1. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively*]

A man cycles at a constant speed u m s^{-1} on level ground and finds that when his velocity is $u\mathbf{j}$ m s^{-1} the velocity of the wind appears to be $v(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}, where v is a positive constant.

When the man cycles with velocity $\frac{1}{5}u(-3\mathbf{i} + 4\mathbf{j})$ m s^{-1}, the velocity of the wind appears to be $w\mathbf{i}$ m s^{-1}, where w is a positive constant.

Find, in terms of u, the true velocity of the wind.

(Total 7 marks)

2. At 12 noon, ship A is 8 km due west of ship B. Ship A is moving due north at a constant speed of 10 km h^{-1}. Ship B is moving at a constant speed of 6 km h^{-1} on a bearing so that it passes as close to A as possible.

(a) Find the bearing on which ship B moves.

(4)

(b) Find the shortest distance between the two ships.

(3)

(c) Find the time when the two ships are closest.

(3)

(Total 10 marks)

3. At noon a motorboat P is 2 km north-west of another motorboat Q. The motorboat P is moving due south at 20 m s^{-1}. The motorboat Q is pursuing motorboat P at a speed of 12 m s^{-1} and sets a course in order to get as close to motorboat P as possible.

(a) Find the course set by Q, giving your answer as a bearing to the nearest degree.

(4)

(b) Find the shortest distance between P and Q.

(3)

(c) Find the distance travelled by Q from its position at noon to the point of closest approach.

(5)

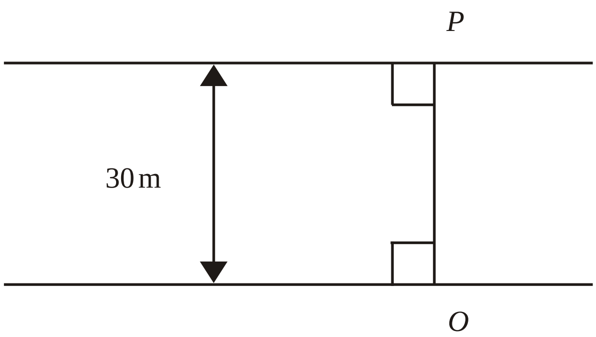
(Total 12 marks)

4. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively*.]

A ship P is moving with velocity $(5\mathbf{i} - 4\mathbf{j})$ km h^{-1} and a ship Q is moving with velocity $(3\mathbf{i} + 7\mathbf{j})$ km h^{-1}. Find the direction that ship Q appears to be moving in, to an observer on ship P, giving your answer as a bearing.

(Total 5 marks)

5.



A river is 30 m wide and flows between two straight parallel banks. At each point of the river, the direction of flow is parallel to the banks. At time $t = 0$, a boat leaves a point O on one bank and moves in a straight line across the river to a point P on the opposite bank. Its path OP is perpendicular to both banks and $OP = 30$ m, as shown in the diagram above. The speed of flow of the river, r m s^{-1}, at a point on OP which is at a distance x m from O, is modelled as

$$r = \frac{1}{10}x, \ \ 0 \le x \le 30\,.$$

The speed of the boat relative to the water is constant at 5 m s^{-1}. At time t seconds the boat is at a distance x m from O and is moving with speed v m s-1 in the direction OP.

(a) Show that

$$100v^2 = 2500 - x^2.$$

(3)

(b) Hence show that

$$\frac{d^2x}{dt^2} + \frac{x}{100} = 0 .$$

(4)

(c) Find the total time taken for the boat to cross the river from O to P.

(9)
(Total 16 marks)

6. At 12 noon, ship A is 20 km from ship B, on a bearing of 300°. Ship A is moving at a constant speed of 15 km h^{-1} on a bearing of 070°. Ship B moves in a straight line with constant speed V km h^{-1} and intercepts A.

(a) Find, giving your answer to 3 significant figures, the minimum possible for V.

(3)

It is now given that $V = 13$.

(b) Explain why there are two possible times at which ship B can intercept ship A.

(2)

(c) Find, giving your answer to the nearest minute, the earlier time at which ship B can intercept ship A.

(8)
(Total 13 marks)

7. At noon, a boat P is on a bearing of 120° from boat Q. Boat P is moving due east at a constant speed of 12 km h^{-1}. Boat Q is moving in a straight line with a constant speed of 15 km h^{-1} on a course to intercept P. Find the direction of motion of Q, giving your answer as a bearing.

(Total 5 marks)

` 88. A cyclist C is moving with a constant speed of 10 m s^{-1} due south. Cyclist D is moving with a constant speed of 16 m s^{-1} on a bearing of 240°.

(a) Show that the magnitude of the velocity of C relative to D is 14 m s^{-1}.

(3)

At 2 pm, D is 4 km due east of C.

(b) Find

(i) the shortest distance between C and D during the subsequent motion,

(ii) the time, to the nearest minute, at which this shortest distance occurs.

(7)
(Total 10 marks)

9. Two ships P and Q are moving with constant velocity. At 3 p.m., P is 20 km due north of Q and is moving at 16 km h^{-1} due west. To an observer on ship P, ship Q appears to be moving on a bearing of 030° at 10 km h^{-1}. Find

(a) (i) the speed of Q,

(ii) the direction in which Q is moving, giving your answer as a bearing to the nearest degree,

(6)

(b) the shortest distance between the ships,

(3)

(c) the time at which the two ships are closest together.

(3)
(Total 12 marks)

10. A cyclist P is cycling due north at a constant speed of 20 km h^{-1}. At 12 noon another cyclist Q is due west of P. The speed of Q is constant at 10 km h^{-1}. Find the course which Q should set in order to pass as close to P as possible, giving your answer as a bearing.

(Total 5 marks)

11. *[In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.]*

An aeroplane makes a journey from a point P to a point Q which is due east of P. The wind velocity is $w(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$, where w is a positive constant. The velocity of the aeroplane relative to the wind is $v(\cos\phi\,\mathbf{i} - \sin\phi\,\mathbf{j})$, where v is a constant and $v > w$. Given that θ and ϕ are both acute angles,

(a) show that $v \sin\phi = w \sin\theta$,

(2)

(b) find, in terms of v, w and θ the speed of the aeroplane relative to the ground.

(4)
(Total 6 marks)

12. At noon, two boats A and B are 6 km apart with A due east of B. Boat B is moving due north at a constant speed of 13 km h^{-1}. Boat A is moving with constant speed 12 km h^{-1} and sets a course so as to pass as close as possible to boat B. Find

(a) the direction of motion of A, giving your answer as a bearing,

(4)

(b) the time when the boats are closest,

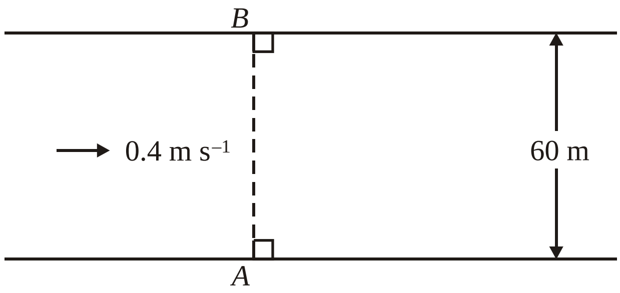
(5)

(c) the shortest distance between the boats.

(2)
(Total 11 marks)

13.

Figure 1



Mary swims in still water at 0.85 m s^{-1}. She swims across a straight river which is 60 m wide and flowing at 0.4 m s^{-1}. She sets off from a point *A* on the near bank and lands at a point *B*, which is directly opposite *A* on the far bank, as shown in Fig. 1.

Find

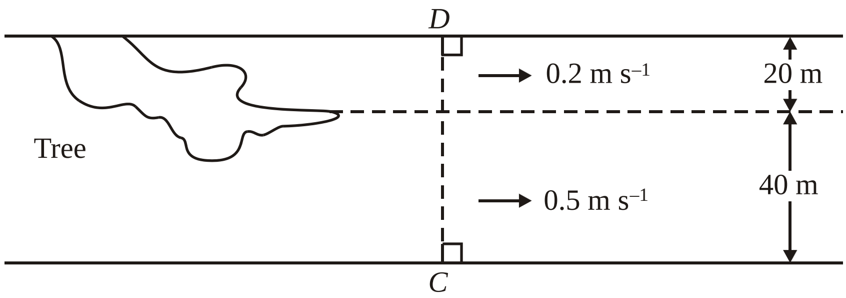
(a) the angle between the near bank and the direction in which Mary swims,

(3)

(b) the time she takes to cross the river.

(3)

Figure 2



A little further downstream a large tree has fallen from the far bank into the river. The river is modelled as flowing at 0.5 m s^{-1} for a width of 40 m from the near bank, and 0.2 m s^{-1} for the 20 m beyond this. Nassim swims at 0.85 m s^{-1} in still water. He swims across the river from a point C on the near bank. The point D on the far bank is directly opposite C, as shown in Fig. 2. Nassim swims at the same angle to the near bank as Mary.

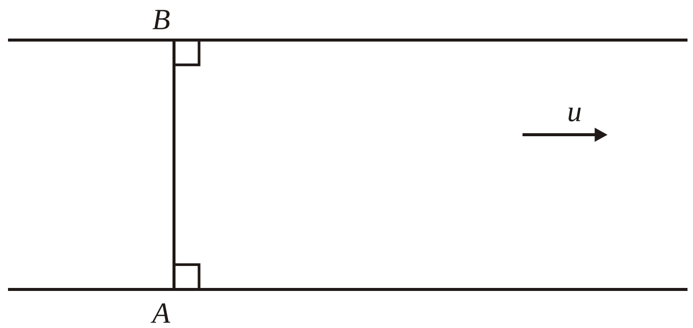
(c) Find the maximum distance, downstream from CD, of Nassim during the crossing.

(5)

(d) Show that he will land at the point D.

(4)
(Total 9 marks)

14.



A man, who rows at a speed v through still water, rows across a river which flows at a speed u. The man sets off from the point A on one bank and wishes to land at the point B on the opposite bank, where AB is perpendicular to both banks, as shown in the diagram above.

(a) Show that, for this to be possible, $v > u$.

(3)

Given that $v < u$ and that he rows from A so as to reach a point C, on the opposite bank, which is as close to B as possible,

(b) find, in terms of u and v, the ratio of BC to the width of the river.

(5)
(Total 8 marks)

15. A man walks due north at a constant speed u and the wind seems to him to be blowing *from* the direction 30° east of north. On his return journey, when he is walking at the same speed u due south, the wind seems to him to be blowing *from* the direction 30° south of east. Assuming that the velocity, $\mathbf{w}$, of the wind relative to the earth is constant, find

(i) the magnitude of $\mathbf{w}$, in terms of u,

(ii) the direction of $\mathbf{w}$.

(Total 9 marks)

16. A boy enters a large horizontal field and sees a friend 100 m due north. The friend is walking in an easterly direction at a constant speed of 0.75 m s^{-1}. The boy can walk at a maximum speed of 1 m s^{-1}.

Find the shortest time for the boy to intercept his friend and the bearing on which he must travel to achieve this.

(Total 6 marks)

17. Boat A is sailing due east at a constant speed of 10 km h^{-1}. To an observer on A, the wind appears to be blowing from due south. A second boat B is sailing due north at a constant speed of 14 km h^{-1}. To an observer on B, the wind appears to be blowing from the south west. The velocity of the wind relative to the earth is constant and is the same for both boats.

Find the velocity of the wind relative to the earth, stating its magnitude and direction.

(Total 7 marks)

18. Two particles P and Q have constant velocity vectors v_P and v_Q respectively. The magnitude of the velocity of P relative to Q is equal to the speed of P. If the direction of motion of one of the particles is reversed, the magnitude of the velocity of P relative to Q is doubled.

Find

(a) the ratio of the speeds of P and Q,

(9)

(b) the cosine of the angle between the directions of motion of P and Q.

(3)
(Total 12 marks)

1.

$$v(3\mathbf{i} - 4\mathbf{j}) = \mathbf{v}_W - u\mathbf{j}$$ M1 A1

$$\mathbf{v}_W = 3v\mathbf{i} + (u - 4v)\mathbf{j}$$

$$w\mathbf{i} = \mathbf{v}_W - \frac{u}{5}(-3\mathbf{i} + 4\mathbf{j})$$ M1 A1

$$\mathbf{v}_W = (w - \frac{3u}{5})\mathbf{i} + \frac{4u}{5}\mathbf{j}$$

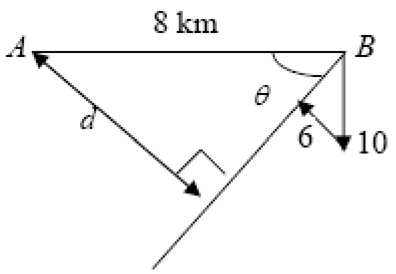
$$(u - 4v) = \frac{4u}{5}$$ M1

$$v = \frac{u}{20}$$ A1

$$\mathbf{v}_W = \frac{3u}{20}\mathbf{i} + \frac{4u}{5}\mathbf{j}$$ A1

[7]

2. (a)



M1

$$\cos\theta = \frac{6}{10} \Rightarrow \theta = 53.1^\circ$$ M1 A1

Bearing is 307° A1 4

(b) $d = 8 \sin \theta$ (= 8 × 0.8) M1 A1

= 6.4 km A1 3

(c)

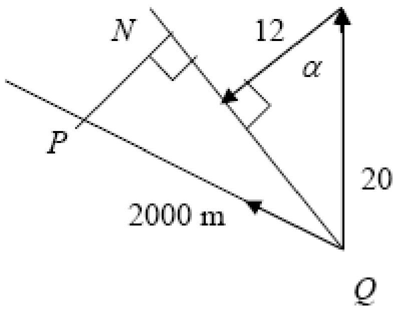
$$T = \frac{8\cos\theta}{\sqrt{10^2 - 6^2}}$$ M1 A1

$= 0.6$ hrs

i.e. the time is 12:36 pm A1 3

[10]

3. (a)



M1

$\cos\alpha = \frac{12}{20}$ M1 A1

Bearing is $180° + \alpha = 233°$ (nearest degree) A1 4

(b) $PN = 2000\cos(135° - \alpha) = 200\sqrt{2}$ m
or decimal equivalent M1A1ft A1 3

(c) $\sqrt{20^2 - 12^2}$ B1

Time to closest approach $= \dfrac{QN}{\sqrt{20^2 - 12^2}}$ M1

$= \dfrac{2000\sin(135° - \alpha)}{16}$ A1

Distance moved by Q = their $t \times 12$ DM1

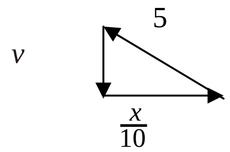
$= 1050\sqrt{2}$
m or decimal equivalent A1 5

[12]

4. ${}_Q\mathbf{V}_P = \mathbf{V}_Q - \mathbf{V}_P = (3\mathbf{i} + 7\mathbf{j}) - (5\mathbf{i} - 4\mathbf{j})$ M1
$= (-2\mathbf{i} + 11\mathbf{j})$ A1
$\tan\theta = \dfrac{11}{2} \Rightarrow \theta = 79.69°...$ M1 A1
Bearing is 350° A1 5

[5]

5. (a)



vector triangle M1

$v^2 + \left(\frac{x}{10}\right)^2 = 5^2$ M1

$\Rightarrow 100v^2 = 2500 - x^2$ A1 3

(b) $200v\frac{dv}{dx} = -2x$ M1 A1

$200\frac{d^2x}{dt^2} + 2x = 0$ DM1

$\frac{d^2x}{dt^2} + \frac{x}{100} = 0*$ A1 4

(c) Aux equn: $m^2 + \frac{1}{100} = 0$ M1

$\Rightarrow m = \pm\frac{i}{10}$ A1

$x = A\sin\frac{t}{10} + B\cos\frac{t}{10}$ A1

$t = 0, x = 0 \Rightarrow B = 0$ B1

$\frac{dx}{dt} = \frac{A}{10}\cos\frac{t}{10}$ M1

$t = 0, x = 0 \Rightarrow v = \frac{dx}{dt} = 5$

$\Rightarrow 5 = \frac{A}{10} \Rightarrow A = 50$ M1

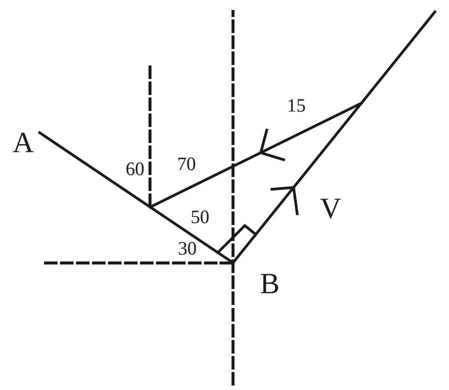
$\Rightarrow x = 50\sin\frac{t}{10}$ A1

$x = 30$: $30 = 50\sin\frac{t}{10}$

$\Rightarrow t = 10\sin^{-1}\left(\frac{3}{5}\right) = 6.44$ s M1A1 9

[16]

6. (a)



Fix A

$v_{\min} = 15\sin 50°$ M1A1

$= 11.5$ km h^{-1} (3 s.f.) A1 3

or: triangle without the right angle identified and $\dfrac{15}{\sin\theta} = \dfrac{v_B}{\sin 50}$

$\Rightarrow v_B = \dfrac{15\sin 50}{\sin\theta}$

minimum value $\Rightarrow \theta = 90$ for M1

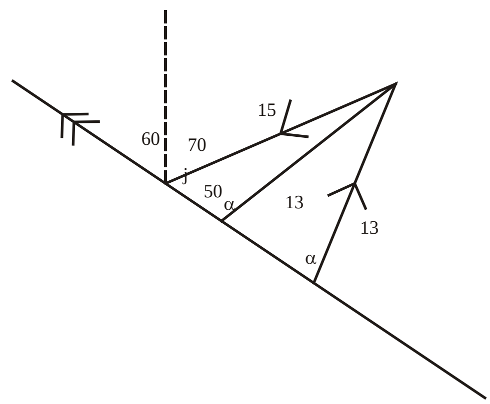
As above for A1A1

M1 Velocity of B relative to A in the direction of the line joining AB.
Minimum V requires a right angled triangle.
Convincing attempt to find the correct side.

A1 15 × sin(their 50°)

A1 Q specifies 3sf, so 11.5 only

(b)



Ambiguous Sine Rule: B1B1 2

2 possible solutions for α

B1B1 Convincing argument

B1B0 Argument with some merit

(c) $\frac{\sin \alpha}{15} = \frac{\sin 50}{13}$ M1A1

$\underline{\alpha = 62.1°}$ (or 118°) A1

(smaller value gives larger relative velocity)

$\Rightarrow$ either

$v = 13 \cos 62.1 + 15 \cos 50 = 15.72 \text{ kmh}^{-1}$ M1A1

Or

$v^2 = 15^2 + 13^2 - 390 \cos 67.9 = 247.27$ M1

$v = 15.7 \text{ kmh}^{-1}$ A1

Time = $\frac{20}{\text{their } 15.72....}$ M1A1

Earliest time is 13.16 hrs or 13.17 hrs A1 8

accept 1.16 (pm) or 1.17 (pm)

M1 Use of Sine Rule

A1 Correct expression

A1 (2 possible values,) pick the correct values.

M1 Use trig. to form an equation in v

A1 correct equation

M1 time = $\frac{\text{distance}}{\text{speed}}$

A1ft correct expression with their v (not necessarily evaluated)

A1 correct time in hours & minutes

Or: M1 Use of cosine rule

A1 $13^2 = 15^2 + v^2 - 2 \times 15 \times v \times \cos 50$

A1 (Award after the next two marks) 15.72 or awrt 15.72

M1 Attempt to solve the equation for v

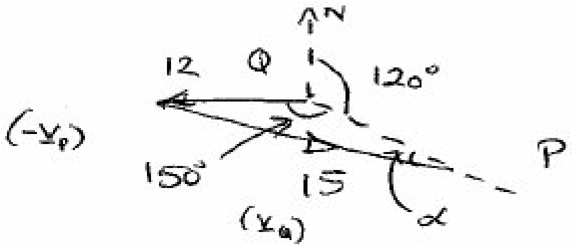
A1 $\frac{30 \cos 50 + \sqrt{(30 \cos 50)^2 - 4 \times 56}}{2}$

(15.72 or 3.562)

Finish as above

[13]

7.



M1

$\frac{\sin\alpha}{12} = \frac{\sin 150^\circ}{15}$ M1 A1

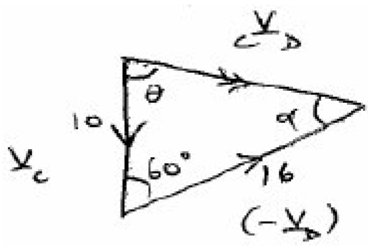
$\Rightarrow \sin\alpha = \frac{6}{15}$

$\Rightarrow \alpha = 23.6^\circ$ A1

$\therefore$ Course is 096 (.4)° A1

[5]

8. (a)



$|{}_C\underline{V}_D|^2 = 10^2 + 16^2 - 2 \times 10 \times 16\cos 60°$ M1 A1

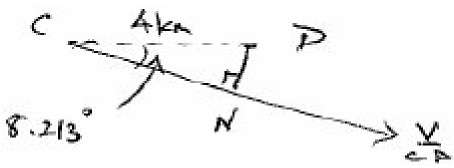
$= 196$

$|{}_C\underline{V}_D| = 14ms^{-1}$ A1 3

(b) α is <u>acute</u> (opposite shortest side)

$\frac{\sin\alpha}{10} = \frac{\sin 60^\circ}{14}$ M1

$\Rightarrow \alpha = 38.213...^\circ$ A1



(i) DN = 4000 sin 8.213 M1

$\simeq$ 571m $\left(\frac{4000}{7}\right)$ A1

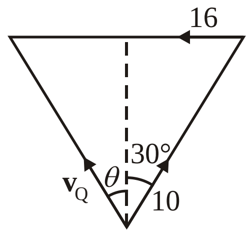
(ii) $t = \dfrac{4000\cos 8.213^{\circ}}{14}$ secs

$\simeq$ 282.78…….secs

Time is 2.05pm (nearest minute) A1 7

[10]

9. (a) (i)



$\mathbf{v}_Q = {}_Q\mathbf{v}_P + \mathbf{v}_P$

$|\mathbf{v}_Q|^2 = (10\cos 30)^2 + (16 - 10\sin 30)^2$ M1 A1

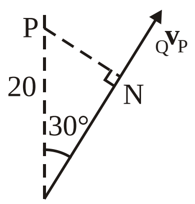
$= 75 + 121$ A1

$\Rightarrow \underline{|\mathbf{v}_Q| = 14\ m\ s^{-1}}$

(ii) $\tan\theta = \dfrac{16 - \sin 30}{10\cos 30}$ (o.e.) M1

$\theta \approx 51.8°$, $\Rightarrow$ bearing <u>308°</u> (nearest degree) A1, A1 6

(b)



At nearest approach: $PN = 20\sin 30$ M1 A1

$= \underline{10\ km}$ A1 3

(c) *Time* = $\frac{20\cos 30}{10}$ ≈ 1.732 hrs M1 A1

$\Rightarrow$ <u>*Time* ≈ 4.44 *pm*</u> (AWRT) A1 3

[12]

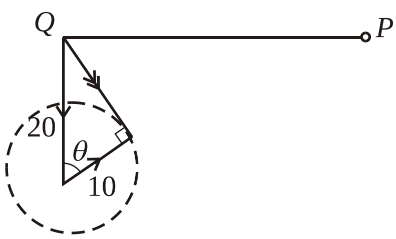
<u>Alternatives</u>

(a) Use of cosine rule in velocity vector triangle.

(b) & (c) Use of scalar product of relative velocity and relative position or differentiating magnitude of relative position vector squared to find the minimum distance and time at which it occurs.

10.

Fix P



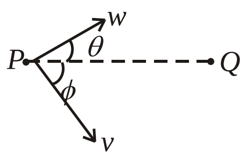
Vector Δ M1 A1

$\cos\theta = \frac{10}{20}$ M1 A1

$\Rightarrow$ <u>θ = 060°</u> A1

[5]

11. (a)



$v_A = v_{AW} + v_W$

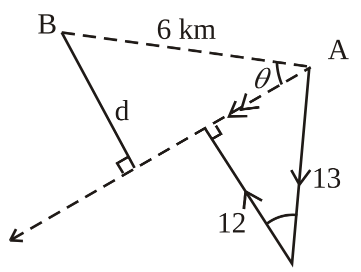
$\Rightarrow \begin{pmatrix} k \\ 0 \end{pmatrix} = \begin{pmatrix} v\cos\phi \\ -v\sin\phi \end{pmatrix} + \begin{pmatrix} w\cos\theta \\ w\sin\theta \end{pmatrix}$ M1 (bottom line)

$\Rightarrow$ <u>$v\sin\phi = w\sin\theta$ (*)</u> A1 2

(b)	$k = v \sin \phi + w \sin \theta$	M1 A1	
	$= \dfrac{v\sqrt{v^2 - w^2 \sin^2 \theta}}{v} + w \cos \theta$	M1	
	$= \underline{\sqrt{v^2 - w^2 \sin^2 \theta} + w\cos\theta}$	A1	4

[6]

12. (a)

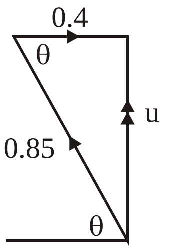


Vector Δ	M1 A1	
$\cos \theta = \dfrac{12}{13}$	M1	
($\theta = 22.6°$)		
Course is $360° - 22.6°$		
$= \underline{337°}$ (AWRT)	A1	4

(b)	$v = \sqrt{13^2 - 12^2} = 5$	M1 A1	
	$t = \dfrac{6\cos\theta}{5} = 1.107$	M1 A1	
	<u>Time is 1.06 p.m.</u> 1.07	A1	5

(c)	$d = 6 \sin \theta = 6 \times \dfrac{5}{13} = 2.31$ km *(AWRT 2.3 km)*	M1 A1	2

[11]

13.	(a)				
		Vector Δ or $\leftarrow$		M1	
		$\cos\theta = \frac{0.4}{0.85}$		M1	
		$\theta \approx 61.9°$ awrt $62°$		A1	3
	(b)	$u = \sqrt{(0.85^2 - 0.4^2)}$ or $u = 0.85\sin\theta$		M1	
		$t = \frac{60}{u} = \frac{60}{0.75} = 80$ (s) cao		M1 A1	3
	(c)	$\mathbf{v}_{N\text{ rel}W} = -0.4\mathbf{i}\ (+0.75\mathbf{j})$	Allow for $\pm 0.4\mathbf{i}$	M1	
		$\mathbf{v}_N = \mathbf{v}_{N\text{ rel }W} + 0.5\mathbf{i} = 0.1\mathbf{i} + (0.75\mathbf{j})$ $0.1\mathbf{i}$		A1	
		$t = \frac{40}{0.75} = \frac{160}{3}$		M1	
		$\delta = 0.1 \times \frac{160}{3} = \frac{16}{3}$ awrt 5.3		M1 A1	5
	(d)	As in(c) $\mathbf{v}_N = -0.2\mathbf{i} + 0.75\mathbf{j}$	$\pm 0.2\mathbf{i}$	M1	
		$t = \frac{20}{0.75} = \frac{80}{3}$		M1	
		$\delta = 0.2 \times \frac{80}{3} = \frac{16}{3}$		M1	
		Hence N lands at D cso		A1	4

[15]

Notes:

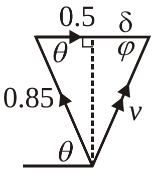
1. In(c) and (d), the candidate can take components without using vectors. Mark as vector method.

2. After the first line in (d), the result is clear by proportion. Allow as long as some explanation given.

3. $\cos\theta = \frac{8}{17} = 0.4705\ldots$, $\sin\theta = \frac{15}{17} = 0.8823\ldots$

Alternatives to(c) and (d)

(c)



$v^2 = 0.5^2 + 0.85^2 - 2 \times 0.5 \times 0.85 \times \cos\theta$

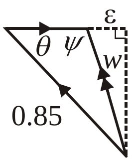
$= 0.5725 \quad \left(v = \dfrac{\sqrt{229}}{20} \approx 0.7566... \right)$ M1

$\dfrac{\sin\varphi}{0.85} = \dfrac{\sin\theta}{v}$ M1

$\sin\varphi = \dfrac{15}{\sqrt{229}} \quad (\approx 0.9912;\ \varphi \approx 82.4°)$ A1

$\dfrac{\delta}{40} = \cot\varphi;\ \delta = 40 \times \dfrac{2}{5} = \dfrac{16}{3}$ awrt 5.3 M1 A1 5

(d)



$w^2 = 0.2^2 + 0.85^2 - 2 \times 0.2 \times 0.85 \times \cos\theta$

$= 0.6025 \quad \left(w = \dfrac{\sqrt{241}}{20} \approx 0.7762... \right)$ M1

$\dfrac{\sin\psi}{0.85} = \dfrac{\sin\theta}{w}$

$\sin\psi = \dfrac{15}{\sqrt{241}} \quad (\approx 0.9662;\ \psi \approx 104.9°)$ M1

$\psi = 75.1°$ gains M1

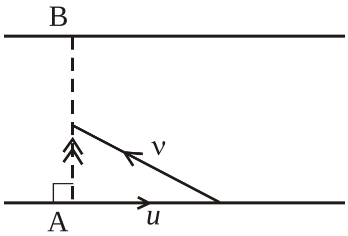
$\dfrac{\varepsilon}{20} = \cot(180° - \psi) = \dfrac{4}{15}$

$\varepsilon = \dfrac{16}{3} = \delta$ M1

Hence N lands at D cso A1 4

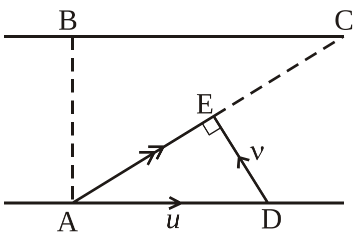
Note: Exact working is needed for final A1 but all previous marks in(c) and (d) may be gained by approximate working.

14. (a)



vector triangle attempted with right angle	M1	
vector triangle correct	A1	
Explanation for $v > u$ (e.g. 'hypotenuse > other sides')	A1	3

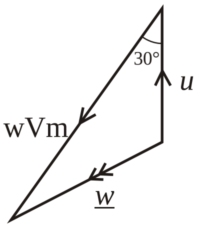
(b)

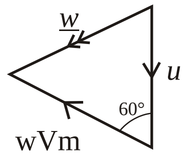


vector triangle attempted	M1	
right angle correctly placed	A1	
$\frac{BC}{AB} = \frac{AE}{ED}$ Use of similar triangles (or 'tanθ=')	M1	
$= \frac{\sqrt{(u^2 - v^2)}}{v}$	M1 A1	5

[8]

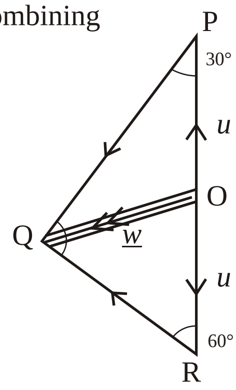
15. $\underline{v}_W = {}_W\underline{v}_M + \underline{v}_M$ (used) M1





(one vector Δ) M1 A1 A1

Combining M1



$P\hat{Q}R = 90°$ M1

$QR = 2\,u \sin 30° = u$ A1

$\Rightarrow$ Δ O Q R equilat

$\Rightarrow$ O Q = $w = u$ A1

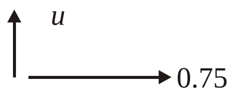
also $\Rightarrow$ $Q\hat{O}R = 60°$

Hence dir^n is from N 60° E A1

or 240°

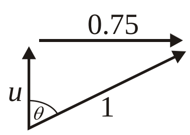
[9]

16. Let boy's velocity be M1



Speed = 1 $\Rightarrow$ $1^2 = u^2 + \dfrac{9}{16}$, $\therefore\ u^2 = \dfrac{7}{16}$ or $u = \dfrac{\sqrt{7}}{4}$ or 0.661… M1 A1

Time = $\dfrac{100}{\sqrt{7}/4}$ = 151.18… = 151s A1



M1

$\sin\theta = \dfrac{0.75}{1}$ $\Rightarrow \theta = 48.6$

$\therefore$ Bearing is 049° or 048.6° A1 6

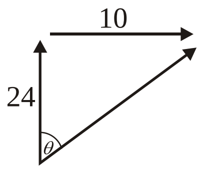
[6]

17. Let wind be $\uparrow W_y$, $\rightarrow W_x$ M1

Relative to A: $\uparrow W_y$, $\rightarrow W_x - 10$ From South, $\Rightarrow W_x = 10$ M1, A1

Relative to B: $\uparrow W_y - 14$, $\rightarrow W_x$ From SW, $\Rightarrow W_y - 14 = W_x \therefore W_y = 24$ M1, A1

$\therefore$ Magnitude of $W = \sqrt{10^2 + 24^2} = 26 \text{ km h}^{-1}$ A1



$\tan \alpha = \dfrac{10}{24} \Rightarrow \alpha = 22.6$

$\therefore$ Bearing 023° or 022.6° A1

[7]

18. (a) $(\mathbf{v}_P - \mathbf{v}_Q)^2 = \mathbf{v}_P{}^2$ ① M1 A1

$(\mathbf{v}_P + \mathbf{v}_Q)^2 = 4\mathbf{v}_P{}^2$ ② M1 A1

$4\mathbf{v}_P \,.\, \mathbf{v}_Q = 3\mathbf{v}_P{}^2$ ② − ① M1 A1

From ①: $2\mathbf{v}_P \,.\, \mathbf{v}_Q = \mathbf{v}_Q{}^2$ ③ M1 A1

$\therefore \sqrt{\dfrac{2}{3}} = \dfrac{|\mathbf{v}_P|}{|\mathbf{v}_Q|}$ A1 9

(b) From ③ above, $2\,|\mathbf{v}_P|\,|\mathbf{v}_Q| \cos\theta = |\mathbf{v}_Q|^2$ M1 A1

$\cos\theta = \dfrac{1}{2}\sqrt{\dfrac{3}{2}} = \dfrac{\sqrt{6}}{4}$ A1 3

[12]

1. By far the most successful approach was to use the vector equation for relative velocity and equate the **j** component of the wind velocity vectors obtained. Candidates who took this approach tended to be successful in obtaining the correct answer.

It was rare for those candidates who attempted to form vector triangles to obtain a correct diagram showing both sets of information and even rarer for them to work correctly from their diagram to find the wind velocity.

A number of candidates spoiled their solutions by losing a factor of u or v from a term in one of their equations at some point, thus producing dimensionally inconsistent equations which could not give the correct answer.

2. This is a fairly standard closest approach question and a great many candidates were able to draw correct diagrams and score full marks. Despite this, however, there were still a large number of candidates who have not understood how to approach such relative velocity questions and were unable to make much progress at all.

Those who attempted to express the relative position and relative velocity as vectors were very rarely successful, usually due to an error in their original diagram.

Finding a bearing still presents problems to a significant number of candidates, despite working from a correct diagram.

Some candidates did not realise that the relative velocity could be found simply from the 3-4-5 triangle, but it was pleasing to see so many candidates answering the question and giving an actual time of the clock as the answer.

3. Clear, correctly orientated diagrams were particularly helpful in this question. Many candidates understood that in order for Q to pass as close as possible to P the course set by Q needed to be perpendicular to the relative velocity. It proved to be more difficult to determine the actual bearing, with many choosing the direction opposite to the required direction. When the working was correct, the final mark was often lost through not expressing the bearing to the nearest whole degree.

Candidates with a good diagram usually used a correct method to determine the shortest distance between P and Q in part (b), but this work was often affected by rounding errors due to premature rounding in calculating the angle in the triangle.

Several correct solutions to (c) were seen, but some candidates were not able to find the time taken to the point of closest approach because they did not realise that they had sufficient information to find the relative velocity.

Some completely correct vector solutions were seen, although candidates using this method were generally less successful.

4. This question was well answered. Candidates generally favoured the relative vector approach and those who adopted this method were usually successful in finding the bearing correctly. Some candidates were able to use the cosine rule and sine rule on a vector triangle to produce a correct solution but there proved to be many more opportunities for errors in this method. The most common error arose from confusion over the direction of the relative velocity, with some candidates selecting the opposite direction.

5. Candidates needed to apply Pythagoras' Theorem to a correct vector triangle and a great many did so successfully to obtain the given result in part (a). Part (b) was answered rather less well, with many candidates not able to apply implicit differentiation correctly. The use of the function of a function rule or demonstration of standard relations between the various expressions for acceleration was not well done. Very few candidates recognised the resulting equation as SHM. Generally candidates choose to solve this second order differential equation in a traditional way, finding the auxiliary equation and hence the general solution. Those that found a correct general solution, tended to be able to complete the problem correctly. A much smaller number of candidates used the result in part (a) to obtain a differential equation in x and t and solved it by separating the variables and using the standard result for arcsine.

6. Relative velocity problems are usually an area of the syllabus in which candidates do not perform well – this question was no exception, with a disappointingly large proportion of candidates being unable to make any progress. Indeed, many demonstrated little appreciation of the fact that if B intercepts A then the velocity of B relative to A would be parallel to the line joining their initial positions.
Many candidates attempted part (a) by forming an expression for the speed of B relative to A and then differentiating with respect to time, rather than direction in their attempt to find the minimum possible value.
Concise and elegant solutions were seen; they were almost invariably preceded by clear and well labelled diagrams in which distances, velocities and relative velocities could easily be distinguished and the required velocity in part (a) could clearly be seen to be perpendicular to AB.
Explanations in part (b) were often not very convincing and in some cases consisted of a poorly drawn diagram with no verbal discussion.
Candidates who attempted vector methods had no success with this problem.
Solutions which were otherwise correct sometimes lost the last mark by not stating the time of day at which interception occurred.

7. This relative velocity question was not well done. Most candidates attempted to draw a vector triangle though the orientation of the components was often wrong, so that although a correct angle in the triangle was found, the correct bearing could not be deduced. A significant number of candidates had Q's velocity directed towards P's initial position rather than that of Q relative to P.

8. (a) This part was well answered, either with the relevant vector triangle drawn and the cosine rule applied or working with the velocities in component form.

(b) There was more success for those using their vector triangle to find the direction of the relative velocity than for those who, working with components, minimised $|{}_C\mathbf{r}_D|^2$ or used the condition ${}_C\mathbf{r}_D \cdot {}_C\mathbf{v}_D = 0$. Calculation of the time taken was frequently incorrect due to the use of 4 (km) rather than 4000 (m). Quite a lot of candidates found the time taken but did not continue to give the actual time at which the closest approach occurred, thus losing the final mark.

9. The inclusion of only one relative velocity question was perhaps a relief to candidates who have difficulty with this topic. Many methods of solution were seen. In part (a), sine and cosine rule or vector methods occurred frequently with candidates often gaining full marks. Those using vector methods could dispense with a diagram – those relying on a diagram were not always successful, having the speed of 16 km per hour in the wrong direction. In parts (b) and (c) the most direct method was rarely seen – using the initial displacement with the relative velocity and forming a right angled triangle. The shortest distance can then be written down directly as 20sin30 and the time taken is the relative distance travelled divided by the relative speed, 20cos30/10**.** Some candidates used differentiation of |relative position| squared and others, the scalar product of relative position and relative velocity**.** These methods were often successful but wasted time because they were significantly longer.

10. This relative velocity problem was not well done. A clear diagram with all velocities shown and with $\underline{v}_Q$ perpendicular to ${}_P\underline{v}_Q$ were rare. The entire problem was frequently reflected about the North-South axis leading to an answer of 300°. Candidates who attempted a vector expression of the problem and the use of a dot product made some progress but often failed to complete the problem. Very few candidates gave the complete three figure bearing 060.

11. This should have been a straightforward starter for most but full marks was not often seen. The first part was usually correct but part (b) proved to be more challenging The most common error was to find the velocity rather than the speed but some were unable to eliminate ▱ despite the hint given by part (a).

12. Many candidates don’t seem to understand relative velocity and resort to M1 methods. This usually makes the problems much longer and harder and this was especially true in the second and third parts of this question. The correct method in (a) was vaguely known by some but there were some strange looking vector triangles. Part (c) tended to be mostly correct but (b) was more of a challenge. Those who understood relative velocity and carried on from their vector triangle from (a) usually picked up all the marks quite easily.

13. Parts (a) and (b) were well done although the number of candidates who found the wrong angle in (a) showed that, even at this level, there are many candidates who lose marks simply by not reading the question accurately. Parts (c) and (d) proved demanding. There are many possible approaches but in (c) the simplest is probably, taking **i** as a unit vector parallel to a bank and **j** as a unit vector perpendicular to a bank, to obtain the velocity of the swimmer relative to a bank as $(0.1\mathbf{i}+0.75\mathbf{j})\text{ms}^{-1}$.

The time of crossing is then $\frac{40}{0.75}\text{s} = \frac{160}{3}\text{s}$ and the distance moved downstream is $0.1\times\frac{160}{3}\text{m} = \frac{16}{3}\text{m}$. A similar method in part (d) will then give, numerically, an identical answer for the distance moved upstream.

Many tried to use the cosine and sine rules in appropriate vector triangles. If the correct triangles were identified, this is possible but the calculations were very susceptible to rounding errors and errors as great as two in the second significant figure were seen. Such errors were treated generously in part (c) but in part (d), for full marks, it was required that the candidate showed that the swimmer landed exactly at the point *D* and, although this can be done using the cosine and sine rules, this proved beyond almost all who attempted the question this way.

14. Answers to this question were generally weak. In part (a) some were able to draw approximately the correct vector diagram of the relevant velocities, though in many cases these were extremely scrappy and/or unlabelled so that it was not easy to see what was being said or claimed. In part (b) very few could even draw an appropriate triangle of velocities with v pointing upstream. The use of similar triangles (or an equivalent method) to find the required ratio defeated all but a very few of the candidates.

15. Again fully correct solutions to this question were not common. Many clearly had learnt the basic equation involving relative velocities, but as often as not, their triangle of velocities failed to reflect this equation. Several tried to draw two triangles and then ‘fit’ one on to the other (with the vector w common to both). Subsequent working to find the magnitude and direction of w was however often extremely unclear. A minority tried a purely algebraic approach, but often went wrong in dealing with the signs of the ratios involved.

16. This proved to be a good first question and most of the candidates applied Pythagoras’ theorem to find the missing component of the boy’s velocity and hence the time taken. A few candidates thought the velocity was $(0.75\mathbf{i} + \mathbf{j})$ m s^{-1} and some had the wrong angle for the bearing.

17. This was a more demanding relative velocity question and only the best candidates were able to progress to a correct solution. A popular approach was to draw separate diagrams for the situation from boat *A* and from boat *B*. These diagrams were then combined and some candidates were then able to use some simple geometry to deduce that the velocity of the wind was $(10\mathbf{i} + 24\mathbf{j})$ m s^{-1}. Others obtained a correct diagram but got lost in some complex calculations involving the sine and cosine rules.

18. No Report available for this question.