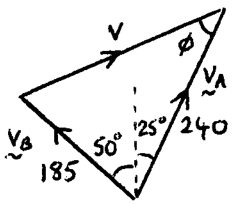
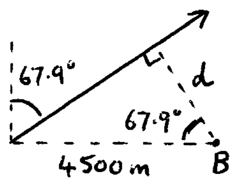
OCR
RECOGNISING ACHIEVEMENT

2640 Mechanics 4

June 2004

Mark Scheme

1	$0.64\times 25 - I\times 30 = (0.64+I)\times 10$ $I = 0.24\text{ kg m}^2$	M1 A1A1 A1 **4**	Use of angular momentum For LHS and RHS
2 (i)	$I = \frac{4}{3}m(\frac{3}{2}a)^2 + \frac{4}{3}m(2a)^2$ $= \frac{25}{3}ma^2$	B1 M1 A1 **3**	For either term Use of perpendicular axes rule
	OR $I = \frac{1}{3}m\{(\frac{3}{2}a)^2 + (2a)^2\} + m(\frac{5}{2}a)^2$ B1 M1 $= \frac{25}{3}ma^2$ A1		For $\frac{1}{3}m\{(\frac{3}{2}a)^2 + (2a)^2\}$ Use of parallel axes rule
(ii)	Period is $2\pi\sqrt{\frac{I}{mgh}}$ $= 2\pi\sqrt{\frac{\frac{25}{3}ma^2}{mg\frac{5}{2}a}}$ $= 2\pi\sqrt{\frac{10a}{3g}}$	M1 A1 ft A1 **3**	or $(-)\, mgh\sin\theta = I\ddot{\theta}$ or $-mg(\frac{5}{2}a)\theta \approx \frac{25}{3}ma^2\ddot{\theta}$
3	$\int \pi xy^2\,\mathrm{d}x = \int_0^3 \pi x^3(3-x)\,\mathrm{d}x$ $= \pi\left[\frac{3}{4}x^4 - \frac{1}{5}x^5\right]_0^3 \quad (=12.15\pi)$ $\int \pi y^2\,\mathrm{d}x = \int_0^3 \pi x^2(3-x)\,\mathrm{d}x$ $= \pi\left[x^3 - \frac{1}{4}x^4\right]_0^3 \quad (=6.75\pi)$ $\bar{x} = \frac{12.15\pi}{6.75\pi}$ $=1.8$	M1 A1 M1 A1 M1 A1 **6**	*(π may be omitted throughout)* *Dependent on previous M1M1*
4 (i)	$I = \frac{2}{5}\times 14\times 0.25^2 \quad (=0.35)$ $4.2 = I\alpha$ $\alpha = 12\text{ rad s}^{-2}$	B1 M1 A1 **3**	
(ii)	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2;\quad 7500 = \omega_1\times 30 + \frac{1}{2}\times 12\times 30^2$ $\omega_1 = 70\text{ rad s}^{-1}$	M1 A1 ft **2**	 Ft $250 - 15\alpha$
(iii)	$\omega_2 = \omega_1 + \alpha t;\quad \omega_2 = 70 + 12\times 30$ $\omega_2 = 430\text{ rad s}^{-1}$	M1 A1 ft **2**	 Ft $250 + 15\alpha$
(iv)	Work done is $L\theta = 4.2\times 7500$ $= 31500\text{ J}$	M1 A1 **2**	Or $\frac{1}{2}I(\omega_2^2 - \omega_1^2)$

5 (i)		B1	Velocity triangle *If wrong triangle used, then* *B0 M1A0 M1A0 for equivalent work* *Full ft in (ii) and (iii)*
	$v^2 = 240^2 + 185^2 - 2 \times 240 \times 185 \cos 75$	M1	
	$v = 262 \text{ m s}^{-1}$	A1	
	$\frac{\sin\phi}{185} = \frac{\sin 75}{262.4}$	M1	
	$\phi = 42.9°$ Bearing is $25 + \phi = 067.9°$	A1 **5**	Accept 68°, 67.8° *Allow other clearly stated descriptions of direction*
	OR $\mathbf{v} = \begin{pmatrix} 240\sin 25 \\ 240\cos 25 \end{pmatrix} - \begin{pmatrix} 185\sin 310 \\ 185\cos 310 \end{pmatrix}$ M1 $= \begin{pmatrix} 243 \\ 98.6 \end{pmatrix}$ A1 M1 $v = \sqrt{243^2 + 98.6^2} = 262$ A1 Bearing is $\tan^{-1}\frac{243}{98.6} = 67.9°$ A1		Finding magnitude *or* angle
(ii)	As viewed from *B* 	M1	Relative displacement diagram
	$d = 4500\cos 67.9$	M1	
	$= 1690$ m	A1 ft **3**	Ft from bearing in (i)
	OR $\overrightarrow{BA} = \begin{pmatrix} 243t - 4500 \\ 98.6t \end{pmatrix}$ M1 $BA^2 = (243t - 4500)^2 + (98.6t)^2$ is minimum when $t = \frac{4500 \times 243}{243^2 + 98.6^2}$ $(= 15.9)$ M1 Then $BA = 1690$ A1 ft		Ft from relative velocity in (i)
(iii)	Bearing is $270 + 67.9$ $= 338°$	M1 A1 ft **2**	Ft from bearing in (i) *SR* B1 ft for 158°
	OR $\overrightarrow{BA} = \begin{pmatrix} -636 \\ 1570 \end{pmatrix}$ Bearing is $360 - \tan^{-1}\frac{636}{1570}$ M1 $= 338°$ A1 ft		Ft from relative velocity in (i)

6 (i)	e.g. R has no weight / mass There is no friction There is no vertical force at R String has minimum length / elastic energy	B1 **1**	For any one contributory reason
(ii)	$V = \frac{1}{2}\left(\frac{mg}{a}\right)(2a\sin\theta)^2 + mga\cos\theta$ $= mga(2\sin^2\theta + \cos\theta)$ $\frac{dV}{d\theta} = mga(4\sin\theta\cos\theta - \sin\theta)$ For equilibrium, $mga\sin\theta(4\cos\theta - 1) = 0$ $\theta = 1.32$ (*or* 75.5°)	B2 B1 ft B1 ft M1 A1 **6**	Give B1 for correct EE or PE Diffn of $\sin^2\theta$ (or $\cos^2\theta$) Diffn of $\cos\theta$ (or $\sin\theta$) Allow $\cos^{-1}\frac{1}{4}$, 76° *Ignore $\theta = 0$ if stated* *Marks can be awarded in (iii) if not earned in (ii)* *SR If done by taking moments, M1A1 for* $\frac{mg(2a\sin\theta)}{a}(2a\cos\theta) = mg(a\sin\theta)$ *A1 for $\theta = 1.32$ (Max 3 out of 6)* *These marks may only **replace** all other marks in (ii)*
(iii)	$\frac{d^2V}{d\theta^2} = mga\cos\theta(4\cos\theta - 1) - 4mga\sin^2\theta$ When $\cos\theta = \frac{1}{4}$, $\frac{d^2V}{d\theta^2} = -4mga\sin^2\theta$ ($= -\frac{15}{4}mga$) < 0 Equilibrium is unstable	B2 ft M1 A1 **4**	Give B1 if just one error *Only B1 ft if work is simpler* Fully correct working only
	OR When $\theta < 1.32$, $\frac{dV}{d\theta} > 0$ B1 ft When $\theta > 1.32$, $\frac{dV}{d\theta} < 0$ B1 ft V has a maximum M1 Equilibrium is unstable A1		Fully correct working, and convincing demonstration of signs above

7 (i)	$I = \sum (\rho\,\delta x)\,x^2$	M1	
	$= \int_{-\frac{2}{3}a}^{\frac{4}{3}a} \frac{m}{2a} x^2\,\mathrm{d}x$	A1	
	$= \left[\frac{mx^3}{6a}\right]_{-\frac{2}{3}a}^{\frac{4}{3}a} = \frac{32}{81}ma^2 + \frac{4}{81}ma^2$	M1	*Dependent on previous M1*
	$= \frac{4}{9}ma^2$	A1 (ag) **4**	
	OR $I_G = \sum (\rho\,\delta x)\,x^2 = \int_{-a}^{a} \frac{m}{2a}x^2\,\mathrm{d}x$ M1A1 $I = \frac{1}{6}ma^2 + \frac{1}{6}ma^2 + m(\frac{1}{3}a)^2$ M1 $= \frac{4}{9}ma^2$ A1 (ag)		Or $2\int_0^a \frac{m}{2a}x^2\,\mathrm{d}x$ *Dependent on previous M1*
(ii)	$mg(\frac{1}{3}a) = (\frac{4}{9}ma^2)\alpha$	M1	Use of $L = I\alpha$
	$\alpha = \frac{3g}{4a}$	A1	
	$mg - R = m(\frac{1}{3}a)\alpha$	M1	For force $= m(\frac{1}{3}a)\alpha$
	$mg - R = \frac{1}{4}mg$	A1 ft	
	$R = \frac{3}{4}mg$ vertically upwards	A1 **5**	Direction must be indicated
(iii)	$\frac{1}{2}(\frac{4}{9}ma^2)\omega^2 = mg(\frac{1}{3}a)$	M1	Use of $\frac{1}{2}I\omega^2$
	$\omega^2 = \frac{3g}{2a}$	A1	
	$S - mg = m(\frac{1}{3}a)\omega^2$	M1	For force $= m(\frac{1}{3}a)\omega^2$
	$S - mg = \frac{1}{2}mg$	A1 ft	
	$S = \frac{3}{2}mg$ vertically upwards	A1 **5**	Direction must be indicated
	Alternative for (ii) and (iii) $\ddot{\theta} = \frac{3g\cos\theta}{4a}$ M1A1 $\dot{\theta}^2 = \frac{3g\sin\theta}{2a}$ M1A1 $mg\cos\theta - Y = m(\frac{1}{3}a)\frac{3g\cos\theta}{4a}$ M1A1 ft $Y = \frac{3}{4}mg\cos\theta$ $X - mg\sin\theta = m(\frac{1}{3}a)\frac{3g\sin\theta}{2a}$ M1A1 ft $X = \frac{3}{2}mg\sin\theta$ When $\theta = 0$, $X = 0$, $Y = \frac{3}{4}mg$ A1 When $\theta = \frac{1}{2}\pi$, $X = \frac{3}{2}mg$, $Y = 0$ A1		(θ measured from horizontal X is radial component Y is transverse component)