1. Two smooth uniform spheres S and T have equal radii. The mass of S is 0.3 kg and the mass of T is 0.6 kg. The spheres are moving on a smooth horizontal plane and collide obliquely. Immediately before the collision the velocity of S is $\mathbf{u}_1$ m s^{-1} and the velocity of T is $\mathbf{u}_2$ m s^{-1}. The coefficient of restitution between the spheres is 0.5. Immediately after the collision the velocity of S is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of T is $(\mathbf{i} + \mathbf{j})$ m s^{-1}. Given that when the spheres collide the line joining their centres is parallel to $\mathbf{i}$,

(a) find

(i) $\mathbf{u}_1$,

(ii) $\mathbf{u}_2$.

(6)

After the collision, T goes on to collide with a smooth vertical wall which is parallel to $\mathbf{j}$. Given that the coefficient of restitution between T and the wall is also 0.5, find

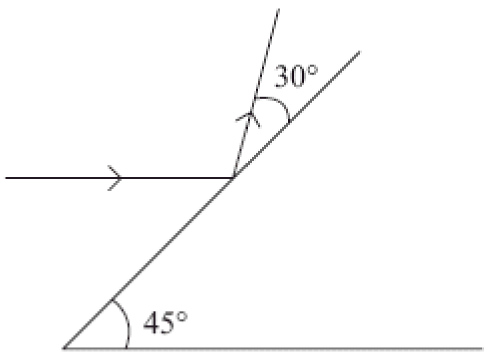
(b) the angle through which the direction of motion of T is deflected as a result of the collision with the wall,

(5)

(c) the loss in kinetic energy of T caused by the collision with the wall.

(3)
(Total 14 marks)

2.



A fixed smooth plane is inclined to the horizontal at an angle of 45°. A particle P is moving horizontally and strikes the plane. Immediately before the impact, P is moving in a vertical plane containing a line of greatest slope of the inclined plane. Immediately after the impact, P is moving in a direction which makes an angle of 30° with the inclined plane, as shown in the diagram above.

Find the fraction of the kinetic energy of P which is lost in the impact.

(Total 6 marks)

3. Two small smooth spheres A and B, of mass 2 kg and 1 kg respectively, are moving on a smooth horizontal plane when they collide. Immediately before the collision the velocity of A is $(\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of B is $-2\mathbf{i}$ m s^{-1}. Immediately after the collision the velocity of A is $\mathbf{j}$ m s^{-1}.

(a) Show that the velocity of B immediately after the collision is $2\mathbf{j}$ m s^{-1}.

(3)

(b) Find the impulse of B on A in the collision, giving your answer as a vector, and hence show that the line of centres is parallel to $\mathbf{i} + \mathbf{j}$.

(4)

(c) Find the coefficient of restitution between A and B.

(6)
(Total 13 marks)

4. A small ball is moving on a horizontal plane when it strikes a smooth vertical wall. The coefficient of restitution between the ball and the wall is e. Immediately before the impact the direction of motion of the ball makes an angle of 60° with the wall. Immediately after the impact the direction of motion of the ball makes an angle of 30° with the wall.

(a) Find the fraction of the kinetic energy of the ball which is lost in the impact.

(6)

(b) Find the value of e.

(4)
(Total 10 marks)

5. A smooth uniform sphere S of mass m is moving on a smooth horizontal plane when it collides with a fixed smooth vertical wall. Immediately before the collision, the speed of S is U and its direction of motion makes an angle α with the wall. The coefficient of restitution between S and the wall is e. Find the kinetic energy of S immediately after the collision.

(Total 6 marks)

6.

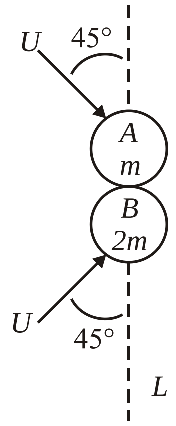


Figure 1

Two small smooth spheres A and B, of equal size and of mass m and $2m$ respectively, are moving initially with the same speed U on a smooth horizontal floor. The spheres collide when their centres are on a line L. Before the collision the spheres are moving towards each other, with their directions of motion perpendicular to each other and each inclined at an angle of 45° to the line L, as shown in Figure 1. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Find the magnitude of the impulse which acts on A in the collision.

(9)

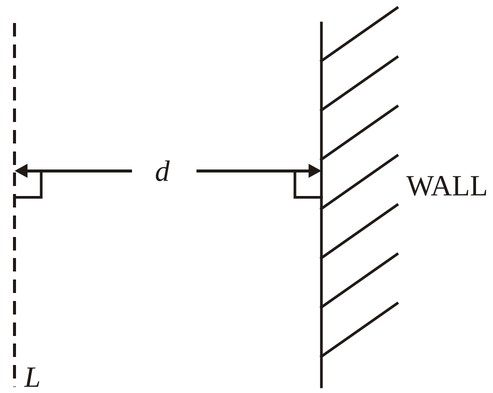


Figure 2

The line L is parallel to and a distance d from a smooth vertical wall, as shown in Figure 2.

(b) Find, in terms of d, the distance between the points at which the spheres first strike the wall.

(5)
(Total 14 marks)

7. A small smooth sphere S of mass m is attached to one end of a light inextensible string of length $2a$. The other end of the string is attached to a fixed point A which is at a distance $a\sqrt{3}$ from a smooth vertical wall. The sphere S hangs at rest in equilibrium. It is then projected horizontally towards the wall with a speed $\sqrt{\left(\frac{37ga}{5}\right)}$.

(a) Show that it strikes the wall with speed $\sqrt{}\sqrt{\left(\frac{27ga}{5}\right)}$

(4)

Given that the loss in kinetic energy due to the impact with the wall is $\frac{3mga}{5}$,

(b) find the coefficient of restitution between S and the wall.

(7)
(Total 11 marks)

8. Two smooth uniform spheres A and B have equal radii. Sphere A has mass m and sphere B has mass km. The spheres are at rest on a smooth horizontal table. Sphere A is then projected along the table with speed u and collides with B. Immediately before the collision, the direction of motion of A makes an angle of 60° with the line joining the centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Show that the speed of B immediately after the collision is $\frac{3u}{4(k+1)}$.

(6)

Immediately after the collision the direction of motion of A makes an angle arctan $(2\sqrt{3})$ with the direction of motion of B.

(b) Show that $k = \frac{1}{2}$.

(6)

(c) Find the loss of kinetic energy due to the collision.

(4)
(Total 16 marks)

9. A small smooth ball of mass $\frac{1}{2}$ kg is falling vertically. The ball strikes a smooth plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. Immediately before striking the plane the ball has speed 10 m s^{-1}. The coefficient of restitution between ball and plane is $\frac{1}{2}$. Find

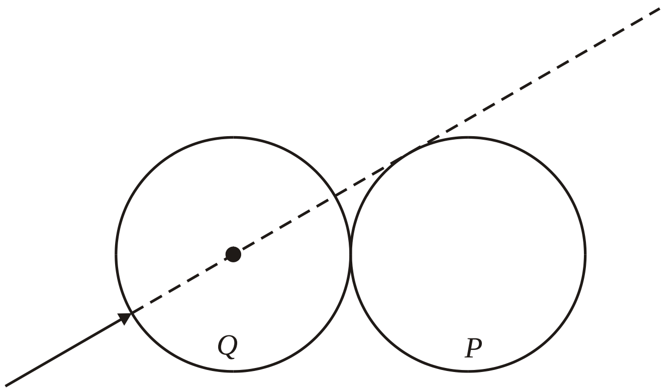
(a) the speed, to 3 significant figures, of the ball immediately after the impact,

(5)

(b) the magnitude of the impulse received by the ball as it strikes the plane.

(2)
(Total 7 marks)

10.

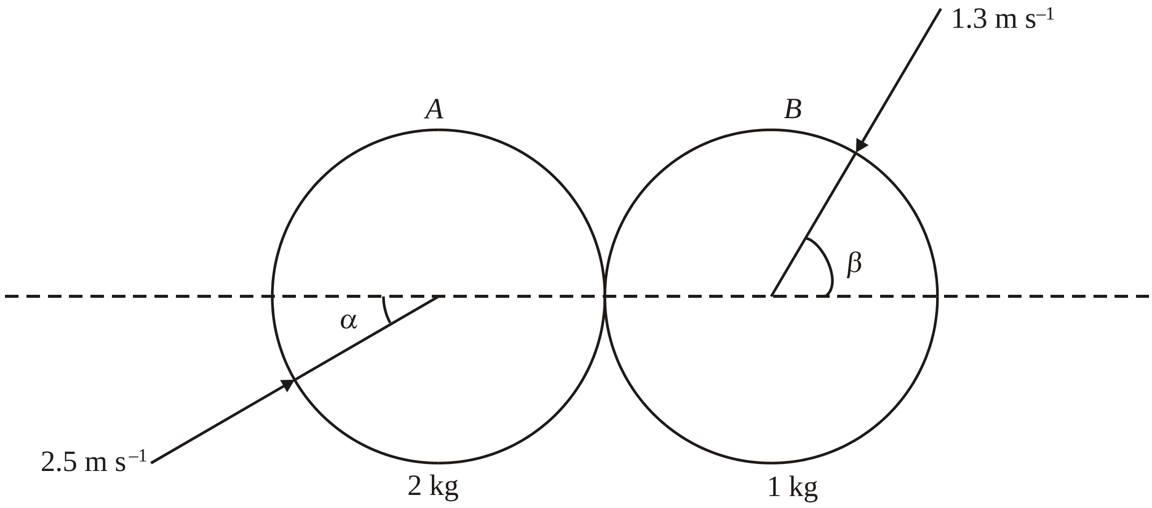


A smooth uniform sphere *P* is at rest on a smooth horizontal plane, when it is struck by an identical sphere *Q* moving on the plane. Immediately before the impact, the line of motion of the centre of *Q* is tangential to the sphere *P*, as shown in the diagram above. The direction of motion of *Q* is turned through 30° by the impact.

Find the coefficient of restitution between the spheres.

(Total 11 marks)

11.



Two smooth uniform spheres *A* and *B* of equal radius have masses 2 kg and 1 kg respectively. They are moving on a smooth horizontal plane when they collide. Immediately before the collision the speed of *A* is 2.5 m s^{-1} and the speed of *B* is 1.3 m s^{-1}. When they collide the line joining their centres makes an angle α with the direction of motion of *A* and an angle β with the direction of motion of *B*, where tan $\alpha = \frac{4}{3}$ and tan $\beta = \frac{12}{5}$ as shown in the diagram above.

(a) Find the components of the velocities of *A* and *B* perpendicular and parallel to the line of centres immediately before the collision.

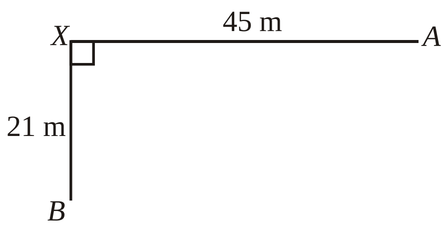
(4)

The coefficient of restitution between *A* and *B* is $\frac{1}{2}$.

(b) Find, to one decimal place, the speed of each sphere after the collision.

(9)
(Total 13 marks)

12.



The diagram above represents the scene of a road accident. A car of mass 600 kg collided at the point X with a stationary van of mass 800 kg. After the collision the van came to rest at the point A having travelled a horizontal distance of 45 m, and the car came to rest at the point B having travelled a horizontal distance of 21 m. The angle AXB is 90°.

The accident investigators are trying to establish the speed of the car before the collision and they model both vehicles as small spheres.

(a) Find the coefficient of restitution between the car and the van.

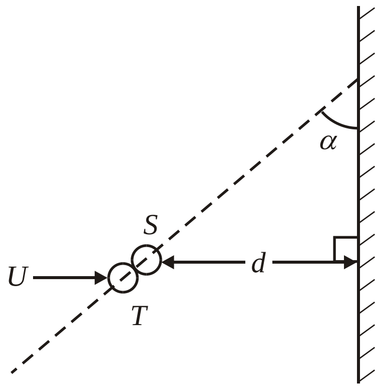
(5)

The investigators assume that after the collision, and until the vehicles came to rest, the van was subject to a constant horizontal force of 500 N acting along AX and the car to a constant horizontal force of 300 N along BX.

(b) Find the speed of the car immediately before the collision.

(9)
(Total 14 marks)

13.



A small smooth uniform sphere S is at rest on a smooth horizontal floor at a distance d from a straight vertical wall. An identical sphere T is projected along the floor with speed U towards S and in a direction which is perpendicular to the wall. At the instant when T strikes S the line joining their centres makes an angle α with the wall, as shown in the diagram above.

Each sphere is modelled as having negligible diameter in comparison with d. The coefficient of restitution between the spheres is e.

(a) Show that the components of the velocity of T after the impact, parallel and perpendicular to the line of centres, are $\frac{1}{2}U(1-e)\sin\alpha$ and $U\cos\alpha$ respectively.

(7)

(b) Show that the components of the velocity of T after the impact, parallel and perpendicular to the wall, are $\frac{1}{2}U(1 + e)\cos\alpha\sin\alpha$ and $\frac{1}{2}U[2 - (1 + e)\sin^2\alpha]$ respectively.

(6)

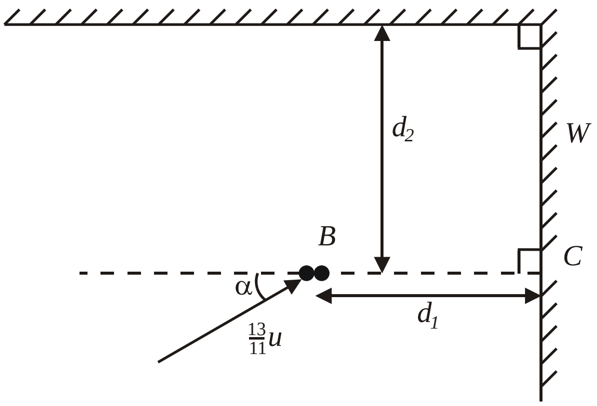
The spheres S and T strike the wall at the points A and B respectively.

Given that $e = \frac{2}{3}$ and $\tan\alpha = \frac{3}{4}$,

(c) find, in terms of d, the distance AB.

(5)
(Total 18 marks)

14.



A small ball Q of mass $2m$ is at rest at the point B on a smooth horizontal plane. A second small ball P of mass m is moving on the plane with speed $\frac{13}{12}u$ and collides with Q. Both the balls are smooth, uniform and of the same radius. The point C is on a smooth vertical wall W which is at a distance d_1 from B, and BC is perpendicular to W. A second smooth vertical wall is perpendicular to W and at a distance d_2 from B. Immediately before the collision occurs, the direction of motion of P makes an angle α with BC, as shown in the diagram above, where $\tan\alpha = \frac{5}{12}$. The line of centres of P and Q is parallel to BC. After the collision Q moves towards C with speed $\frac{3}{5}u$.

(a) Show that, after the collision, the velocity components of P parallel and perpendicular to CB are $\frac{1}{5}u$ and $\frac{5}{12}u$ respectively.

(4)

(b) Find the coefficient of restitution between P and Q.

(2)

(c) Show that when Q reaches C, P is at a distance $\frac{4}{3}d_1$ from W.

(3)

For each collision between a ball and a wall the coefficient of restitution is $\frac{1}{2}$.

Given that the balls collide with each other again,

(d) show that the time between the two collisions of the balls is $\dfrac{15d_1}{u}$,

(4)

(e) find the ratio $d_1:d_2$.

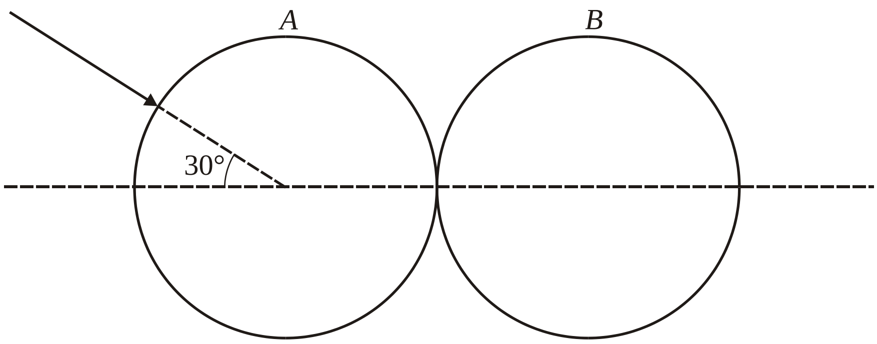
(5)
(Total 18 marks)

15. A smooth sphere S is moving on a smooth horizontal plane with speed u when it collides with a smooth fixed vertical wall. At the instant of collision the direction of motion of S makes an angle of 30° with the wall. The coefficient of restitution between S and the wall is $\frac{1}{3}$.

Find the speed of S immediately after the collision.

(Total 6 marks)

16.



A smooth uniform sphere A, moving on a smooth horizontal table, collides with a second identical sphere B which is at rest on the table. When the spheres collide the line joining their centres makes an angle of 30° with the direction of motion of A, as shown in the diagram above. The coefficient of restitution between the spheres is e. The direction of motion of A is deflected through an angle θ by the collision.

Show that $\tan\theta = \dfrac{(1+e)\sqrt{3}}{5-3e}$.

(Total 10 marks)

1. (a)

$\uparrow 2$ $\qquad \uparrow 1$

$1 \leftarrow$ $\qquad \rightarrow 1$

S 0.3 kg $\quad T$ 0.6 kg

$2 \uparrow$ $\qquad \uparrow 1$

$\rightarrow v$ $\qquad w \leftarrow$

$$0.3v - 0.6w = 0.3$$ M1 A1

$$v - 2w = 1$$

$$\tfrac{1}{2}\ (v + w) = 2$$ M1 A1

$$v + w = 4$$

$$w = 1, v = 3$$

(i) $\mathbf{u_1} = 3\mathbf{i} + 2\mathbf{j}$ (ii) $\mathbf{u}_2 = -\mathbf{i} + \mathbf{j}$ A1 A1 6

(b)

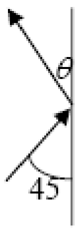
$\uparrow 1$

$v \leftarrow$

$v = 0.5$ B1

$1 \uparrow$

$\rightarrow 1$



$\tan\theta = 0.5$ — tan θ = their v — M1

$\theta = 26.6$ — A1

their θ + 45° — M1

Defln angle $= 45 + 26.6 = 71.6^{\circ}$ A1 5

(c) KE Loss = $\tfrac{1}{2}$ x 0.6 x $\left\{(1^2 + 1^2) - (1^2 + v^2)\right\}$ M1 A1

$= 0.225$ J A1 3

[14]

2. CLM along plane: $v \cos 30° = u \cos 45°$ M1 A1

$$v = u\sqrt{\frac{2}{3}}$$ A1

Fraction of KE Lost $= \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}m\frac{2}{3}u^2}{\frac{1}{2}mu^2} = \dfrac{1}{3}$ M1 M1 A1

[6]

3. (a) CLM: $2(\mathbf{i} + 2\mathbf{j}) + -2\mathbf{i} = 2\,\mathbf{j} + \mathbf{v}$ M1 A1

	$\mathbf{v} = 2\mathbf{j}$ m s^{-1}	A1	3
(b)	$\mathbf{I} = 2(\mathbf{j} - (\mathbf{i} + 2\mathbf{j}))$	M1 A1	
	$= (-2\mathbf{i} - 2\mathbf{j})$ Ns	A1	
	Since **I** acts along l.o.c.c., l.o.c.c is parallel to $\mathbf{i} + \mathbf{j}$	B1	4

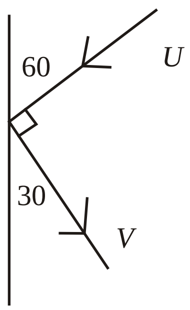
(c)

$$\left.\begin{array}{lll} \text{Before} & A: & (\mathrm{i}+2\mathrm{j}).\frac{1}{\sqrt{2}}(\mathrm{i}+\mathrm{j}) = \frac{3}{\sqrt{2}} \\ & B: & -2\mathrm{j}.\frac{1}{\sqrt{2}}(\mathrm{i}+\mathrm{j}) + \frac{-2}{\sqrt{2}} \\ \text{After} & A: & \mathrm{j}.\frac{1}{\sqrt{2}}(\mathrm{i}+\mathrm{j}) = \frac{1}{\sqrt{2}} \\ & B: & 2\mathrm{j}.\frac{1}{\sqrt{2}}(\mathrm{i}+\mathrm{j}) + \frac{2}{\sqrt{2}} \end{array}\right\}$$ M1 A3

NIL: $$e = \frac{\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\frac{3}{\sqrt{2}} - \frac{-2}{\sqrt{2}}} = \frac{1}{5}$$ DM1 A1 6

[13]

4. (a)



$u \cos 60° = v \cos 30°$	M1A1	
$u = v\sqrt{3}$	A1	
KE lost = $\frac{1}{2}m(u^2 - v^2)$	M1	
Fraction of KE lost = $1 - \left(\frac{v}{u}\right)^2$	DM1	
$= 1 - \frac{1}{3} = \frac{2}{3}$ or at least 3sf ending in 7	A1	6
or $\frac{3}{4}(1 - \mathrm{e}^2)$		

M1 Resolve parallel to the wall
Alt: reasonable attempt at equation connecting two variables

A1 Correct as above or equivalent
equation correct

A1 u in terms of v or v.v. – not necessarily simplified.
or ration of the two variables correct

M1 expression for KE lost

DM1 expression in one variable for fraction of KE lost – could be u/v as above

A1 cao

The first three marks can be awarded in (b) if not seen in (a)

(b) $e = \dfrac{v\sin 30°}{u\sin 60°}$ M1A1

$= \dfrac{v}{u}.\dfrac{1}{\sqrt{3}}$ DM1

$= \dfrac{1}{3}$ A1 4

M1 Use NIL perpendicular to the wall and form equation in e

A1 Correct unsimplified expression as above or
eu sin 60°= v sin 30° or equivalent

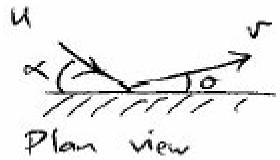
DM1 Substitute values for trig functions or use relationship from (a) and rearrange to e = ...

A1 cao accept decimals to at least 3sf

The first two marks can be awarded in (a)

[10]

5.



$(\rightarrow)\ \ u\cos\alpha = v\cos\theta$ M1 A1

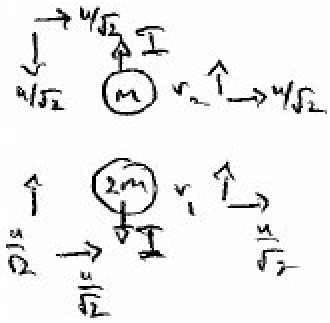
$(\uparrow)\ \ eu\sin\alpha - v\sin\theta$ M1 A1

$\Rightarrow v^2 = u^2\left(\cos^2\alpha + e^2\sin^2\alpha\right)$ M1

$\text{KE} = \dfrac{1}{2}\text{m}u^2\left(\cos^2\alpha + e^2\sin^2\alpha\right)$ A1

[6]

6. (a)



Form: $I = m(v_2 + \frac{u}{\sqrt{2}})$ M1 A1

$CLM\left(\uparrow\right): \frac{2mu}{\sqrt{2}} - \frac{mu}{\sqrt{2}} = 2mv_1 + mv_2$

$\frac{u}{\sqrt{2}} = 2v_1 + v_2$ ① M1 A1

$NIL: e\frac{2u}{\sqrt{2}} = \frac{u}{\sqrt{2}} = -v_1 + v_2$ ② M1 A1

$\Rightarrow \frac{\cancel{3}u}{\sqrt{2}} = \cancel{3}v_2$ M1 A1

$\Rightarrow I = m(\frac{u}{\sqrt{2}} + \frac{u}{\sqrt{2}})$

$= mu\sqrt{2}$ A1 9

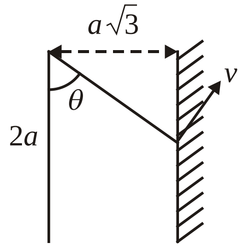
(b) $v_2 - v_1 = \frac{u}{\sqrt{2}}$ (separation speed) M1

Time to wall $= \frac{d}{\frac{u}{\sqrt{2}}} = \frac{d\sqrt{2}}{u}$ M1 A1

$\therefore$ Separation $= \frac{d\sqrt{2}}{u} \times \frac{u}{\sqrt{2}} = d$ M1 A1 5

[14]

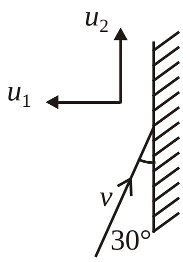
7. (a)



Energy: $\frac{1}{2}m\left(\frac{37ga}{5} - v^2\right) = mg.2a(1 - \cos\theta)$ M1 A1

Using $\theta = \frac{\pi}{3}$ & solve: $\rightarrow$ $\underline{v = \sqrt{\frac{27ga}{5}}}$ M1 A1 4

(b)



Impact: $u_1 = ev\sin 30$ M1 A1

KE loss = $\frac{1}{2}m\,(v^2 \sin^2 30 - e^2v^2 \sin^2 30)$

$\left[+\frac{1}{2}mv^2\cos^2 30 - \frac{1}{2}mu_2{}^2\right] = \frac{3mga}{5}$ M1 A1

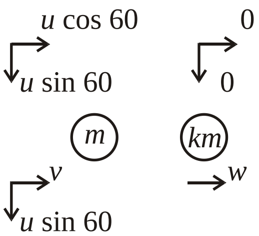
[Using $u_2 = v\cos 30$ if necessary & reducing to equation in (m, g, a) e alone]

$\frac{3mga}{5} = \frac{1}{2}m.\frac{27ga}{5}.\frac{1}{4}(1 - e^2)$ A1

Solve for e: $\rightarrow$ $\underline{e = \frac{1}{3}}$ M1 A1 7

[11]

8. (a)



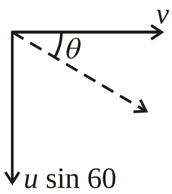
$CLM(\leftrightarrow)$: $mu\cos 60 = mv + kmw$ M1 A1

NLI: $\frac{1}{2}u\cos 60 = w - v$ M1 A1

Solve for w: $(1 + k)w = \frac{1}{2}u(1 + \frac{1}{2})$ M1

$\Rightarrow w = \frac{3u}{4(k+1)}$ A1 6

(b)



Solve for $v \to v = \frac{u(2-k)}{4(k+1)}$ M1 A1

$\tan\theta = 2\sqrt{3} = \frac{u\sin 60}{v}$ M1 A1

$= \frac{u\sqrt{3}}{2}.\frac{4(k+1)}{u(2-k)}$

Solve k: $\to \underline{k = \frac{1}{2}}$ M1 A1 6

(c) $k = \frac{1}{2} \Rightarrow v = \frac{u}{4}, w = \frac{u}{2}$ B1

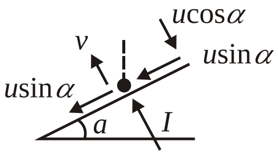
KE loss $= \frac{1}{2}mu^2 - \left(\frac{1}{2}m.\frac{u^2}{16} + \frac{1}{2}m.\frac{3u^2}{4} + \frac{1}{2}.\frac{1}{2}m.\frac{u^2}{4}\right)$ M1 A1

$= \frac{1}{2}mu^2\left(1 - \frac{1}{16} - \frac{3}{4} - \frac{1}{8}\right)$

$= \underline{\frac{1}{32}mu^2}$ A1 4

[16]

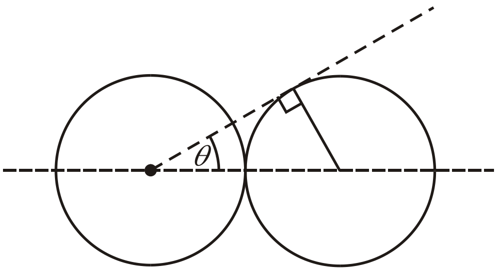
9. (a)



Cpt along plane $= 10\sin\alpha$	B1		
after impact $= 10 \times \frac{3}{5}$			
$= 6$			
$V = e \times 10\cos\alpha$	M1 A1		
$\left(= \frac{1}{2} \times 10 \times \frac{4}{5} = 4\right)$			
Speed $= \sqrt{4^2 + 6^2}$ $=$ <u>7.21 ms^{-1} (3 sf)</u>	M1 A1	5	

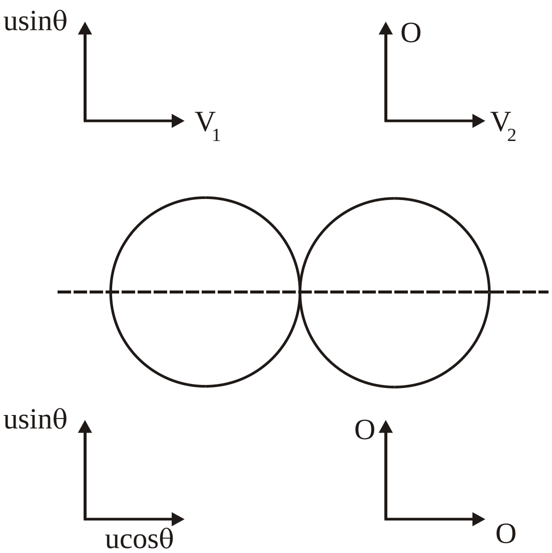
(b) $I = \frac{1}{2}(4 - -8) = \underline{6\text{Ns}}$ M1 A1 2

[7]



10.

$\sin\theta = \frac{a}{2a}$	M1
$\Rightarrow \theta = 30°$	A1



$v_1 + v_2 = u \cos \theta$ M1 A1

$-v_1 + v_2 = eu \cos \theta$ M1 A1

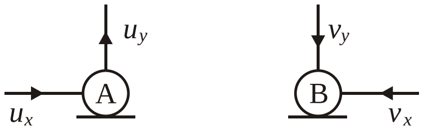
$\dfrac{u \sin \theta}{v_1} = \tan (\theta + 30°)$ (or equivalent) M1 A1

Producing an equation in e only M1 (dep. On all previous M's)

$\underline{e = \dfrac{1}{3}}$ A1 c.s.o

[11]

11. (a) Before



A: ↑ $u_y = 2.5 \sin \alpha = 2.5 \times \dfrac{4}{5} = 2$ (ms^{-1}) either M1

→ $u_x = 2.5 \cos \alpha = 2.5 \times \dfrac{3}{5} = 1.5$ (ms^{-1}) both A1

B: ↓ $v_y = 1.3 \sin \beta = 1.3 \times \dfrac{12}{13} = 1.2$ (ms^{-1}) either M1

← $v_x = 1.3 \cos \beta = 1.3 \times \dfrac{5}{13} = 0.5$ (ms^{-1}) both A1 4

(b) After

LM $2x + w = 3 - 0.5$ (= 2.5) M1 A1 ft

NEL $w - x = \dfrac{1}{2} \times 2$ (=1) M1 A1 ft

Solving $x = 0.5$, $y = 1.5$ M1 solving for either M1 A1

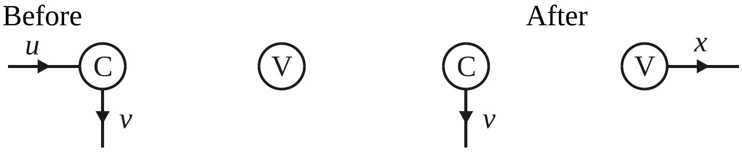
Speed of A is $\sqrt{(2^2 + 0.5^2)} = \sqrt{4.25} \approx 2.1$ (ms^{-1}) M1 either M1 A1

Speed of B is $\sqrt{(1.2^2 + 1.5^2)} = \sqrt{3.69} \approx 1.9$ (ms^{-1}) A1 9

[13]

Note: Not 1 d.p. loses maximum of one mark

12. (a) Before After



→ LM $600u = 800x$ M1 A1

→ NEL $x = eu$ M1 A1

$e = 0.75$ A1 5

(b) Van N2L – 500 = 800a M1

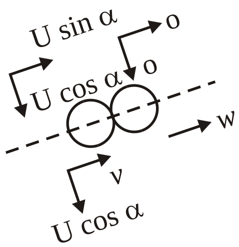
$0^2 = x^2 - 2 \times 0.625 \times 45$, $x^2 = 56.25$ (x = 7.5) M1, A1

Car N2L –300 = 600a M1

$0^2 = v^2 - 2 \times 0.5 \times 21,\ v^2 = 21$	M1, A1	
From (a) NEL $u = \dfrac{4}{3} \times 7.5 = 10$	M1	
$V^2 = 10^2 + 21, \Rightarrow V = 11$ (ms^{-1}) cao	M1, A1	9

[14]

13. (a)

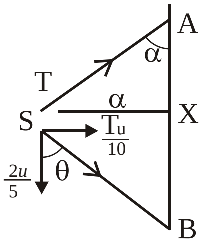


No impulse perpendicular to line of centres		
$\Rightarrow$ velocity perpendicular to line of centres unchanged = $U \cos \alpha$ (*)	B1	
$(\leftrightarrow)$: CLM $U \sin \alpha = v + w$	M1 A1	
NLI $eU \sin \alpha = w - v$	M1 A1	
$\Rightarrow v = \frac{1}{2} U \sin \alpha (1 - e)$	M1 A1	7

(b)

Component perpendicular to wall = $v \sin \alpha + U \cos \alpha \cos \alpha$	M1	
$= \frac{1}{2} U \sin^2 \alpha (1 - e) + U \cos^2 \alpha$		
$= \frac{1}{2} U(\sin^2 \alpha - e \sin^2 \alpha + 2 - 2 \sin^2 \alpha)$	M1	
$= \frac{1}{2} U [2 - \sin^2 \alpha (1 + e)]$ (*)	A1	
Component parallel to wall = $U \cos \alpha \sin \alpha - v \cos \alpha$	M1	
$= U \cos \alpha \sin \alpha - \frac{1}{2} U \sin \alpha \cos \alpha (1 - e)$	A1	
$= \frac{1}{2} U \cos \alpha \sin \alpha (1 + e)$ (*)	A1	6

(c)



$e = \frac{2}{3}$, $\tan\alpha = \frac{3}{4}$

Component perpendicular to wall $= \frac{1}{2}U\,(2 - \frac{9}{25} \times \frac{5}{3}) = \frac{7u}{10}$ B1

Component parallel to wall $= \frac{1}{2}U \times \frac{4}{5} \times \frac{3}{12} \times \frac{5}{3} = \frac{2u}{5}$

Distance of A from $X = d\cot\theta = \frac{4d}{3}$ B1

$BX = d\cot\theta$ M1

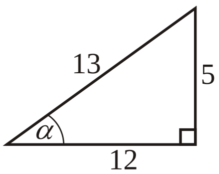
$\cot\theta = \frac{2u}{5} \times \frac{10}{7u} = \frac{4}{7}$ A1

$\therefore$ Total distance $AB = \frac{4d}{3} + \frac{4d}{7}$

$= \frac{40d}{21}$ A1 5

[18]

14. (a)



P before: $\rightarrow \frac{13u}{12}\cos\alpha = u$, $\uparrow \frac{13u}{12}\sin\alpha = \frac{5u}{12}$ B1, B1

$\rightarrow u$ $\quad \rightarrow 0$ $\quad \quad \rightarrow v$ $\quad \rightarrow \frac{3u}{5}$

$\bullet$ $\quad \bullet$ $\quad \rightarrow$ $\quad \bullet$ $\quad \bullet$

m $\quad 2m$ $\quad \quad m$ $\quad 2m$

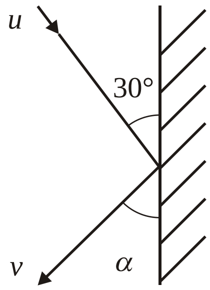
PCLM ($\rightarrow$) $\quad mu = mv + 2m\,\frac{3u}{5}, \Rightarrow v = \frac{-u}{5}$, i.e. $\frac{u}{5}$ // CB M1 A1 4

(b) NLI $\rightarrow eu = v_2 - v_1 \Rightarrow eu = \frac{3u}{5} - \frac{u}{5}$, i.e. $e = \frac{4}{5}$ M1, A1 2

(c)	$Q \to C\ t_1 = \dfrac{d_1}{3u/5} = \dfrac{5d_1}{3u}$	B1	
	P travels $\dfrac{u}{5} \times \dfrac{5d_1}{3u} = \dfrac{d_1}{3}$ in direction CB	M1	
	$\therefore\ P$ is $d_1 + \dfrac{d_1}{3} = \dfrac{4d_1}{3}$ from w (*)	A1 c.s.o	3
(d)	A fter hitting w, Q has speed $\dfrac{3u}{10}$ in direction CB	B1	
	Velocity of Q relative to P in direction CB is $\dfrac{u}{10}$	M1	
	Time for Q to travel $\dfrac{4}{3}d_1$ is: $\dfrac{4d_1}{3u} \times 10 = \dfrac{40d_1}{3u}$	A1	
	Total time between collisions is: $\dfrac{40d_1}{3u} + \dfrac{5d_1}{3u} = \dfrac{15d_1}{u}$ (*)	A1 c.s.o	4
(e)	For collision to occur P must travel $\uparrow d_2$ and $\downarrow d_2$ in time $\dfrac{15d_1}{u}$		
	$d_2\uparrow\ t_2 = \dfrac{d_2}{5u/12} = \dfrac{12d_2}{5u}$	B1	
	$\downarrow d_2$ velocity $\downarrow$ is $\dfrac{5u}{24}$, $\therefore\ t_3 = \dfrac{d_2}{5u/24} = \dfrac{24d_2}{5u}$	B1, B1	
	Total time is $\dfrac{36d_2}{5u} = \dfrac{15d_1}{u}$,	M1	
	$\therefore\ 12d_2 = 25d_1$, i.e. $d_1{:}d_2 = 12{:}25$	A1	5

[18]

15.



$v \cos \alpha = u \cos 30°$ M1 A1

$v \sin \alpha = \frac{1}{3} u \sin 30°$ M1 A1

squaring and adding, M1

$v^2 = u^2(\frac{3}{4} + \frac{1}{36})$

$v = \dfrac{u\sqrt{7}}{3}$ A1

[6]

16. $\rightarrow$ u cos 30° $\rightarrow 0$ $v_1 + v_2 = u \cos 30°$ M1 A1

$\downarrow A$ (m) $\downarrow B$ (m) $-v_1 + v_2 = eu \cos 30°$ M1 A1

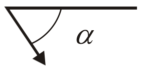
$u \sin 30°$ 0

v_1 α $\rightarrow v_2$

$\downarrow$ $\downarrow$

$u \sin 30°$ 0

subtracting, $v_1 = \dfrac{u\sqrt{3}}{4}(1 - e)$ A1



$\tan\,\theta = \tan\,(\alpha - 30°) = \dfrac{\tan\alpha - \tan 30°}{1 + \tan\alpha\tan 30°}$ M1

$\tan\,\alpha = \dfrac{u\sin 30°}{v_1} = \dfrac{2}{\sqrt{3}(1-e)}$ M1 A1

$\tan\,\theta = \dfrac{\dfrac{2}{\sqrt{3}(1-e)} - \dfrac{1}{\sqrt{3}}}{1 + \dfrac{2}{\sqrt{3}(1-e)}\dfrac{1}{\sqrt{3}}}$ M1

$= \dfrac{(1+e)\sqrt{3}}{5-3e}$ (*) A1 10

[10]

1. In part (a) many candidates were able to obtain correct equations by applying the conservation of momentum and Newton's experimental law parallel to the line of centres. A common mistake, however, was to solve the equations and give the speed of the spheres rather than the velocities as required in the question.

The need to move from a question posed in vectors to scalar equations caused difficulties for some candidates. Many produced a momentum equation in **i** and **j**, rather than confining themselves to consideration of the components parallel to the line of centres. The j component did not always cancel out. Those candidates who introduced vectors to Newton's Experimental law were penalised for this significant error.

In part (b) the majority of candidates were able to use the coefficient of restitution to find the velocity of the sphere after the collision with the wall and to find the angle between the wall and the path after impact. Far fewer identified and found the correct angle of deflection.

In part (c) the majority of candidates found the loss in kinetic energy correctly. Errors were usually due to failure to find v^2 correctly from their vectors, or the use of v rather than v^2 in attempting to find the kinetic energy.

2. Most candidates started by applying CLM parallel to the plane to obtain an equation connecting the speed of approach and the speed of rebound, which enabled them to form an expression for the fraction of kinetic energy lost. Many candidates, who also considered the components of the speeds perpendicular to the surface and used the impact law to find the value of e, did eventually reach a correct conclusion, but in most instances this method took longer because they were doing more work than necessary. Some candidates found the amount of kinetic energy lost and did not go on to complete the question. There was also evidence of some confusion amongst candidates who worked only with the components of velocity perpendicular to the plane and did not express their change in kinetic energy as a fraction of the *total* initial kinetic energy – candidates should be discouraged from thinking of energy as resolvable, as this has clearly led to some misconceptions.

3. In part (a) the majority of candidates demonstrated their dislike of vectors by calculating the **i** and **j** components of the velocity of B in two separate equations. Many candidates lost marks because they only considered the **j** component. In their attempts to derive the given result without the use of vectors some candidates created some unconvincing notation and arguments.

Most candidates in part (b) understood that impulse causes a change in momentum, but there were many sign errors in finding the impulse, with large numbers of candidates arriving at the negative of the required result when considering the impulse on A. Equally, some candidates started by finding the impulse on B and did not take the final step to answer the question. When candidates had found an impulse acting in the right direction they were usually able to draw the correct conclusion about the line of centres of the two spheres, although here too some of the presentation was rather muddled and the examiner was often left to interpret the candidate's use of the word 'it' to apply to whatever the candidate had in mind.

Part (c) challenged many candidates, and was a good indicator of the level of understanding of what was happening in this collision. Only the stronger candidates appreciated the need to resolve their velocities parallel to the line of centres, and for these candidates the initial velocity of A proved the most difficult to deal with. This was largely because the usual approach was to draw a diagram and use a trigonometry, rather than using the scalar product of the vectors.

4. Candidates were generally able to obtain a correct relationship between the speed of the ball before and after the collision by resolving parallel to the wall. The vast majority then went on to find the loss in KE in one variable, but a significant proportion of these candidates did not then go on to find the fraction of KE lost. Some candidates found the loss in KE using only the components perpendicular to the wall. In most cases, however, these candidates then incorrectly went on to find the fraction of KE lost by using the perpendicular component of the initial velocity rather than the initial speed in their expression for the initial KE. Most candidates were able to obtain a correct equation in e and substitute to find a correct value for e.

5. Completely correct solutions to this question were common. Even those who initially decided that momentum might be conserved perpendicular to the wall usually realised the error of their ways and corrected their solutions. A few candidates correctly used conservation of momentum and Newton's experimental law but then were unable to proceed to an expression for the kinetic energy.

6. (a) This was a standard question that many candidates completed successfully. Many however forced the problem to conform to their preconceived idea of this type of question. A diagram, preferably in the orientation given in the question, with velocity components clearly shown, would have helped candidates to get the signs correct in their equations. Some candidates used the velocities in their calculation of impulse, rather than the components along the line of centres.

(b) Despite problems with part (a), most candidates used a correct method with often only a single mark being lost due to errors from part (a).

7. This was a challenging question for all but the strongest candidates. Part (a) was accessible although some did not realise it was a circular motion/energy question. Part (b) was not recognised as an oblique impact – on most scripts, there was no attempt to resolve the before and after velocities into components and candidates used v = eu with resultant velocities instead of components perpendicular to the wall.

8. This question was a very good source of marks with many completely correct solutions. Candidates were confident in the use of conservation of momentum and Newton's law of restitution and coped well with the resulting algebra. There were some errors in (c) with candidates using m for both masses instead of m and km with k = ½ as printed. Several saved some calculation by realising that all the energy loss came from the components of velocity along the line of centres

9. (a) Candidates could usually draw the plane as described but often misplaced the angle α □between the vertical and the normal to the plane. Many candidates drew the problem with the plane horizontal or used components in undefined directions leaving it to the examiner to work out what was intended. Finding components along and perpendicular to the plane was attempted by most candidates and those who squared the components to find the resultant were usually more successful than those who tried to find the angle to the plane at which the ball rebounded.

(b) This part was very badly done. Most candidates did not use components of the velocity perpendicular to the plane using instead the initial and final speeds of the particle. Candidates who realised that components were needed often failed to take into account the directions involved resulting in an answer of 2Ns rather than 6Ns.

10. Most could score something on this unstructured question but only the best were able to get all the way through to a correct answer. The main problem was finding the initial angle of incidence but weaker candidates failed to observe that velocities perpendicular to the line of centres were unchanged by the impact.

11. Nearly all candidates completed part (a) successfully and writing out the initial components of velocity in this way helped candidates to sort out the directions required in part (b). Apart from a few sign errors seen in the equations for conservation of energy and for Newton's experimental law, the majority of candidates could find correctly the components of the velocity parallel to the line of centres after the impact. A few candidates stopped there, failing to understand that finding speeds requires combining two components of velocity.

12. This question, although manipulatively quite straight forward, was in an unusual form and it was noticeable that many candidates needed two or three starts before recognising that this was an oblique impact question and reducing the problem to a number of variables that could be solved. However the great majority were able to do this and full marks were often gained for the question. In part (b) the majority used equations of motion with $v^2 = u^2 + 2as$ to obtain the two components of velocity after impact but correct work-energy equations were also used.

13. Part (a) was generally well done: this was a fairly routine problem and candidates showed confidence and good knowledge in the principles involved. Part (b) was a slightly unusual question and in fact only required resolving the two components of the velocity found in (a), but the majority of candidates failed to see this: many simply left this part blank, others tried to find the velocity of the other ball. In part (c) many failed to see the connection with part (b) and made little progress. A few managed to get through to the end correctly, but the majority either gave up or made little progress.

14. The first two parts of this question were usually answered well. Candidates knew how to resolve the velocity of *P* into two components and they applied the principle of conservation of linear momentum in (a) and Newton's law of impact in (b) along the line of centres. Sometimes the direction of the velocity component of *P* after the impact was not clearly explained and this sometimes led to an incorrect answer of 0.4 instead of 0.8 in part (b). Most candidates knew how to answer part (c) but the last two parts were more demanding and required clear and careful thinking to achieve the correct answers. A common error was to assume that the velocity of *P* perpendicular to *CB* was the same both before and after *P* hit the second wall.

15. No Report available for this question.

16. No Report available for this question.