

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MATHEMATICS**

**2640**

**Mechanics 4**

Friday

**28 MAY 2004**

Afternoon

1 hour 20 minutes

Additional materials:

Answer booklet

Graph paper

List of Formulae (MF8)

**TIME** 1 hour 20 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use  $9.8 \text{ m s}^{-2}$ .
- You are permitted to use a graphic calculator in this paper.

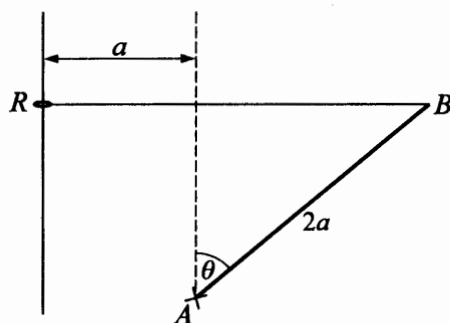
**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

---

**This question paper consists of 3 printed pages and 1 blank page.**

- 1 Two flywheels  $P$  and  $Q$  are rotating, in opposite directions, about the same fixed axis. The angular speed of  $P$  is  $25 \text{ rad s}^{-1}$  and the angular speed of  $Q$  is  $30 \text{ rad s}^{-1}$ . The flywheels lock together, and after this they both rotate with angular speed  $10 \text{ rad s}^{-1}$  in the direction in which  $P$  was originally rotating. The moment of inertia of  $P$  about the axis is  $0.64 \text{ kg m}^2$ . Find the moment of inertia of  $Q$  about the axis. [4]
- 2 A uniform rectangular lamina has mass  $m$  and sides of length  $3a$  and  $4a$ , and rotates freely about a fixed horizontal axis. The axis is perpendicular to the lamina and passes through a corner. The lamina makes small oscillations in its own plane, as a compound pendulum.
- (i) Find the moment of inertia of the lamina about the axis. [3]
- (ii) Find the approximate period of the small oscillations. [3]
- 3 The region between the curve  $y = x\sqrt{3-x}$  and the  $x$ -axis for  $0 \leq x \leq 3$  is rotated through  $2\pi$  radians about the  $x$ -axis to form a uniform solid of revolution. Find the  $x$ -coordinate of the centre of mass of this solid. [6]
- 4 A uniform solid sphere, of mass  $14 \text{ kg}$  and radius  $0.25 \text{ m}$ , is rotating about a fixed axis which is a diameter of the sphere. A couple of constant moment  $4.2 \text{ N m}$  about the axis, acting in the direction of rotation, is applied to the sphere.
- (i) Find the angular acceleration of the sphere. [3]
- During a time interval of 30 seconds the sphere rotates through 7500 radians.
- (ii) Find the angular speed of the sphere at the start of the time interval. [2]
- (iii) Find the angular speed of the sphere at the end of the time interval. [2]
- (iv) Find the work done by the couple during the time interval. [2]
- 5 Two aircraft  $A$  and  $B$  are flying horizontally at the same height.  $A$  has constant velocity  $240 \text{ m s}^{-1}$  in the direction with bearing  $025^\circ$ , and  $B$  has constant velocity  $185 \text{ m s}^{-1}$  in the direction with bearing  $310^\circ$ .
- (i) Find the magnitude and direction of the velocity of  $A$  relative to  $B$ . [5]
- Initially  $A$  is  $4500 \text{ m}$  due west of  $B$ . For the instant during the subsequent motion when  $A$  and  $B$  are closest together, find
- (ii) the distance between  $A$  and  $B$ , [3]
- (iii) the bearing of  $A$  from  $B$ . [2]



A uniform rod  $AB$ , of mass  $m$  and length  $2a$ , is free to rotate in a vertical plane about a fixed horizontal axis through  $A$ . A light elastic string has natural length  $a$  and modulus of elasticity  $mg$ ; one end is attached to  $B$  and the other end is attached to a light ring  $R$  which can slide along a smooth vertical wire. The wire is in the same vertical plane as  $AB$ , and is at a distance  $a$  from  $A$ . The rod  $AB$  makes an angle  $\theta$  with the upward vertical, where  $0 < \theta < \frac{1}{2}\pi$  (see diagram).

(i) Give a reason why the string  $RB$  is always horizontal. [1]

(ii) By considering potential energy, find the value of  $\theta$  for which the system is in equilibrium. [6]

(iii) Determine whether this position of equilibrium is stable or unstable. [4]

7 A uniform rod  $AB$  has mass  $m$  and length  $2a$ . The point  $P$  on the rod is such that  $AP = \frac{2}{3}a$ .

(i) Prove by integration that the moment of inertia of the rod about an axis through  $P$  perpendicular to  $AB$  is  $\frac{4}{9}ma^2$ . [4]

The axis through  $P$  is fixed and horizontal, and the rod can rotate without resistance in a vertical plane about this axis. The rod is released from rest in a horizontal position. Find, in terms of  $m$  and  $g$ ,

(ii) the force acting on the rod at  $P$  immediately after the release of the rod, [5]

(iii) the force acting on the rod at  $P$  at an instant in the subsequent motion when  $B$  is vertically below  $P$ . [5]