RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4731:** Mechanics 4

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

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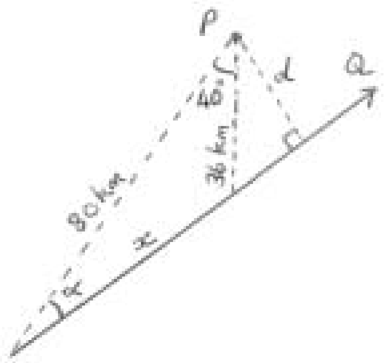
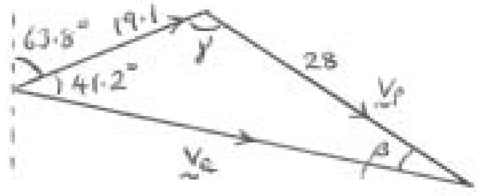
1 (i)	Using $\omega_2 = \omega_1 + \alpha t$, $750 = 950 - 0.8t$ Time taken is 250 s	M1 A1 **[2]**	
(ii)	Using $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, $200^2 = 220^2 - 1.6\theta$ Angle is 5250 rad	M1 A1 **[2]**	
(iii)	Angle is 20π rad Using $\theta = \omega_2 t - \frac{1}{2}\alpha t^2$, $20\pi = 0 + 0.4t^2$ Time taken is 12.5 s (3 sf)	B1 M1 A1 **[3]**	or equivalent; e.g. finding $\omega_1 = 10.03$ and then $t = \omega_1 \div 0.8$ *Accept* $\sqrt{50\pi}$ or $5\sqrt{2\pi}$

2	$m = \int_0^a k\,\mathrm{e}^{-\frac{x}{a}}\,\mathrm{d}x$	M1	For $\int \mathrm{e}^{-\frac{x}{a}}\,\mathrm{d}x$
	$= k\left[-a\,\mathrm{e}^{-\frac{x}{a}}\right]_0^a \quad \left(= ka(1-\mathrm{e}^{-1})\right)$	A1	For $-a\,\mathrm{e}^{-\frac{x}{a}}$
	$m\bar{x} = \int_0^a x\,k\,\mathrm{e}^{-\frac{x}{a}}\,\mathrm{d}x$	M1	For $\int x\,\mathrm{e}^{-\frac{x}{a}}\,\mathrm{d}x$
	$= k\left[-ax\,\mathrm{e}^{-\frac{x}{a}} - a^2\mathrm{e}^{-\frac{x}{a}}\right]_0^a$	M1 A1	Integration by parts For $-ax\,\mathrm{e}^{-\frac{x}{a}} - a^2\mathrm{e}^{-\frac{x}{a}}$
	$= ka^2(1-2\mathrm{e}^{-1})$	A1	For $a^2(1-2\mathrm{e}^{-1})$ or exact equivalent
	$\bar{x} = \frac{ka^2(1-2\mathrm{e}^{-1})}{ka(1-\mathrm{e}^{-1})}$ $= \frac{a(1-2\mathrm{e}^{-1})}{1-\mathrm{e}^{-1}} = \frac{a(\mathrm{e}-2)}{\mathrm{e}-1}$	A1 [7]	

3 (i)	WD by couple is $C \times \frac{\pi}{2}$ Change in PE is $5 \times 9.8 \times 0.9$ By conservation of energy, $C \times \frac{\pi}{2} = 5 \times 9.8 \times 0.9$ Moment of couple is 28.1 Nm (3 sf)	B1 B1 M1 A1 **[4]**	 Must clearly be PE (not moment) Equation involving WD and PE
(ii) (a)	$I = \frac{4}{3} \times 5 \times 0.9^2 \quad (= 5.4)$ $28.075 = 5.4\alpha$ Angular acceleration is $5.20\ \mathrm{rad\,s^{-2}}$ (3 sf)	B1 M1 A1 ft **[3]**	*Can be earned anywhere in the question* Applying $C = I\alpha$ ft is $C \div I$
(ii) (b)	$28.075 - 5 \times 9.8 \times 0.9 = 5.4\alpha$ Angular acceleration is (–) $2.97\ \mathrm{rad\,s^{-2}}$ (3 sf)	M1 A1 [2]	Rotational equation of motion (3 terms) (*Allow 1.8 instead of 0.9 etc*)

4 (i)	GPE is $-mg(\frac{1}{2}a\sin 2\theta)$ EPE is $\frac{3mg}{2a}AD^2 + \frac{4mg}{2a}BD^2$ $= \frac{3mg}{2a}(2a\cos\theta)^2 + \frac{4mg}{2a}(2a\sin\theta)^2$ $= mga(6\cos^2\theta + 8\sin^2\theta)$ $= mga(3 + 3\cos 2\theta + 4 - 4\cos 2\theta)$ $= mga(7 - \cos 2\theta)$ Total PE is $V = mga(7 - \cos 2\theta) - \frac{1}{2}mga\sin 2\theta$ $= \frac{1}{2}mga(14 - 2\cos 2\theta - \sin 2\theta)$	B1 M1 A1 M1 A1 ag **[5]**	*Negative sign is essential, but may be implied later* Any correct form Expressing EPE in terms of $\cos 2\theta$
(ii)	$\frac{dV}{d\theta} = \frac{1}{2}mga(4\sin 2\theta - 2\cos 2\theta)$ $\frac{dV}{d\theta} = 0$ when $4\sin 2\theta = 2\cos 2\theta$ $\tan 2\theta = 0.5$ $\theta = 0.232$ (3 sf)	B1 M1 A1 **[3]**	 Equating to zero and solving Accept 13.3°
(iii)	$\frac{d^2V}{d\theta^2} = \frac{1}{2}mga(8\cos 2\theta + 4\sin 2\theta)$ When $\theta = 0.232$, $\frac{d^2V}{d\theta^2} > 0$ So the equilibrium is stable	 M1 A1 **[2]**	 Fully correct working only

5 (i)	$(\frac{4}{3}\pi a^3)\rho = 10M$, so $\rho = \frac{15M}{2\pi a^3}$	M1	
	$I = \sum \frac{1}{2}(\rho\pi y^2 \delta x)y^2 = \frac{1}{2}\rho\pi \int y^4 \, dx$	M1	For $\int y^4 \, dx$
	$= \frac{1}{2}\rho\pi \int_{-a}^{a} (a^2 - x^2)^2 \, dx$	A1	Correct integral expression including limits
	$= \frac{1}{2}\rho\pi \left[a^4 x - \frac{2}{3}a^2 x^3 + \frac{1}{5}x^5 \right]_{-a}^{a}$	A1	For $a^4 x - \frac{2}{3}a^2 x^3 + \frac{1}{5}x^5$
	$= \frac{1}{2}\rho\pi \left(a^5 - \frac{2}{3}a^5 + \frac{1}{5}a^5 \right) \times 2$		
	$= \frac{8}{15}\rho\pi a^5$	A1	
	$= \frac{8}{15} \times \frac{15M}{2\pi a^3} \times \pi a^5 = 4Ma^2$	A1 ag [6]	
(ii)	MI is $4Ma^2 + Ma^2$	M1	
	$= 5Ma^2$	A1	
	$-Mga\sin\theta = 5Ma^2\ddot{\theta}$	M1	Equation of motion
	$\ddot{\theta} \approx -\frac{g}{5a}\theta$		
	Period is $2\pi\sqrt{\frac{5a}{g}}$	M1 A1 [5]	Obtaining period
	Alternative for last 3 marks of (ii)		
	$11M\,\bar{x} = 10M(0) + Ma$ M1		Finding centre of mass
	$\bar{x} = \frac{1}{11}a$		
	Period is $2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{5Ma^2}{11Mg\frac{1}{11}a}}$ M1		Using formula *Dependent on previous M1*
	$= 2\pi\sqrt{\frac{5a}{g}}$ A1		*Note* $2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{5Ma^2}{Mga}}$ *is M0*

6 (i)	As viewed from *P*  $x^2 = 80^2 + 36^2 - 2\times 80\times 36\cos 40°$ $x = 57.30$ Relative velocity has magnitude $\frac{x}{3} = 19.1\ \text{km h}^{-1}$ $\frac{\sin\alpha}{36} = \frac{\sin 40°}{57.30}$ $\alpha = 23.82°$ Relative velocity has bearing $40 + \alpha = 063.8°$	M1 M1 A1 ag M1 A1 ag **[5]**	Suitable diagram showing relative velocity *May be implied* Or other valid method for finding a relevant angle
	OR, using components, Diagram M1 East $\frac{80\sin 40°}{3}$ $(= 17.14)$ M1 North $\frac{80\cos 40° - 36}{3}$ $(= 8.428)$ M1 Speed $\sqrt{17.14^2 + 8.428^2} = 19.1$ A1 ag Bearing $\tan^{-1}\frac{17.14}{8.428} = 063.8°$ A1 ag		Implied by both components correct
(ii)	Shortest distance $d = 80\sin 23.82°$ $= 32.3$ km (3 sf)	M1 A1 **[2]**	or $36\sin 63.8°$
(iii)	 $\frac{\sin\beta}{19.10} = \frac{\sin 41.18°}{28}$ $\beta = 26.69°$ Bearing of *P* is $105 + \beta = 131.7°$ (1 dp)	M1 M1 A1 **[3]**	Velocity diagram *May be implied* (28 opposite a known angle between sides with positive and negative slopes) *Using components for (iii) and (iv)* *M2A1 for* $\theta = 131.7°$ *or* $v = 39.4$ *M1A1 for other quantity*
(iv)	$\frac{v_Q}{\sin 112.13°} = \frac{28}{\sin 41.18°}$ Speed of *Q* is $39.4\ \text{km h}^{-1}$ (3 sf)	M1 A1 **[2]**	Or other valid method for finding speed

7 (i)	$XG=\sqrt{5}a$	B1	For $I_G=\frac{1}{3}m\{a^2+(3a)^2\}$
	$I=\frac{1}{3}m\{a^2+(3a)^2\}+m(\sqrt{5}a)^2$	M1	Using parallel axes rule
	$=\dfrac{25}{3}ma^2$	A1 [**3**]	
	OR, other complete method, e.g. M1 $\frac{4}{3}(\frac{1}{6}m)\left((\frac{1}{2}a)^2+a^2\right)+\frac{4}{3}(\frac{5}{6}m)\left((\frac{5}{2}a)^2+a^2\right)$ A1 $I=\dfrac{25}{3}ma^2$ A1		Correct expression for I
(ii)	$mg(\sqrt{5}a)=I\alpha$	M1	*Allow, e.g.* $mg(2a)=I\alpha$
	$\sqrt{5}mga=\dfrac{25}{3}ma^2\alpha$		
	$\alpha=\dfrac{3\sqrt{5}g}{25a}$	A1 ag [**2**]	
(iii)	$\frac{1}{2}I\omega^2=mga$	M1	Equation involving KE and PE
	$\dfrac{25}{6}ma^2\omega^2=mga$	A1 ft	
	$\omega=\sqrt{\dfrac{6g}{25a}}$	A1 [**3**]	
(iv)	$H=m(XG)\omega^2$	M1 A1	For using acceleration $r\omega^2$ Or (F parallel to BA, θ is angle GXB) $F-mg\sin\theta=m\left((AG)\omega^2\cos\theta-(AG)\alpha\sin\theta\right)$
	$=m(\sqrt{5}a)\left(\dfrac{6g}{25a}\right)$		
	$=\dfrac{6\sqrt{5}}{25}mg$	A1 ft	ft from incorrect ω only Or $F=\dfrac{mg(2\sqrt{5}+12)}{25}$
	$mg-V=m(XG)\alpha$	M1 A1	For using acceleration $r\alpha$ Or (R parallel to AD) $mg\cos\theta-R=m\left((AG)\omega^2\sin\theta+(AG)\alpha\cos\theta\right)$
	$V=mg-m(\sqrt{5}a)\left(\dfrac{3\sqrt{5}g}{25a}\right)$		
	$=\dfrac{2}{5}mg$	A1	Or $R=\dfrac{mg(4\sqrt{5}-6)}{25}$
	Force has magnitude $\sqrt{H^2+V^2}$		Or $\sqrt{F^2+R^2}$
	$=\dfrac{2}{25}mg\sqrt{(3\sqrt{5})^2+5^2}$	M1	
	$=\dfrac{2\sqrt{70}}{25}mg$	A1 ag [**8**]	

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