OCR
RECOGNISING ACHIEVEMENT

# Monday 25 June 2012 – Afternoon

## A2 GCE MATHEMATICS

**4731** Mechanics 4

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4731
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

### INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.



Turn over

1 A uniform square lamina, of mass 4.5 kg and side 0.6 m, is rotating about a fixed vertical axis which is perpendicular to the lamina and passes through its centre. A stationary particle becomes attached to the lamina at one of its corners, and this causes the angular speed of the lamina to change instantaneously from $2.2\,\text{rad}\,\text{s}^{-1}$ to $1.5\,\text{rad}\,\text{s}^{-1}$.

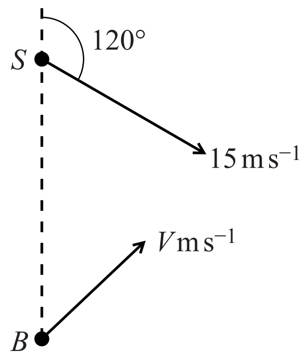
**(i)** Find the mass of the particle. **[4]**

The lamina then slows down with constant angular deceleration. It turns through 36 radians as its angular speed reduces from $1.5\,\text{rad}\,\text{s}^{-1}$ to zero.

**(ii)** Find the time taken for the lamina to come to rest. **[2]**

2 A uniform solid of revolution is formed by rotating the region bounded by the $x$-axis and the curve $y = x\left(1 - \frac{x^2}{a^2}\right)$ for $0 \leqslant x \leqslant a$, where $a$ is a constant, about the $x$-axis. Find the $x$-coordinate of the centre of mass of this solid. **[7]**

3




A ship $S$ is travelling with constant velocity $15\,\text{m}\,\text{s}^{-1}$ on a course with bearing 120°. A patrol boat $B$ observes the ship when $S$ is due north of $B$. The patrol boat $B$ then moves with constant speed $V\,\text{m}\,\text{s}^{-1}$ in a straight line (see diagram).

**(i)** Given that $V = 18$, find the bearing of the course of $B$ such that $B$ intercepts $S$. **[3]**

**(ii)** Given instead that $V = 9$, find the bearing of the course of $B$ such that $B$ passes as close as possible to $S$. **[5]**

**(iii)** Find the smallest value of $V$ for which it is possible for $B$ to intercept $S$. **[2]**

4 A uniform lamina of mass 18 kg occupies the region bounded by the $x$-axis, the $y$-axis, the line $x = \ln 9$ and the curve $y = e^{\frac{1}{2}x}$ for $0 \leqslant x \leqslant \ln 9$. The unit of length is the metre. Find the moment of inertia of this lamina about the $x$-axis. **[7]**

**5** A uniform rod of mass 4 kg and length 2.4 m can rotate in a vertical plane about a fixed horizontal axis through one end of the rod. The rod is released from rest in a horizontal position and a frictional couple of constant moment 20 N m opposes the motion.

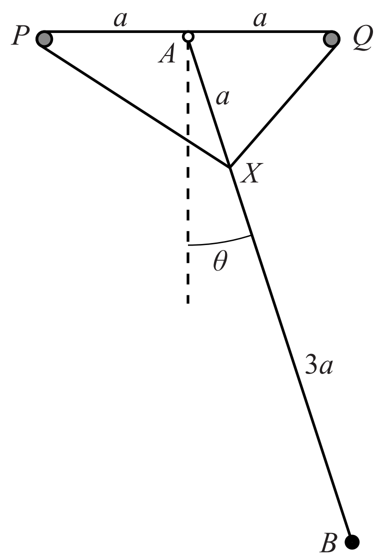
**(i)** Find the angular acceleration of the rod immediately after it is released. **[4]**

**(ii)** Find the angle that the rod makes with the horizontal when its angular acceleration is zero. **[3]**

**(iii)** Find the maximum angular speed of the rod. **[5]**

**(iv)** The rod first comes to instantaneous rest after rotating through an angle $\theta$ radians from its initial position. Find an equation for $\theta$, and verify that $2.0 < \theta < 2.1$. **[3]**

**6**




Two small smooth pegs $P$ and $Q$ are fixed at a distance $2a$ apart on the same horizontal level, and $A$ is the mid-point of $PQ$. A light rod $AB$ of length $4a$ is freely pivoted at $A$ and can rotate in the vertical plane containing $PQ$, with $B$ below the level of $PQ$. A particle of mass $m$ is attached to the rod at $B$. A light elastic string, of natural length $2a$ and modulus of elasticity $\lambda$, passes round the pegs $P$ and $Q$ and its two ends are attached to the rod at the point $X$, where $AX = a$. The angle between the rod and the downward vertical is $\theta$, where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ (see diagram). You are given that the elastic energy stored in the string is $\lambda a(1 + \cos\theta)$.

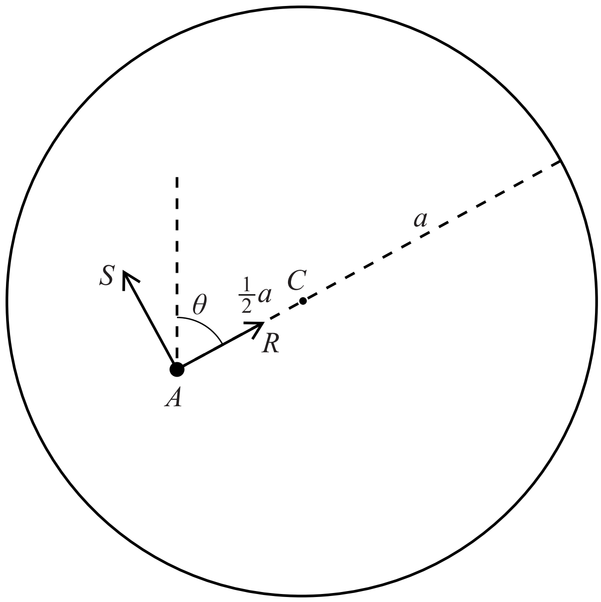
**(i)** Show that $\theta = 0$ is a position of equilibrium, and show that the equilibrium is stable if $\lambda < 4mg$. **[6]**

**(ii)** Given that $\lambda = 3mg$, show that $\ddot{\theta} = -k\frac{g}{a}\sin\theta$, stating the value of the constant $k$. Hence find the approximate period of small oscillations of the system about the equilibrium position $\theta = 0$. **[6]**



**Turn over**

7




A uniform circular disc with centre $C$ has mass $m$ and radius $a$. The disc is free to rotate in a vertical plane about a fixed horizontal axis passing through a point $A$ on the disc, where $AC = \frac{1}{2}a$. The disc is slightly disturbed from rest in the position with $C$ vertically above $A$. When $AC$ makes an angle $\theta$ with the upward vertical the force exerted by the axis on the disc has components $R$ parallel to $AC$ and $S$ perpendicular to $AC$ (see diagram).

**(i)** Show that the angular speed of the disc is $\sqrt{\dfrac{4g(1-\cos\theta)}{3a}}$. **[4]**

**(ii)** Find the angular acceleration of the disc, in terms of $a$, $g$ and $\theta$. **[2]**

**(iii)** Find $R$ and $S$, in terms of $m$, $g$ and $\theta$. **[6]**

**(iv)** Find the magnitude of the force exerted by the axis on the disc at an instant when $R = 0$. **[3]**