RECOGNISING ACHIEVEMENT

ADVANCED GCE **4731/01**

MATHEMATICS

Mechanics 4

WEDNESDAY 18 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 Two flywheels F and G are rotating freely, about the same axis and in the same direction, with angular speeds $21\,\text{rad s}^{-1}$ and $36\,\text{rad s}^{-1}$ respectively. The flywheels come into contact briefly, and immediately afterwards the angular speeds of F and G are $28\,\text{rad s}^{-1}$ and $34\,\text{rad s}^{-1}$, respectively, in the same direction. Given that the moment of inertia of F about the axis is $1.5\,\text{kg m}^2$, find the moment of inertia of G about the axis. [4]

2 A rotating turntable is slowing down with constant angular deceleration. It makes 16 revolutions as its angular speed decreases from $8\,\text{rad s}^{-1}$ to rest.

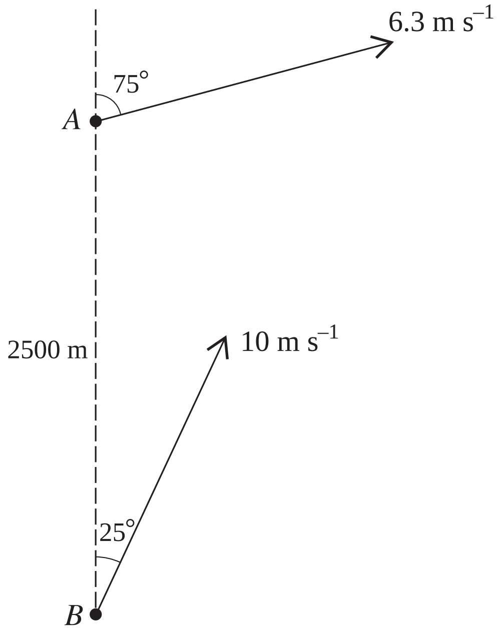
(i) Find the angular deceleration of the turntable. [2]

(ii) Find the angular speed of the turntable at the start of its last complete revolution before coming to rest. [2]

(iii) Find the time taken for the turntable to make its last complete revolution before coming to rest. [2]

3 The region bounded by the curve $y = 2x + x^2$ for $0 \leqslant x \leqslant 3$, the x-axis, and the line $x = 3$, is occupied by a uniform lamina. Find the coordinates of the centre of mass of this lamina. [9]

4



A boat A is travelling with constant speed $6.3\,\text{m s}^{-1}$ on a course with bearing 075°. Boat B is travelling with constant speed $10\,\text{m s}^{-1}$ on a course with bearing 025°. At one instant, A is 2500 m due north of B (see diagram).

(i) Find the magnitude and bearing of the velocity of A relative to B. [5]

(ii) Find the shortest distance between A and B in the subsequent motion. [3]

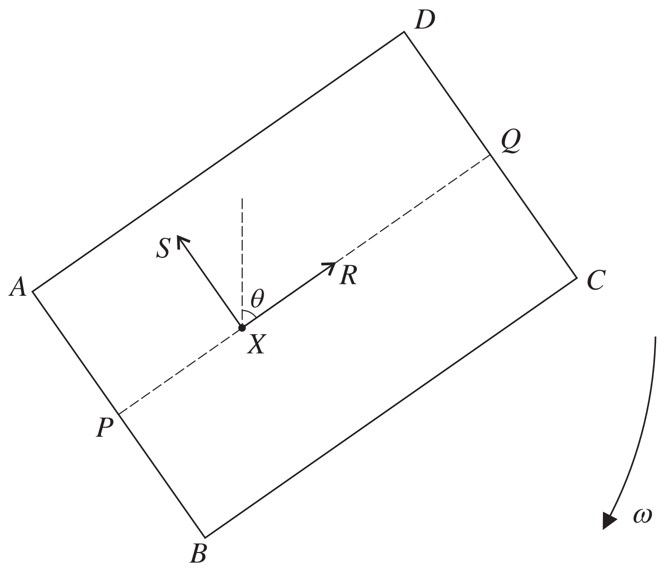
5 The region bounded by the curve $y = \sqrt{ax}$ for $a \leqslant x \leqslant 4a$ (where a is a positive constant), the x-axis, and the lines $x = a$ and $x = 4a$, is rotated through 2π radians about the x-axis to form a uniform solid of revolution of mass m.

(i) Show that the moment of inertia of this solid about the x-axis is $\frac{7}{5}ma^2$. [8]

The solid is free to rotate about a fixed horizontal axis along the line $y = a$, and makes small oscillations as a compound pendulum.

(ii) Find, in terms of a and g, the approximate period of these small oscillations. [4]

6



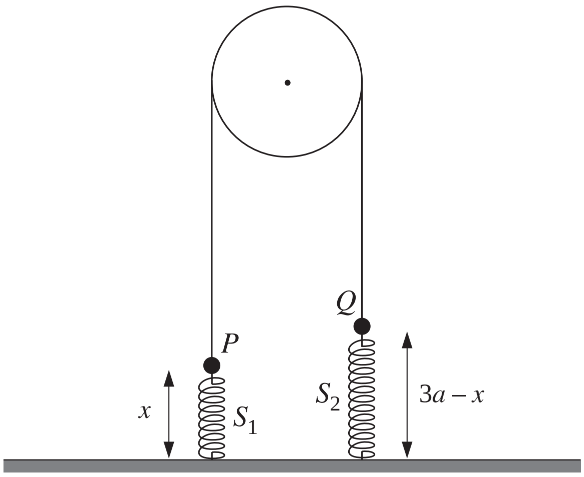
A uniform rectangular lamina $ABCD$ has mass m and sides $AB = 2a$ and $BC = 3a$. The mid-point of AB is P and the mid-point of CD is Q. The lamina is rotating freely in a vertical plane about a fixed horizontal axis which is perpendicular to the lamina and passes through the point X on PQ where $PX = a$. Air resistance may be neglected. When Q is vertically above X, the angular speed is $\sqrt{\frac{9g}{10a}}$. When XQ makes an angle θ with the upward vertical, the angular speed is ω, and the force acting on the lamina at X has components R parallel to PQ and S parallel to BA (see diagram).

(i) Show that the moment of inertia of the lamina about the axis through X is $\frac{4}{3}ma^2$. [3]

(ii) At an instant when $\cos\theta = \frac{3}{5}$, show that $\omega^2 = \frac{6g}{5a}$. [3]

(iii) At an instant when $\cos\theta = \frac{3}{5}$, show that $R = 0$, and given also that $\sin\theta = \frac{4}{5}$ find S in terms of m and g. [9]

7



Particles P and Q, with masses $3m$ and $2m$ respectively, are connected by a light inextensible string passing over a smooth light pulley. The particle P is connected to the floor by a light spring S_1 with natural length a and modulus of elasticity mg. The particle Q is connected to the floor by a light spring S_2 with natural length a and modulus of elasticity $2mg$. The sections of the string not in contact with the pulley, and the two springs, are vertical. Air resistance may be neglected. The particles P and Q move vertically and the string remains taut; when the length of S_1 is x, the length of S_2 is $(3a - x)$ (see diagram).

(i) Find the total potential energy of the system (taking the floor as the reference level for gravitational potential energy). Hence show that $x = \frac{4}{3}a$ is a position of stable equilibrium. [9]

(ii) By differentiating the energy equation, and substituting $x = \frac{4}{3}a + y$, show that the motion is simple harmonic, and find the period. [9]