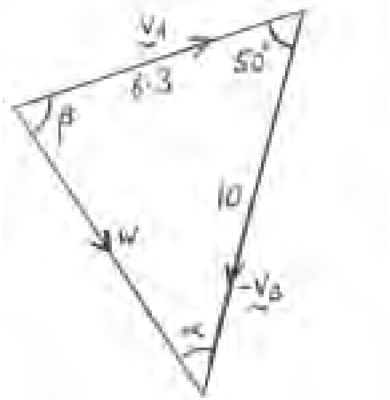
4731 Mechanics 4

1	By conservation of angular momentum $1.5\times 21+I_G\times 36=1.5\times 28+I_G\times 34$ $I_G=5.25\text{ kg m}^2$	M1 A1A1 A1 **4**	Give A1 for each side of the equation or $1.5(28-21)=I_G(36-34)$
2 (i)	Using $\omega_1^2=\omega_0^2+2\alpha\theta$, $0^2=8^2+2\alpha(2\pi\times 16)$ $\alpha=-\frac{1}{\pi}=-0.318$ Angular deceleration is 0.318 rad s^{-2}	M1 A1 **2**	*Accept* $-\frac{1}{\pi}$
(ii)	Using $\omega_1^2=\omega_0^2+2\alpha\theta$, $\omega^2=8^2+2\alpha(2\pi\times 15)$ $\omega=2\text{ rad s}^{-1}$	M1 A1 ft **2**	or $0^2=\omega^2+2\alpha(2\pi)$ ft is $\sqrt{64-60\pi\lvert\alpha\rvert}$ or $\sqrt{4\pi\lvert\alpha\rvert}$ *Allow A1 for* $\omega=2$ *obtained using* $\theta=16$ *and* $\theta=15$ *(or* $\theta=1$*)*
(iii)	Using $\omega_1=\omega_0+\alpha t$, $0=\omega+\alpha t$ $t=2\pi=6.28\text{ s}$	M1 A1 ft **2**	or $2\pi=0t-\frac{1}{2}\alpha t^2$ ft is $\frac{\omega}{\lvert\alpha\rvert}$ or $\sqrt{\frac{4\pi}{\lvert\alpha\rvert}}$ *Accept* 2π
3	$A=\int_0^3 (2x+x^2)\,dx$ $=\left[x^2+\frac{1}{3}x^3\right]_0^3=18$ $A\bar{x}=\int_0^3 x(2x+x^2)\,dx$ $=\left[\frac{2}{3}x^3+\frac{1}{4}x^4\right]_0^3=\frac{153}{4}=38.25$ $\bar{x}=\frac{38.25}{18}=\frac{17}{8}=2.125$ $A\bar{y}=\int_0^3 \frac{1}{2}(2x+x^2)^2\,dx$ $=\int_0^3 (2x^2+2x^3+\frac{1}{2}x^4)\,dx$ $=\left[\frac{2}{3}x^3+\frac{1}{2}x^4+\frac{1}{10}x^5\right]_0^3=82.8$ $\bar{y}=\frac{82.8}{18}=4.6$	M1 A1 M1 M1 A1 M1 M1 M1 A1 **9**	*Definite integrals may be evaluated by calculator (i.e with no working shown)* Integrating and evaluating *(dependent on previous M1)* or $\int_0^{15}\left(3-(\sqrt{y+1}-1)\right)y\,dy$ Arranging in integrable form Integrating and evaluating *SR If ½ is missing, then M0M1M1A0 can be earned for* $\bar{y}$

4 (i)		B1	Correct velocity triangle
	$w^2 = 6.3^2 + 10^2 - 2 \times 6.3 \times 10 \cos 50°$	M1	
	$w = 7.66 \text{ m s}^{-1}$	A1	
	$\frac{\sin\alpha}{6.3} = \frac{\sin 50°}{w}$	M1	*This mark cannot be earned from work done in part (ii)*
	$\alpha = 39.04°$ $(\beta = 90.96°)$		
	Bearing is $205 - \alpha = 166°$	A1 **5**	
	OR $\begin{pmatrix} 6.3\sin 75 \\ 6.3\cos 75 \end{pmatrix} - \begin{pmatrix} 10\sin 25 \\ 10\cos 25 \end{pmatrix} = \begin{pmatrix} 1.859 \\ -7.433 \end{pmatrix}$ M1A1		
	M1		Finding magnitude or direction
	$w = \sqrt{1.859^2 + 7.433^2} = 7.66$ A1		
	Bearing is $180 - \tan^{-1}\frac{1.859}{7.433} = 166°$ A1		
(ii)	As viewed from *B* 	B1 ft	Diagram showing path of *A* as viewed from *B* *May be implied* Or B1 for a correct (ft) expression for d^2 in terms of *t*
	$d = 2500\sin 14.04$	M1	or other complete method
	$= 607$ m	A1 **3**	Accept 604.8 to 609 *SR* If $\beta = 89°$ is used, give A1 for 684.9 to 689.1

5 (i)	$V=\int_a^{4a}\pi(a\,x)\,\mathrm{d}x$	M1	*(Omission of π is an accuracy error)*
	$=\left[\tfrac{1}{2}\pi a x^2\right]_a^{4a}=\tfrac{15}{2}\pi a^3$	M1	
	Hence $m=\tfrac{15}{2}\pi a^3\rho$	M1	
	$I=\sum\tfrac{1}{2}(\rho\pi y^2\delta x)y^2=\int\tfrac{1}{2}\rho\pi y^4\,\mathrm{d}x$	M1 A1	For $\int y^4\,\mathrm{d}x$
	$=\int_a^{4a}\tfrac{1}{2}\rho\pi a^2x^2\,\mathrm{d}x$	A1 ft	Substitute for y^4 and correct limits
	$=\left[\tfrac{1}{6}\rho\pi a^2x^3\right]_a^{4a}=\tfrac{21}{2}\rho\pi a^5$	A1	
	$=\tfrac{7}{5}(\tfrac{15}{2}\pi a^3\rho)a^2=\tfrac{7}{5}ma^2$	A1 (ag) **8**	
(ii)	MI about axis, $I_A=\tfrac{7}{5}ma^2+ma^2$	M1	Using parallel axes rule
	$=\tfrac{12}{5}ma^2$	A1	
	Period is $2\pi\sqrt{\dfrac{I}{mgh}}$	M1	
	$=2\pi\sqrt{\dfrac{\frac{12}{5}ma^2}{mga}}=2\pi\sqrt{\dfrac{12a}{5g}}$	A1 ft **4**	ft from any I with $h=a$
6 (i)	$I=\tfrac{1}{3}m\{a^2+(\tfrac{3}{2}a)^2\}+m(\tfrac{1}{2}a)^2$ $=\tfrac{13}{12}ma^2+\tfrac{1}{4}ma^2=\tfrac{4}{3}ma^2$	M1 M1 A1 (ag) **3**	MI about perp axis through centre Using parallel axes rule
(ii)	By conservation of energy	M1	Equation involving KE and PE
	$\tfrac{1}{2}(\tfrac{4}{3}ma^2)\omega^2-\tfrac{1}{2}(\tfrac{4}{3}ma^2)\dfrac{9g}{10a}=mg(\tfrac{1}{2}a-\tfrac{1}{2}a\times\tfrac{3}{5})$	A1	
	$\tfrac{2}{3}ma^2\omega^2-\tfrac{3}{5}mga=\tfrac{1}{5}mga$		
	$\omega^2=\dfrac{6g}{5a}$	A1 (ag) **3**	
(iii)	$mg\cos\theta-R=m(\tfrac{1}{2}a)\omega^2$	M1	Acceleration $r\omega^2$ and three terms (one term must be R)
	$mg\times\tfrac{3}{5}-R=\tfrac{3}{5}mg$	A1	*SR* $mg\cos\theta+R=m(\tfrac{1}{2}a)\omega^2\Rightarrow R=0$ earns M1A0A1
	$R=0$	A1 (ag)	
	$mg(\tfrac{1}{2}a\sin\theta)=I\alpha$	M1A1	Applying $L=I\alpha$
	$\alpha=\dfrac{3g}{10a}$	A1	
	$mg\sin\theta-S=m(\tfrac{1}{2}a)\alpha$	M1A1	Acceleration $r\alpha$ and three terms (one term must be S)
	$S=\tfrac{4}{5}mg-\tfrac{3}{20}mg$		or $S(\tfrac{1}{2}a)=I_G\alpha=\tfrac{13}{12}ma^2\alpha$
	$=\tfrac{13}{20}mg$	A1 **9**	

7 (i)	$U = 3mgx + 2mg(3a - x)$ $+\frac{mg}{2a}(x-a)^2 + \frac{2mg}{2a}(2a-x)^2$ $= \frac{mg}{2a}(3x^2 - 8ax + 21a^2)$ $\frac{dU}{dx} = 3mg - 2mg + \frac{mg}{a}(x-a) - \frac{2mg}{a}(2a-x)$ $= \frac{3mgx}{a} - 4mg$ When $x = \frac{4}{3}a$, $\frac{dU}{dx} = 4mg - 4mg = 0$ so this is a position of equilibrium $\frac{d^2U}{dx^2} = \frac{3mg}{a}$ > 0, so equilibrium is stable	B1B1 B1B1 M1 A1 A1 (ag) M1 A1 (ag) **9**	*Can be awarded for terms listed separately* Obtaining $\frac{dU}{dx}$ *(or any multiple of this)*
(ii)	KE is $\frac{1}{2}(3m)v^2 + \frac{1}{2}(2m)v^2$ Energy equation is $U + \frac{5}{2}mv^2 = \text{constant}$ Differentiating with respect to t $\left(\frac{3mgx}{a} - 4mg\right)\frac{dx}{dt} + 5mv\frac{dv}{dt} = 0$ $\frac{3gx}{a} - 4g + 5\frac{d^2x}{dt^2} = 0$ Putting $x = \frac{4}{3}a + y$, $\frac{3gy}{a} + 5\frac{d^2y}{dt^2} = 0$ $\frac{d^2y}{dt^2} = -\frac{3g}{5a}y$ Hence motion is SHM with period $2\pi\sqrt{\frac{5a}{3g}}$	M1A1 M1 A1 ft A1 ft M1A1 ft A1 (ag) A1 **9**	 Differentiating the energy equation (with respect to t or x) *Condone $\ddot{x}$ instead of $\ddot{y}$* *Award M1 even if KE is missing* *Must have $\ddot{y} = -\omega^2 y$ or other satisfactory explanation*