg(L - 2r\cos\theta)$ B1

For R: $-mg\, 2r\cos^2\theta$ M1 A1

$V = 2mgr(\cos\theta - \cos^2\theta) - mgL$ (*) M1 A1 6

(b) $\frac{dV}{d\theta} = 2mgr(-\sin\theta + 2\cos\theta \sin\theta)$ M1 A1

$= 2mgr\sin\theta(2\cos\theta - 1)$ A1

$0 = 2mgr\sin\theta(2\cos\theta - 1)$ M1

$\sin\theta = 0$ or $\cos\theta = \frac{1}{2}$

$\theta = 0$ or $\theta = \frac{\pi}{3}$ A1 A1 6

(c) $\frac{d^2V}{d\theta^2} = 2mgr(-\cos\theta + 2\cos 2\theta)$ M1 A1

$\theta = 0, \frac{d^2V}{d\theta^2} = 2mgr > 0 \Rightarrow$ STABLE M1 A1

$\theta = \frac{\pi}{3}, \frac{d^2V}{d\theta^2} = -3mgr < 0 \Rightarrow$ UNSTABLE A1 5

[17]

1. For part (a) many candidates identified the need to find the distance of the mass below the pulley in terms of an unknown, the length of the string. These candidates found the distance from *B* to the pulley correctly and were able to proceed correctly to obtain an expression for the potential energy of the system which simplified to the given result. There were a disappointing number of candidates who could barely attempt this part of the question and either tried to fudge the result (including the sign of the term relating to the mass) or who offered no attempt to this part of the question at all. A small minority of candidates complained that they could not proceed without being told the length of the string.

For part (b) a large proportion of candidates knew they needed to differentiate the expression for potential energy and set it equal to zero. Many were able to go on to obtain the given equation correctly, but there were a number of algebraic errors and a few candidates were not able to achieve an expression in $\sin\theta$ alone.

For part (c) the majority of candidates understood that they should find the sign of $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}$ in order to determine the stability of the system and most correctly interpreted the sign they obtained. Some candidates made errors in their differentiation but a large number coped very well with differentiating the complicated product/quotient. A significant number differentiated the equation obtained in part (b) rather than their expression for $\frac{\mathrm{d}V}{\mathrm{d}\theta}$. A small number of candidates opted for the alternative route of considering the sign of $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ on either side of $\theta = \frac{\pi}{6}$.

2. Although this was a standard problem, many candidates lost marks in part (a) because they fudged the result for the lighter particle rather than calculate the distance of the particle below *B*. Some candidates offered no attempt to part (a) and simply went on to use the given result. In parts (b) and (c) the majority of candidates were confident in using the given expression for the potential energy to identify the position of equilibrium of the system and to conclude that this was a point of stable equilibrium. The problems occurred in trying to explain why there was only one solution for $\cos\theta$ when their quadratic equation had two roots. The given condition on *L* was often interpreted as an upper bound rather than a lower bound leading to some spurious reasons for rejecting one of the roots. Some responses demonstrated little understanding of the physical properties of the model, and of strings in particular.

3. This standard problem about equilibrium positions and their stability was well done by most candidates. The vast majority were able to obtain the potential energy of the rod and most could also find *BP* and hence obtain a correct expression for the potential energy of the mass, leading to the given result. Generally the methods required for finding a position of equilibrium and determining its stability were well understood by candidates. Parts (b) and (c) were therefore answered well in most cases. In part (d) most candidates appreciated the need to find the second differential and consider its sign for the two values of cos θ. There were a great many mistakes in calculating the values of the second differential, however, although most candidates did make correct inferences about stability from the signs of their answers.

4. This was a very straightforward question which posed few difficulties for most candidates. Finding the potential function in part (a) caused the greatest number of problems. Many demonstrations of the given result left a lot for the examiner to infer from diagrams which were often poorly drawn. Methods involving $a\sqrt{2}$, the length of the light rod, or $a\sqrt{5}$, the distance of the centre of mass of *BC* from *A*, were seen and were often successful but much longer than direct calculation of the distances of the centres of mass of the rods below the level of *A*. Having been given the potential function, parts (b) and (c) were routine and generally done well. Some candidates working with the centre of mass of the framework started with part (b), usually successfully, but did not then go on to complete the rest of the question.

5. (a) This was a routine type of question that most candidates were happy to tackle. However the zero position for GPE was occasionally chosen as the 'variable point' Q, instead of using a fixed level (PR), which then resulted in a sign error – with the benefit of the printed answer, many making this error adjusted their solutions either legitimately (without penalty) or by faking with a loss of 2 marks. It was pleasing to note that nearly all candidates found the EPE correctly and followed this with accurate algebra.

(b) Candidates who do not reach a printed answer should be instructed to proceed with the printed answer, not with their incorrect expression. This part was less routine, and although most candidates differentiated V and put $V' = 0$, the convincing use of $\cos\theta < 1$ to obtain the given result was rarer.

6. If candidates spotted that $\angle$OAB was $(45 - \theta)$ and $\angle$OBR was $(45 + \theta)$ then part (a) was reasonably straightforward. Unfortunately, many did not, and used the cosine rule which resulted in expressions involving $\sin 2\theta$ as well as $\cos 2\theta$. Credit was given for this as the trigonometry was correct but only the strongest candidates managed to obtain the printed answer from this approach. Those who did used some ingenious methods and are to be congratulated on the standard of their trigonometry! However, since the answer was given, all could progress to the next part of the question. The first three marks in (b) were almost always obtained but then some cancelled the $\sin\theta$ factor and lost one solution, others gave 0 and π, others gave just +0.723 and many gave a solution in degrees instead of radians. In questions involving calculus, candidates need to be reminded that they <u>must</u> use radians. Part (c) was usually correct.

7. (a) Candidates found it difficult to draw a clear diagram with the angle θ correctly placed. Many candidates failed to see that the triangle PBC was isosceles and so used the cosine rule to find PC which resulted in many errors. Trigonometric manipulation was not well handled.

(b) This part of the question was answered much better than the first part. Most candidates knew the required method and obtained correct solutions although θ = 0 was often omitted on the wrong grounds that the string was slack. Another error was to give only θ = π/3 and to omit θ = -π/3.

8. This question was well-answered and there were many fully correct solutions. Parts (b) and (c) were usually correct but the geometry/trigonometry in (a) caused problems for some candidates with many failing to exploit the isosceles triangle. A few candidates, in part (b), tried to solve rather than verify and some, in part (c), left out -*mga*/√3 from the derivative and consequently got the answer wrong.

9. This question was well done and full marks were common. A few candidates did, however, make the fundamental error in part (b) of attempting to measure potential energies from the level of *A*, a point which has a variable height above the smooth horizontal surface.

10. This question proved to be a good source of marks for many. Most were well versed in the methods required. A few ‘fudges’ of the signs occurred in part (a) but the general principles were generally well understood. So too in part (b) most realised that they had to differentiate the expression for *V* and equate it to zero, though a number failed to realize that sin *q* = 0 gives rise to two solutions. In part (c) the general methods were well known; however, a number gave no figures at all for their values of *V*″ at the relevant values of the angle and simply stated without justification that this was positive or negative. Candidates must realise that they must show their working to justify the claims they make.

11. Most candidates realised that the gravitational potential energy of the rod and the elastic potential energy of the string were required but some could not find *AC* correctly. Those that did were usually able to complete part (a) satisfactorily. The differentiation in part (b) was usually carried out correctly but some candidates tried to solve to find the equilibrium position rather than demonstrating a change of sign using the given values. Few realised that having shown that the gradient changed from negative to positive in part (b) that was sufficient to establish a minimum point and hence stable equilibrium. The preferred approach involved a second derivative and use of the given values but this was often spoilt by errors in accuracy.

12. No Report available for this question.