1. A particle of mass m is projected vertically upwards, at time $t = 0$, with speed U. The particle is subject to air resistance of magnitude $\frac{mgv^2}{k^2}$, where v is the speed of the particle at time t and k is a positive constant.

(a) Show that the particle reaches its greatest height above the point of projection at time $\frac{k}{g}\tan^{-1}\left(\frac{U}{k}\right)$.

(6)

(b) Find the greatest height above the point of projection attained by the particle.

(6)
(Total 12 marks)

2. At time $t = 0$, a particle P of mass m is projected vertically upwards with speed $\sqrt{\frac{g}{k}}$, where k is a constant. At time t the speed of P is v. The particle P moves against air resistsance whose magnitude is modelled as being mkv^2 when the speed of P is v. Find, in terms of k, the distance travelled by P until its speed first becomes half of its initial speed.

(Total 9 marks)

3. At time $t = 0$, a particle of mass m is projected vertically downwards with speed U from a point above the ground. At time t the speed of the particle is v and the magnitude of the air resistance is modelled as being mkv, where k is a constant.

Given that $U < \frac{g}{2k}$, find, in terms of k, U and g, the time taken for the particle to double its speed.

(Total 8 marks)

4. A lorry of mass M moves along a straight horizontal road against a constant resistance of magnitude R. The engine of the lorry works at a constant rate RU, where U is a constant. At time t, the lorry is moving with speed v.

(a) Show that $Mv\frac{dv}{dt} = R(U - v)$.

(3)

At time $t = 0$, the lorry has speed $\frac{1}{4}U$ and the time taken by the lorry to attain a speed of $\frac{1}{3}U$ is $\frac{kMU}{R}$, where k is a constant.

(b) Find the exact value of k.

(7)
(Total 10 marks)

1. (a)

$-mg(1+\frac{v^2}{k^2}) = m\frac{dv}{dt}$	M1 A1	
$g\int_0^T dt = \int_U^0 \frac{-k^2 dv}{(k^2+v^2)}$	DM1	
$T = \frac{k}{g}\left[\tan^{-1}\frac{v}{k})\right]_0^U$	A1	
$= \frac{k}{g}\tan^{-1}\frac{U}{k}$	DM1 A1	6

(b)

$-mg(1+\frac{v^2}{k^2}) = mv\frac{dv}{dx}$	M1 A1	
$g\int_0^H dx = \int_U^0 \frac{-k^2 v dv}{(k^2+v^2)}$	DM1	
$H = \frac{k^2}{2g}\left[\ln(k^2+v^2\right]_0^U$	A1	
$H = \frac{k^2}{2g}\ln\frac{(k^2+U^2)}{k^2}$	DM1 A1	6

[12]

2.

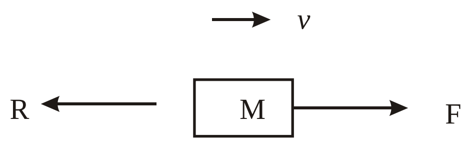
$-mg - mkv^2 = ma$	M1 A1
$-(g+kv^2) = v = \frac{dv}{dx}$	M1
$\pm\int_0^X dx = \int_{\sqrt{\frac{g}{k}}}^{\frac{1}{2}\sqrt{\frac{g}{k}}} \frac{-v dv}{\left(g+kv^2\right)}$	DM1 A1
	(both previous)
$X = \frac{1}{2k}\left[\ln\left(g+kv^2\right)\right]_{\frac{1}{2}\sqrt{\frac{g}{k}}}^{\sqrt{\frac{g}{k}}}$	M1 A1
$= \frac{1}{2k}\left(\ln 2g - \ln\frac{5g}{4}\right)$	M1
$= \frac{1}{2k}\ln\frac{8}{5}$	A1

[9]

3.	$mg - mkv = m\dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1*A1A1	
	$\int \mathrm{d}t = \int \dfrac{\mathrm{d}v}{g - kv}$	DM1*	
	$t = -\dfrac{1}{k}\ln(g - kv) + c$	A1cao	
	$t = 0, v = u \Rightarrow c = \dfrac{1}{k}\ln(g - ku)$	M1†	
	$T = \dfrac{1}{k}\ln(g - ku) - \dfrac{1}{k}\ln(g - 2ku)$	DM1†	
	$= \dfrac{1}{k}\ln\left(\dfrac{g - ku}{g - 2ku}\right)$	A1	8

[8]

4. (a)



$F = \dfrac{Ru}{v}$	B1	
R(→), $\dfrac{Ru}{v} - R = M\dfrac{dv}{dt}$	M1	
$R(u - v) = Mv\dfrac{dv}{dt}$ *	A1	3

B1 Correct expression involving the driving force.

M1 Use of F = ma to form a differential equation.
Condone sign errors.
a must be expressed as a derivative, but could be any valid form.

A1 Rearrange to **<u>given form</u>**.

(b) $\int_0^T dt = \frac{M}{R}\int_{\frac{1}{4}U}^{\frac{1}{3}U} \frac{v\,dv}{u-v}$ M1A1

$\Rightarrow T = \frac{M}{R}\int_{\frac{1}{4}U}^{\frac{1}{3}U} -1 + \frac{u}{u-v}\,dv$ DM1

$= \frac{M}{R}[-v - u\ln(u-v)]_{\frac{1}{4}U}^{\frac{1}{3}U}$ A1

$= \frac{M}{R}\left[-\frac{u}{3} - u\ln\left(\frac{2u}{3}\right) + \frac{u}{4} + u\ln\left(\frac{3u}{4}\right)\right]$ $\left(C = -\frac{Mu}{R}\left(\ln\frac{3u}{4} + \frac{1}{4}\right)\right)$ M1

$= \frac{Mu}{R}\left(-\frac{1}{12} + \ln\frac{9}{8}\right)$ M1

Hence $k = \ln\frac{9}{8} - \frac{1}{2}$ A1 7

M1 Separate the variables

A1 Separation correct (limits not necessarily seen at this stage)

DM1 Attempt a complete integration process

A1 Integration correct

M1 Correct use of both limits – substitute and subtract.
Condone wrong order.

M1 Simplify to find k from an expression involving a logarithm

A1 Answer as given, or exact equivalent. Need to see $k = \ln A + B$

[10]

1. This question followed a standard format and was answered very well by a large proportion of candidates. A majority took the constant of integration approach rather than the use of definite integration, but generally did so successfully in both parts (a) and (b). A small number of candidates were confused about the directions involved and often tried to justify a change of sign part way through their working in order to arrive at the given answer in part (a).

A small number of candidates attempted to use their answer to part (a) to generate a differential equation in $\frac{dx}{dt}$ for part (b) rather than the simpler method of restarting from the beginning and using $v\frac{dv}{dx}$. These candidates were rarely able to obtain an expression for v correctly, or to integrate their expression successfully, and tended to grind to a halt with little progress made.

2. This proved to be a particularly accessible question, with many candidates offering completely correct solutions. The most common errors involved incorrect signs in the initial differential equation, or problems in substituting the limits and simplifying the answer. Some candidates integrated $\frac{v}{-g-kv^2}$ to obtain $-\frac{1}{2k}\ln\left(-g-kv^2\right)$ and went on to work with the logarithms of negative quantities with no indication at all of any recognition that this should have concerned them.

A few candidates started with a differential equation in v and t. Although it is possible to complete the solution by this route, very few of these attempts got as far as a correct first integral, and almost all stopped work before achieving a differential equation involving x.

3. This was a standard question and candidates demonstrated a sound understanding of how to solve this sort of problem. Most were able to obtain a differential equation in v and t, recognised the need to separate the variables and integrated to find an expression for t in terms of v. A few attempts considered upwards as positive but then failed to use $-U$ and $-2U$ as the values of v, but most were competent and successful. Most attempts to solve the problem by writing the equation of motion as a second order differential equation in x and t and then going on to differentiate x failed because they ignored the need for a particular integral as part of the general solution.

4. The vast majority of candidates found the driving force, correctly applied Newton's second law and were able to rearrange to obtain the given result in part (a).

In part (b) candidates were generally able to separate the variables correctly. The most popular method for integrating the expression in v was to use partial fractions, although this caused a number of candidates to make sign errors. The minority who used substitution (e.g. $x = u - v$) were generally successful. There was a significant number of candidates who integrated incorrectly, or who attempted to use integration by parts only to grind to a halt because they did not know how to complete the second stage of the integration, or worse still did not show any indication of the need for a second step in the integration. The vast majority of candidates preferred finding a constant of integration to using definite integration, but those who integrated correctly tended to obtain the final simplified result successfully.