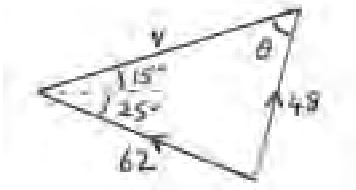
4731 Mechanics 4

1 (i)	Using $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, $67^2 = 83^2 + 2\alpha \times 1000$ $\alpha = -1.2$ Angular deceleration is 1.2 rad s^{-2}	M1 A1 [**2**]	
(ii)	Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, $400 = 83t - 0.6t^2$ $t = 5$ *or* $133\frac{1}{3}$ Time taken is 5 s	M1 A1ft M1 A1 [**4**]	 Solving to obtain a value of t
	Alternative for (ii) $\omega_2^2 = 83^2 - 2 \times 1.2 \times 400$ M1A1 ft $\omega_2 = 77$ $77 = 83 - 1.2t$ M1 $t = 5$ A1		*(M0 if $\omega = 67$ is used in (ii))*

2	Volume $V = \int \pi y^2 \text{d}x = \int_a^{2a} \pi \frac{a^6}{x^4} \text{d}x$	M1	π *may be omitted throughout*
	$= \pi \left[-\frac{a^6}{3x^3} \right]_a^{2a} = \frac{7}{24}\pi a^3$	A1	For integrating x^{-4} to obtain $-\frac{1}{3}x^{-3}$
	$V\bar{x} = \int \pi x y^2 \text{d}x$	M1	for $\int xy^2 \text{d}x$
	$= \int_a^{2a} \pi \frac{a^6}{x^3} \text{d}x$	A1	Correct integral form (including limits)
	$= \pi \left[-\frac{a^6}{2x^2} \right]_a^{2a} = \frac{3}{8}\pi a^4$	A1	For integrating x^{-3} to obtain $-\frac{1}{2}x^{-2}$
	$\bar{x} = \dfrac{\frac{3}{8}\pi a^4}{\frac{7}{24}\pi a^3}$	M1	*Dependent on previous M1M1*
	$= \frac{9a}{7}$	A1 [7]	

3 (i)	$I = \frac{1}{2}(4m)(2a)^2 + (4m)a^2$ $+ m(3a)^2$ $= 21ma^2$	M1 A1 B1 A1 [**4**]	Applying parallel axes rule
(ii)	From P, $\bar{x} = \frac{(4m)a + m(3a)}{5m}$ $\left(= \frac{7a}{5}\right)$ Period is $2\pi\sqrt{\frac{21ma^2}{5mg(\frac{7}{5}a)}}$ $= 2\pi\sqrt{\frac{3a}{g}}$	M1 M1 A1 ft A1 [**4**]	Correct formula $2\pi\sqrt{\frac{I}{mgh}}$ seen *or* using $L = I\ddot{\theta}$ and period $2\pi/\omega$
	Alternative for (ii) $-4mga\sin\theta - mg(3a)\sin\theta = (21ma^2)\ddot{\theta}$ M1 M1 Period is $2\pi\sqrt{\frac{21ma^2}{7mga}} = 2\pi\sqrt{\frac{3a}{g}}$ A1 ft A1		Using $L = I\ddot{\theta}$ with three terms Using period $2\pi/\omega$

4 (i)	 $\frac{\sin\theta}{62}=\frac{\sin 40}{48}$ $\theta = 56.1°$ *or* $123.9°$ Bearings are $018.9°$ and $311.1°$	M1 M1 A1 A1A1 **[5]**	Velocity triangle One value sufficient Accept $19°$ and $311°$
(ii)	Shorter time when $\theta = 56.1°$ $\frac{v}{\sin 83.87}=\frac{48}{\sin 40}$ Relative speed is $v = 74.25$ Time to intercept is $\frac{3750}{74.25}$ $= 50.5\text{ s}$	B1 ft M1 M1 A1 **[4]**	 Or $v^2 = 62^2 + 48^2 - 2\times 62\times 48\cos 83.87$ *Dependent on previous M1*
	Alternative for (i) and (ii) $\begin{pmatrix}48\sin\phi\\48\cos\phi\end{pmatrix}t=\begin{pmatrix}3750\sin 75\\3750\cos 75\end{pmatrix}+\begin{pmatrix}62\sin 295\\62\cos 295\end{pmatrix}t$ M1 M1 $3.732\cos\phi - \sin\phi = 3.208$ A1 M1 M1 $\phi = 18.9°$ and $311.1°$ A1A1 B1 ft $t = 50.5$ A1		component eqns (displacement or velocity) obtaining eqn in ϕ *or* t *or* v $(=3750/t)$ correct simplified equation or $t^2 - 231.3t + 9131.5 = 0$ $[t = 50.5,\ 180.8]$ or $v^2 - 94.99v + 1540 = 0$ $[v = 74.25,\ 20.74]$ solving to obtain a value of ϕ solving to obtain a value of t *(max A1 if any extra values given)* appropriate selection for shorter time

5 (i)	Area is $\int_0^2 (8-x^3)\,dx = \left[8x - \frac{1}{4}x^4 \right]_0^2 = 12$ Mass per m^2 is $\rho = \frac{63}{12} = 5.25$ $I_y = \sum(\rho y \delta x)x^2 = \rho \int x^2 y\,dx$ $= \rho \int_0^2 (8x^2 - x^5)\,dx$ $= \rho \left[\frac{8}{3}x^3 - \frac{1}{6}x^6 \right]_0^2 = \frac{32}{3}\rho$ $= \frac{32}{3} \times \frac{63}{12} = 56 \text{ kg m}^2$	B1 M1 M1 A1 A1 A1 AG **[6]**	for $\int x^2 y\,dx$ or $\int x^3\,dy$ or $\frac{1}{3}\rho \int_0^8 (8-y)\,dy$ for $\frac{32}{3}$
(ii)	Anticlockwise moment is $800 - 63 \times 9.8 \times \frac{4}{5}$ $= 306.08 \text{ N m} > 0$ so it will rotate anticlockwise	M1 A1 **[2]**	Full explanation is required; (anti)clockwise should be mentioned before the conclusion
(iii)	$I = I_x + I_y = 1036.8 + 56 \quad (= 1092.8)$ WD by couple is $800 \times \frac{1}{2}\pi$ Change in PE is $63 \times 9.8 \times \left(\frac{24}{7} - \frac{4}{5}\right)$ $800 \times \frac{1}{2}\pi = \frac{1}{2}I\omega^2 - 63 \times 9.8 \times \left(\frac{24}{7} - \frac{4}{5}\right)$ $1256.04 = 546.4\omega^2 - 1622.88$ $\omega = 2.30 \text{ rad s}^{-1}$	B1 B1 B1 M1 A1 A1 **[6]**	Equation involving WD, KE and PE *May have an incorrect value for I; other terms and signs are cao*

6 (i)	GPE is $mg(a\sin 2\theta)$ $\text{AB} = 2a\cos\theta$ *or* $\text{AB}^2 = a^2 + a^2 - 2a^2\cos(\pi - 2\theta)$ EPE is $\frac{\sqrt{3}mg}{2a}(2a\cos\theta)^2$ $= \sqrt{3}mga(1+\cos 2\theta)$ Total PE is $V = \sqrt{3}mga(1+\cos 2\theta) + mga\sin 2\theta$ $= mga(\sqrt{3} + \sqrt{3}\cos 2\theta + \sin 2\theta)$	B1 B1 M1 A1 AG [**4**]	Or $mg(2a\cos\theta\sin\theta)$ Any correct form Expressing EPE and GPE in terms of $\cos 2\theta$ and $\sin 2\theta$
(ii)	$\frac{dV}{d\theta} = mga(-2\sqrt{3}\sin 2\theta + 2\cos 2\theta)$ $= 0$ when $2\sqrt{3}\sin 2\theta = 2\cos 2\theta$ $\tan 2\theta = \frac{1}{\sqrt{3}}$ $\theta = \frac{\pi}{12},\ -\frac{5\pi}{12}$	B1 M1 M1 A1A1 [**5**]	(B0 for $\frac{dV}{d\theta} = -2\sqrt{3}\sin 2\theta + 2\cos 2\theta$) Solving to obtain a value of θ Accept $0.262,\ -1.31$ or $15°,\ -75°$
(iii)	$\frac{d^2V}{d\theta^2} = mga(-4\sqrt{3}\cos 2\theta - 4\sin 2\theta)$ When $\theta = \frac{\pi}{12}$, $\frac{d^2V}{d\theta^2} = -8mga < 0$ so this position is unstable When $\theta = -\frac{5\pi}{12}$, $\frac{d^2V}{d\theta^2} = 8mga > 0$ so this position is stable	B1ft M1 A1 A1 [**4**]	 Determining the sign of V'' *or* M2 for alternative method for max / min

7 (i)	Initially $\cos\theta = \frac{0.6}{1.5} = 0.4$ $\frac{1}{2}\times 4.9\omega^2 = 6\times 9.8(0.5\times 0.4 - 0.5\cos\theta)$ $\omega^2 = 12(0.4-\cos\theta)$ $\omega^2 = 4.8 - 12\cos\theta$	M1 A1 A1 AG [**3**]	Equation involving KE and PE
(ii)	$6\times 9.8\times 0.5\sin\theta = 4.9\alpha$ $\alpha = 6\sin\theta \quad (\text{rad s}^{-2})$	M1 A1 [**2**]	or $2\omega\frac{d\omega}{d\theta} = 12\sin\theta$ or $2\omega\frac{d\omega}{dt} = 12\sin\theta\frac{d\theta}{dt}$
(iii)	$6\times 9.8\cos\theta - F = 6\times 0.5\omega^2$ $58.8\cos\theta - F = 14.4 - 36\cos\theta$ $F = 94.8\cos\theta - 14.4$ $6\times 9.8\sin\theta - R = 6\times 0.5\alpha$ $58.8\sin\theta - R = 18\sin\theta$ $R = 40.8\sin\theta$	M1 M1 A1 AG M1 M1 A1 [**6**]	for radial acceleration $r\omega^2$ radial equation of motion *Dependent on previous M1* for transverse acceleration $r\alpha$ transverse equation of motion *Dependent on previous M1*
(iv)	If B reaches the ground, $\cos\theta = -0.4$ $F = -52.32$ $\sin\theta = \sqrt{0.84}$ [$\theta = 1.982$ *or* $113.6°$] $R = 37.39$ Since $\frac{52.32}{37.39} = 1.40 > 0.9$, this is not possible	M1 A1 M1 A1 [**4**]	*Allow M1A0 if* $\cos\theta = +0.4$ *is used* Obtaining a value for R Or $\mu R = 33.65$, and $52.32 > 33.65$
	Alternative for (iv) Slips when $F = -0.9R$ $94.8\cos\theta - 14.4 = -36.72\sin\theta$ M1 $\theta = 1.798$ [$103.0°$] A1 B reaches the ground when $\cos\theta = -0.4$ M1 $\theta = 1.982$ [$113.6°$] so it slips before this A1		 *Allow M1A0 if* $F = +0.9R$ *is used* *Allow M1A0 if* $\cos\theta = +0.4$ *is used*