1. A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass m kg. The spheres are moving on a smooth horizontal plane when they collide. At the instant of collision the line joining the centres of the spheres is parallel to **j**. Immediately **after** the collision, the velocity of A is $(3\mathbf{i} - \mathbf{j})$ m s^{-1} and the velocity of B is $(2\mathbf{i} + \mathbf{j})$ m s^{-1}. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Find the velocities of the two spheres immediately before the collision.

(7)

(b) Find the magnitude of the impulse in the collision.

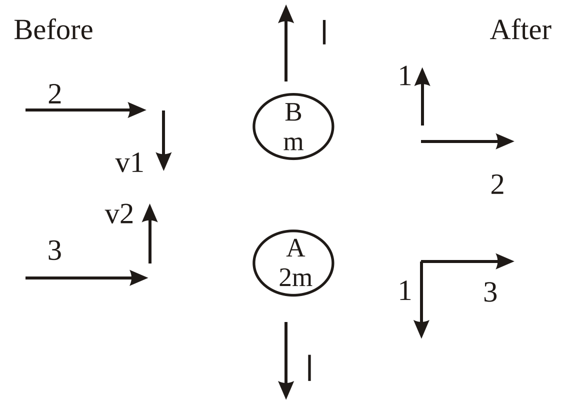
(2)

(c) Find, to the nearest degree, the angle through which the direction of motion of A is deflected by the collision.

(4)

(Total 13 marks)

1. (a)



CLM: $2v_2 - v_1 = 1 - 2 = -1$		M1A1	
NIL: $1 + 1 = \frac{1}{2}(v_1 + v_2)$		M1A1	
$\therefore\ v_2 = 1,\ v_1 = 3$	*Dependent on both M's above*	DM1	
Horizontal components unchanged (i.e. 2 & 3)	*Independent of all other marks*	A1	
$\mathbf{v}_A = 3\mathbf{i} + \mathbf{j};\ \mathbf{v}_B = 2\mathbf{i} - 3\mathbf{j}$		A1	7

M1 Conservation of momentum along the line of centres.
Condone sign errors

A1 equation correct

M1 Impact law along the line of centres.
e must be used correctly, but condone sign errors.

A1 equation correct. The signs need to be consistent between the two equations

M1 Solve the simultaneous equations for their v_1 and v_2.

A1 **i** components correct – independent mark

A1 $\mathbf{v}_A$ & $\mathbf{v}_B$ correct

(b)

For B: I = m(1 – (–3)) = 4m	M1A1	2
(Or For A: –I = 2m(–1 – 1) $\therefore$ I = 4m)		

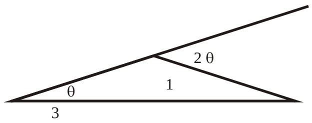
M1 Impulse = change in momentum for one sphere.
Condone order of subtraction.

A1 Magnitude correct.

(c)

$\begin{pmatrix}3\\1\end{pmatrix}.\begin{pmatrix}3\\-1\end{pmatrix} = \sqrt{3^2+1^2}.\sqrt{3^2+(-1)^2}\cos\theta$	M1A1	
$\Rightarrow 8 = 10\cos\theta$	M1	
$\theta = 37°$	A1	4

Alternative:



M1

where $\tan\theta = \frac{1}{3}$ A1

required angle is 2θ M1A1

M1 Any complete method to find the trig ratio of a relevant angle.

A1 $\cos\theta = \frac{4}{5}, \tan\frac{\theta}{2} = \frac{1}{3}, ...$

Or M1 find angle of approach to the line centres and angle after collision.

A1 values correct. (both 71.56....)

M1 solve for θI

A1 37° (Q specifies nearest degree)

Special case: candidates who act as if the line of centres is in the direction of **i**:

CLM $u + 2v = 8$
NIL $v - u = 2$

$u = 4/3$, $v = 10/3$

$4/3\mathrm{i} + \mathbf{j}$; $10/3\mathrm{i} - \mathbf{j}$

Impulse $2m - 4/3m = 2/3m$

$\dfrac{10+1}{\sqrt{10}\sqrt{\dfrac{109}{9}}} = \cos\theta$ $\theta = 1.70°$

Work is equivalent, so treat as a MR:
M1A0M1A0M1A1A1 M1A1 M1A1M1A1

[13]

1. Candidates often failed to draw diagrams accurately showing *A* and *B* in the correct positions relative to each other; the mis-read directing the collision along **i,** where candidates expected it to be, instead of along **j**, as given in the question, was common. Many candidates who did read the question correctly then rotated the situation to conform to their standard model for the problem rather than dealing with the given situation from first principles. Some candidates who read the question correctly then drew diagrams with *A* and *B* in incorrect positions relative to each other, representing an impossible situation in terms of the final velocities.

A significant minority of candidates assumed and found a change in the velocity perpendicular to the line of centres of the two spheres.

Sign errors were frequent and were the main source of errors in part (a). They often remained uncorrected even when the resulting velocity components implied that there would be no collision.

Candidates often thought that both spheres needed to be considered together to find the impulse between them, rather than considering the change of momentum for just one sphere. Another common error was to combine the components of the velocity to find speed before attempting to find the impulse.

Most candidates made some progress in finding the angle of deflection of the direction of motion of *A* by finding a relevant angle but many gave the obtuse angle between the directions rather than the deflection. The dot product method was extremely rare.