

**Tuesday 24 June 2014 – Morning**

**A2 GCE MATHEMATICS**

**4731/01 Mechanics 4**

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

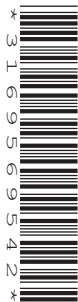
**OCR supplied materials:**

- Printed Answer Book 4731/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

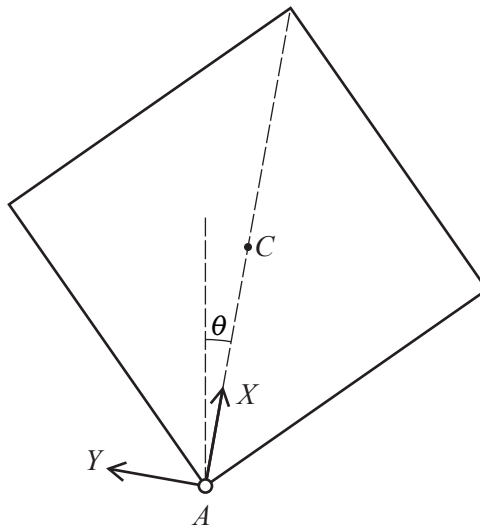
**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 Alan is running in a straight line on a bearing of  $090^\circ$  at a constant speed of  $4 \text{ m s}^{-1}$ . Ben sees Alan when they are 50 m apart and Alan is on a bearing of  $060^\circ$  from Ben. Ben sets off immediately to intercept Alan by running at a constant speed of  $6 \text{ m s}^{-1}$ .
- (i) Calculate the bearing on which Ben should run to intercept Alan. [3]
- (ii) Calculate the magnitude of the velocity of Ben relative to Alan and find the time it takes, from the moment Ben sees Alan, for Ben to intercept Alan. [4]
- 2 A uniform solid circular cone has mass  $M$  and base radius  $R$ .
- (i) Show by integration that the moment of inertia of the cone about its axis of symmetry is  $\frac{3}{10}MR^2$ . (You may assume the standard formula  $\frac{1}{2}mr^2$  for the moment of inertia of a uniform disc about its axis and that the volume of a cone is  $\frac{1}{3}\pi r^2 h$ .) [6]
- The axis of symmetry of the cone is fixed vertically and the cone is rotating about its axis at an angular speed of  $6 \text{ rad s}^{-1}$ . A frictional couple of constant moment  $0.027 \text{ N m}$  is applied to the cone bringing it to rest. Given that the mass of the cone is  $2 \text{ kg}$  and its base radius is  $0.3 \text{ m}$ , find
- (ii) the constant angular deceleration of the cone, [3]
- (iii) the time taken for the cone to come to rest from the instant that the couple is applied. [2]
- 3 The region bounded by the  $y$ -axis and the curves  $y = \sin 2x$  and  $y = \sqrt{2} \cos x$  for  $0 \leq x \leq \frac{1}{4}\pi$  is occupied by a uniform lamina. Find the exact value of the  $x$ -coordinate of the centre of mass of the lamina. [8]

4 A uniform square lamina has mass  $m$  and sides of length  $2a$ .

- (i) Calculate the moment of inertia of the lamina about an axis through one of its corners perpendicular to its plane. [3]

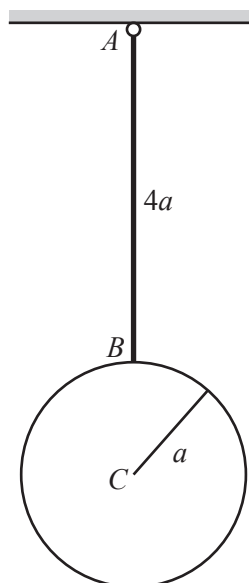


The uniform square lamina has centre  $C$  and is free to rotate in a vertical plane about a fixed horizontal axis passing through one of its corners  $A$ . The lamina is initially held such that  $AC$  is vertical with  $C$  above  $A$ . The lamina is slightly disturbed from rest from this initial position. When  $AC$  makes an angle  $\theta$  with the upward vertical, the force exerted by the axis on the lamina has components  $X$  parallel to  $AC$  and  $Y$  perpendicular to  $AC$  (see diagram).

- (ii) Show that the angular speed,  $\omega$ , of the lamina satisfies  $a\omega^2 = \frac{3}{4}g\sqrt{2}(1 - \cos\theta)$ . [4]

- (iii) Find  $X$  and  $Y$  in terms of  $m$ ,  $g$  and  $\theta$ . [6]

**Question 5 begins on page 4.**



A pendulum consists of a uniform rod  $AB$  of length  $4a$  and mass  $4m$  and a spherical shell of radius  $a$ , mass  $m$  and centre  $C$ . The end  $B$  of the rod is rigidly attached to a point on the surface of the shell in such a way that  $ABC$  is a straight line. The pendulum is initially at rest with  $B$  vertically below  $A$  and it is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through  $A$  (see diagram).

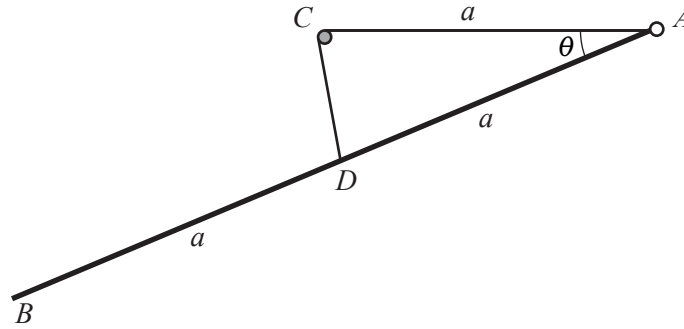
(i) Show that the moment of inertia of the pendulum about the axis of rotation is  $47ma^2$ . [4]

A particle of mass  $m$  is moving horizontally in the plane in which the pendulum is free to rotate. The particle has speed  $\sqrt{kga}$ , where  $k$  is a positive constant, and strikes the rod at a distance  $3a$  from  $A$ . In the subsequent motion the particle adheres to the rod and the combined rigid body  $P$  starts to rotate.

(ii) Show that the initial angular speed of  $P$  is  $\frac{3}{56}\sqrt{\frac{kg}{a}}$ . [4]

(iii) For the case  $k = 4$ , find the angle that  $P$  has turned through when  $P$  first comes to instantaneous rest. [4]

(iv) Find the least value of  $k$  such that the rod reaches the horizontal. [2]



A uniform rod  $AB$  has mass  $m$  and length  $2a$ . The rod can rotate in a vertical plane about a smooth fixed horizontal axis passing through  $A$ . One end of a light elastic string of natural length  $a$  and modulus of elasticity  $\sqrt{3}mg$  is attached to  $A$ . The string passes over a small smooth fixed pulley  $C$ , where  $AC$  is horizontal and  $AC = a$ . The other end of the string is attached to the rod at its mid-point  $D$ . The rod makes an angle  $\theta$  below the horizontal (see diagram).

- (i) Taking  $A$  as the reference level for gravitational potential energy, show that the total potential energy  $V$  of the system is given by

$$V = mga(\sqrt{3} - \sin \theta - \sqrt{3} \cos \theta). \quad [4]$$

- (ii) Show that  $\theta = \frac{1}{6}\pi$  is a position of stable equilibrium for the system. [5]

The system is making small oscillations about the equilibrium position.

- (iii) By differentiating the energy equation with respect to time, show that

$$\frac{4}{3}a\ddot{\theta} = g(\cos \theta - \sqrt{3} \sin \theta). \quad [4]$$

- (iv) Using the substitution  $\theta = \phi + \frac{1}{6}\pi$ , show that the motion is approximately simple harmonic, and find the approximate period of the oscillations. [6]

**END OF QUESTION PAPER**

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