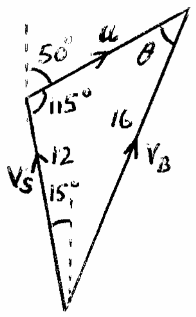
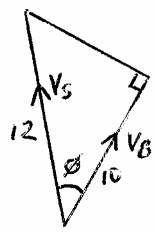
1 (i)	Using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, $56 = 0 + \frac{1}{2}\alpha \times 8^2$ $\alpha = 1.75 \text{ rad s}^{-2}$	M1 A1	**2**	
(ii)	Using $\omega_1^2 = \omega_0^2 + 2\alpha\theta$, $36^2 = 20^2 + 2 \times 1.75\theta$ $\theta = 256 \text{ rad}$	M1 A1 ft	**2**	ft is $448 \div \alpha$
2	Volume is $\int_0^a \pi(4a^2 - x^2)\,dx = \pi\left[4a^2x - \frac{1}{3}x^3\right]_0^a$ $= \frac{11}{3}\pi a^3$ $\int_0^a \pi x(4a^2 - x^2)\,dx$ $= \pi\left[2a^2x^2 - \frac{1}{4}x^4\right]_0^a$ $= \frac{7}{4}\pi a^4$ $\bar{x} = \dfrac{\frac{7}{4}\pi a^4}{\frac{11}{3}\pi a^3}$ $= \dfrac{21}{44}a$	M1 A1 M1 A1 A1 M1 A1	**7**	*π may be omitted throughout* (Limits not required) (Limits not required) for $\dfrac{\int xy^2\,dx}{\int y^2\,dx}$
3 (i)	$I = 6.2 + 2.8 = 9.0 \text{ kg m}^2$	B1	**1**	
(ii)	WD against frictional couple is $L \times \frac{1}{2}\pi$ Loss of PE is $6 \times 9.8 \times 1.3$ $(= 76.44)$ Gain of KE is $\frac{1}{2} \times 9.0 \times 2.4^2$ $(= 25.92)$ By work-energy principle, $L \times \frac{1}{2}\pi = 76.44 - 25.92$ $L = 32.2 \text{ N m}$	B1 B1 B1 ft M1 A1	**5**	Equation involving WD, KE and PE Accept 32.1 to 32.2
(iii)	$6 \times 9.8 \times 0.8 - L = I\alpha$ $\alpha = 1.65 \text{ rad s}^{-2}$	M1 A1 ft A1	**3**	Moments equation

4 (i)	MI of elemental disc about a diameter is $\frac{1}{4}\left(\frac{M}{3a}\delta x\right)a^2$ MI of elemental disc about *AB* is $\frac{1}{4}\left(\frac{M}{3a}\delta x\right)a^2 + \left(\frac{M}{3a}\delta x\right)x^2$ $I = \frac{M}{3a}\int_0^{3a}(\frac{1}{4}a^2 + x^2)\,\mathrm{d}x$ $= \frac{M}{3a}\left[\frac{1}{4}a^2x + \frac{1}{3}x^3\right]_0^{3a}$ $= \frac{M}{3a}(\frac{3}{4}a^3 + 9a^3)$ $= M(\frac{1}{4}a^2 + 3a^2)$ $= \frac{13}{4}Ma^2$	B1 M1 A1 M1 A1 M1 A1 (ag) **7**	$\frac{M}{3a}$ may be $\rho\pi a^2$ throughout *(condone use of $\rho = 1$)* Using parallel axes rule (can award A1 for $\frac{1}{4}ma^2 + mx^2$) Integrating MI of disc *about AB* Correct integral expression for *I* Obtaining an expression for *I* in terms of *M* and *a* *Dependent on previous M1*
(ii)	Period is $2\pi\sqrt{\frac{I}{Mgh}}$ $= 2\pi\sqrt{\frac{\frac{13}{4}Ma^2}{Mg\frac{3}{2}a}}$ $= 2\pi\sqrt{\frac{13a}{6g}}$	M1 A1 A1 **3**	or $-Mgh\sin\theta = I\ddot{\theta}$

5 (i)	 $\frac{\sin\theta}{12} = \frac{\sin 115}{16}$ $\theta = 42.8°$ Bearing of $\mathbf{v}_B$ is 007.2° $\frac{u}{\sin 22.2} = \frac{16}{\sin 115}$ $u = 6.66$ Time taken is $\frac{2400}{6.664} = 360$ s	M1 A1 M1 A1 M1 A1 M1*A1 ft **8**	Relative velocity on bearing 050 Correct velocity diagram; or $\begin{pmatrix} u\sin 50 \\ u\cos 50 \end{pmatrix} = \begin{pmatrix} 16\sin\alpha \\ 16\cos\alpha \end{pmatrix} - \begin{pmatrix} 12\sin 345 \\ 12\cos 345 \end{pmatrix}$ or eliminating u (or α) or obtaining equation for u (or α) *For equations in α and t* *M1*M1A1 for equations* *M1 for eliminating t (or α)* *A1 for $\alpha = 7.2$* *M1A1 ft for equation for t (or α)* *A1 cao for $t = 360$*
(ii)	 $\cos\phi = \frac{10}{12}$ $\phi = 33.6°$ Bearing of $\mathbf{v}_B$ is 018.6°	M1 A1 M1 A1 **4**	Relative velocity perpendicular to $\mathbf{v}_B$ Correct velocity diagram For alternative methods: M2 for a completely correct method A2 for 018.6 (give A1 for a correct relevant angle)

6 (i)	$I = \frac{1}{3}ma^2 + m(\frac{1}{3}a)^2$	M1	Using parallel axes rule
	$= \frac{4}{9}ma^2$	A1	
	$mg(\frac{1}{3}a\cos\theta) = I\alpha$	M1	
	$\alpha = \dfrac{\frac{1}{3}mga\cos\theta}{\frac{4}{9}ma^2} = \dfrac{3g\cos\theta}{4a}$	A1 (ag) **4**	
(ii)	By conservation of energy,	M1	
	$\frac{1}{2}I\omega^2 = mg(\frac{1}{3}a\sin\theta)$	A1 ft	
	$\frac{2}{9}ma^2\omega^2 = \frac{1}{3}mga\sin\theta$		
	$\omega = \sqrt{\dfrac{3g\sin\theta}{2a}}$	A1 **3**	*Condone* $\omega^2 = \dfrac{3g\sin\theta}{2a}$
	OR $\omega\dfrac{d\omega}{d\theta} = \dfrac{3g\cos\theta}{4a}$ M1		
	$\frac{1}{2}\omega^2 = \int \dfrac{3g\cos\theta}{4a}d\theta$		
	$= \dfrac{3g\sin\theta}{4a}$ $(+C)$ A1		
	$\omega = \sqrt{\dfrac{3g\sin\theta}{2a}}$ A1		
(iii)	Acceleration parallel to rod is $(\frac{1}{3}a)\omega^2$	B1	
	$F - mg\sin\theta = m(\frac{1}{3}a)\omega^2$	M1	Radial equation with 3 terms
	$F - mg\sin\theta = \frac{1}{2}mg\sin\theta$		
	$F = \frac{3}{2}mg\sin\theta$	A1	
	Acceleration perpendicular to rod is $(\frac{1}{3}a)\alpha$	B1 ft	ft is $r\alpha$ with r the same as before
	$mg\cos\theta - R = m(\frac{1}{3}a)\alpha$	M1	Transverse equation with 3 terms
	$mg\cos\theta - R = \frac{1}{4}mg\cos\theta$		
	$R = \frac{3}{4}mg\cos\theta$	A1 **6**	
	OR $R(\frac{1}{3}a) = I_G\alpha$ M1		Must use I_G
	$R(\frac{1}{3}a) = (\frac{1}{3}ma^2)\left(\dfrac{3g\cos\theta}{4a}\right)$ A1		
	$R = \frac{3}{4}mg\cos\theta$ A1		
(iv)	On the point of slipping, $F = \mu R$		
	$\frac{3}{2}mg\sin\theta = \mu(\frac{3}{4}mg\cos\theta)$	M1	
	$\tan\theta = \frac{1}{2}\mu$	A1 (ag) **2**	Correctly obtained *Dependent on 6 marks earned in (iii)*

7 (i)	GPE $=(-)\, mg(2a\cos\theta)\cos\theta$ EPE $=\dfrac{\frac{1}{2}mg}{2a}(AR-a)^2$ $=\dfrac{\frac{1}{2}mg}{2a}(2a\cos\theta-a)^2$ $V=\frac{1}{4}mga(2\cos\theta-1)^2-2mga\cos^2\theta$ $=mga(\cos^2\theta-\cos\theta+\frac{1}{4}-2\cos^2\theta)$ $=mga(\frac{1}{4}-\cos\theta-\cos^2\theta)$	B1 M1 A1 A1 (ag) **4**	or $(-)\, mg(a+a\cos 2\theta)$
(ii)	$\dfrac{\mathrm{d}V}{\mathrm{d}\theta}=mga(\sin\theta+2\cos\theta\sin\theta)$ $=mga\sin\theta(1+2\cos\theta)$ Equilibrium when $\dfrac{\mathrm{d}V}{\mathrm{d}\theta}=0$ ie when $\theta=0$	B1 M1 A1 (ag) **3**	
(iii)	KE is $\frac{1}{2}m(2a\dot{\theta})^2$ $2ma^2\dot{\theta}^2+V=\text{constant}$ Differentiating with respect to t, $4ma^2\dot{\theta}\ddot{\theta}+\dfrac{\mathrm{d}V}{\mathrm{d}\theta}\dot{\theta}=0$ $4ma^2\dot{\theta}\ddot{\theta}+mga\sin\theta(1+2\cos\theta)\dot{\theta}=0$ $\ddot{\theta}=-\dfrac{g}{4a}\sin\theta(1+2\cos\theta)$	B1 M1 M1 A1 ft A1 (ag) **5**	(can award this M1 if no KE term) *SR* B2 (replacing the last 3 marks) for the given result correctly obtained by differentiating w.r.t. θ
(iv)	When θ is small, $\sin\theta\approx\theta$, $\cos\theta\approx 1$ $\ddot{\theta}\approx-\dfrac{g}{4a}\theta(1+2)=-\dfrac{3g}{4a}\theta$ Period is $2\pi\sqrt{\dfrac{4a}{3g}}$	M1 A1 A1 **3**	