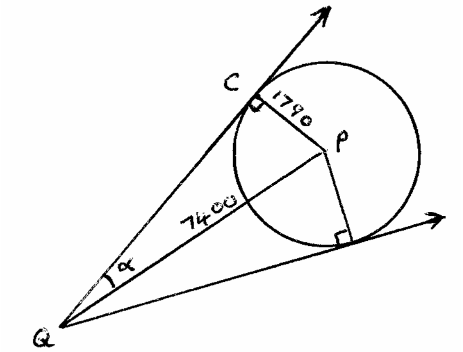
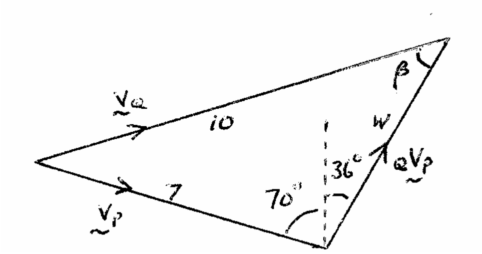
MARKING SCHEME

1	$\int x\rho\,dx = \int_0^a k(a+2x)\,x\,dx$	M1	for $\int ...(a+2x)x\,dx$
	$= k\left[\tfrac{1}{2}ax^2 + \tfrac{2}{3}x^3\right]_0^a \quad (=\tfrac{7}{6}ka^3)$	A1	
	$\int \rho\,dx = k\int_0^a (a+2x)\,dx \quad = k\left[ax + x^2\right]_0^a$	B1	for $...\left[ax+x^2\right]_0^a$
	$= 2ka^2$		
	$\bar{x} = \dfrac{\tfrac{7}{6}ka^3}{2ka^2}$	M1	*Dependent on first M1*
	$= \tfrac{7}{12}a$	A1 **5**	Accept $0.583a$
2 (i)	$I = \tfrac{1}{2}\times 8\times 0.15^2 \quad (=0.09\text{ kg m}^2)$	B1	
	Using $\omega_2^{\,2} = \omega_1^{\,2} + 2\alpha\theta$ $25^2 = 10^2 + 2\alpha\times 75$	M1A1	
	$\alpha = 3.5\text{ rad s}^{-2}$		
	Couple is $I\alpha = 0.09\times 3.5$	M1	
	$= 0.315\text{ N m}$	A1 ft **5**	ft from wrong I and / or α, but ft requires M1M1
	OR Increase in KE is $\tfrac{1}{2}\times 0.09\times(25^2 - 10^2)$ M1A1 ft $= 23.625\text{ J}$		
	M1		WD by couple is $L\times 75$
	Couple is $\dfrac{23.625}{75} = 0.315\text{ N m}$ A1 ft		ft requires M1M1
(ii)	By conservation of angular momentum	M1	Using angular momentum
	$(0.09 + I_2)\times 9 = 0.09\times 25$	A1 ft	
	$I_2 = 0.16\text{ kg m}^2$	A1 **3**	
3	$\int_1^2 \dfrac{1}{x^2}\,dx = \left[-\dfrac{1}{x}\right]_1^2$	M1	
	$= \tfrac{1}{2}$	A1	
	Mass per unit area $\rho = 48\text{ kg m}^{-2}$	B1	
	$I = \int \tfrac{4}{3}(\rho y\,\delta x)(\tfrac{1}{2}y)^2$	M1	For integral of y^3
	$= \int \tfrac{1}{3}\rho y^3\,dx$	A1	
	$= \tfrac{1}{3}\rho\int_1^2 \dfrac{1}{x^6}\,dx$	A1 ft	
	$= \tfrac{1}{3}\rho\left[-\dfrac{1}{5x^5}\right]_1^2$	A1	For correct integration of $\dfrac{1}{x^6}$
	$= \dfrac{31}{480}\rho = \dfrac{31}{480}\times 48$		
	$= 3.1\text{ kg m}^2$	A1 **8**	

4 (i)	$RC = 2a\cos\theta$	B1	or $RC^2 = 2a^2 + 2a^2\cos 2\theta$
	EPE $= \dfrac{5mg}{2a}(2a\cos\theta)^2$	M1	
	GPE $= mga\sin 2\theta + 2mg(2a\sin 2\theta)$	M1	One term sufficient for M1
	$V = 10mga\cos^2\theta + 5mga\sin 2\theta$	A1	
	$\dfrac{dV}{d\theta} = -20mga\cos\theta\sin\theta + 10mga\cos 2\theta$ $= -10mga\sin 2\theta + 10mga\cos 2\theta$	B1	Correct differentiation of $\cos^2\theta$ (or $\cos 2\theta$) and $\sin 2\theta$
	For equilibrium, $10mga(\cos 2\theta - \sin 2\theta) = 0$ $\tan 2\theta = 1$	M1	For using $\dfrac{dV}{d\theta} = 0$
	$\theta = \frac{1}{8}\pi$	A1 **7**	Accept $22\frac{1}{2}°$, 0.393
(ii)	$\dfrac{d^2V}{d\theta^2} = -20mga\cos 2\theta - 20mga\sin 2\theta$	B1 ft	
	When $\theta = \frac{1}{8}\pi$, $\dfrac{d^2V}{d\theta^2}$ $(= -20\sqrt{2}\,mga\ ) < 0$	M1	Determining the sign of V''
	Hence the equilibrium is unstable	A1 **3**	Correctly shown
	OR Other method for determining whether V has a maximum or a minimum — M1 Correct determination — A1 ft Equilibrium is unstable — A1		Correctly shown
5 (i)	$I = \frac{1}{3}(20)(0.3^2 + 0.9^2) + 20\times 0.9^2$	M1 M1	MI of lamina about any axis Use of parallel (or perp) axes rule
	$= 22.2\ \text{kg m}^2$	A1 (ag) **3**	Correctly obtained
	OR $I = \frac{1}{3}\times 20\times 0.3^2 + \frac{4}{3}\times 20\times 0.9^2$ — M1M1 $= 22.2\ \text{kg m}^2$ — A1		As above
(ii)	Total moment is $20\times 9.8\times 0.9\cos\theta - 44.1$ Angular acceleration is zero when moment is zero	M1 M1	
	$\cos\theta = \dfrac{44.1}{20\times 9.8\times 0.9} = 0.25$	A1 (ag) **3**	
(iii)	Maximum angular speed when $\cos\theta = 0.25$	M1	
	$\theta = 1.318$ Work done against couple is 44.1×1.318	A1	
	By work energy principle, $\frac{1}{2}I\omega^2 = 20\times 9.8\times 0.9\sin\theta - 44.1\theta$	M1 A1 ft	Equation involving work, KE and PE
	$\omega = 3.19\ \text{rad s}^{-1}$	A1 **5**	

6 (i)	As viewed from P  $\sin\alpha = \dfrac{1790}{7400}$ $\alpha = 14.0°$ Bearing of relative velocity is $50 - \alpha = 036°$ or $50 + \alpha = 064°$	M1 A1 (ag) B1 ft **3**	For 64 or ft $50 + \alpha$
(ii)	Velocity diagram  $\dfrac{\sin\beta}{7} = \dfrac{\sin 106}{10}$ $\beta = 42.3°$ Bearing of $\mathbf{v}_Q$ is $36 + \beta = 078.3°$	B1 M1 A1 A1 **4**	Correct diagram *(may be implied)* *Correct triangle must be intended* Accept 78°
(iii)	$\dfrac{w}{\sin 31.7} = \dfrac{10}{\sin 106}$ $w = 5.47 \text{ m s}^{-1}$	M1 A1 **2**	*If cosine rule is used, M1 also requires an attempt at solving the quadratic*
	Alternative for (ii) and (iii) $\begin{pmatrix} w\sin 36 \\ w\cos 36 \end{pmatrix} = \begin{pmatrix} 10\sin\theta \\ 10\cos\theta \end{pmatrix} - \begin{pmatrix} 7\sin 110 \\ 7\cos 110 \end{pmatrix}$ B1 Obtaining an equation in θ only, and solving it M1 $\theta = 78.3°$ A2 Obtaining an equation in w only, and solving it M1 $w = 5.47 \text{ m s}^{-1}$ A1		e.g. $10\sin\theta - 7.2654\cos\theta = 8.3173$ or A1A1 if another angle found first
(iv)	$QC = \sqrt{7400^2 - 1790^2} = 7180$ m Time taken is $\dfrac{7180}{5.468}$ $= 1310$ s	M1 M1 A1 ft **3**	(Or M2 for other complete method for finding the time) For attempt at relative distance $\div w$ (not awarded for $7400 \div w$) or 21.9 minutes ft is $7180 \div w$
(v)	Bearing of CP is $90 + 36 = 126°$	B1 **1**	

7 (i)	$I = \frac{1}{3}m(3a)^2 + m(2a)^2$ $= 7ma^2$ $mg(2a\sin\theta) = I\alpha$ $\alpha = \dfrac{2g\sin\theta}{7a}$	M1 A1 M1 A1 **4**	Using parallel axes rule
(ii)	By conservation of energy $\frac{1}{2}I\omega^2 = mg(2a\cos\frac{1}{3}\pi - 2a\cos\theta)$ $\frac{7}{2}ma^2\omega^2 = mga(1-2\cos\theta)$ $\omega = \sqrt{\dfrac{2g(1-2\cos\theta)}{7a}}$	M1 A1 A1 (ag) **3**	Equation involving KE and PE Need to see how $\frac{1}{3}\pi$ is used Correctly obtained
(iii)	$mg\cos\theta - R = m(2a\omega^2)$ $R = mg\cos\theta - \frac{4}{7}mg(1-2\cos\theta)$ $= \frac{1}{7}mg(15\cos\theta - 4)$	M1 A1 A1	For radial acceleration $r\omega^2$
	$mg\sin\theta - S = m(2a\alpha)$ $S = mg\sin\theta - \frac{4}{7}mg\sin\theta$ $= \frac{3}{7}mg\sin\theta$	M1 A1 A1 **6**	For transverse acceleration $r\alpha$
	OR $S(2a) = I_G\alpha = (3ma^2)\alpha$ M1A1 $S = \frac{3}{7}mg\sin\theta$ A1		Must use I_G
(iv)	When $\cos\theta = \frac{1}{3}$, $\sin\theta = \frac{\sqrt{8}}{3}$, $\tan\theta = \sqrt{8}$ $R = \frac{1}{7}mg$, $S = \frac{\sqrt{8}}{7}mg$ Angle with R is $\tan^{-1}\dfrac{S}{R} = \tan^{-1}\sqrt{8} = \theta$ so the resultant force is vertical Magnitude is $\sqrt{R^2 + S^2}$ $= \frac{1}{7}mg\sqrt{1+8} = \frac{3}{7}mg$	M1 A1 M1 A1 **4**	
	OR When resultant force is F vertically upwards $S = F\sin\theta$, hence $F = \frac{3}{7}mg$ M1A1 $R = F\cos\theta$, so $\frac{1}{7}mg(15\cos\theta - 4) = \frac{3}{7}mg\cos\theta$ M1 $\cos\theta = \frac{1}{3}$ A1		
	OR Horizontal force is $R\sin\theta - S\cos\theta$ $= \frac{1}{7}mg(15\cos\theta - 4)\sin\theta - \frac{3}{7}mg\sin\theta\cos\theta$ M1 $= \frac{1}{7}mg\sin\theta(12\cos\theta - 4)$ $= 0$ when $\cos\theta = \frac{1}{3}$ A1 Vertical force is $R\cos\theta + S\sin\theta$ $= \frac{1}{7}mg \times \frac{1}{3} + \frac{3}{7}mg \times \frac{8}{9} = \frac{3}{7}mg$ M1A1		