OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2640

Mechanics 4

Thursday **22 MAY 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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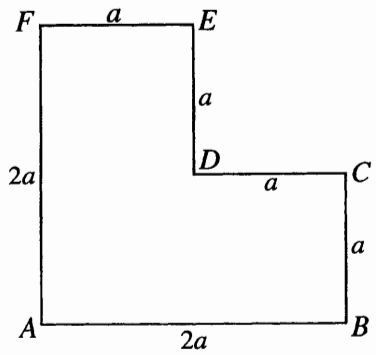
1 A propeller shaft has constant angular acceleration. It turns through 160 radians as its angular speed increases from 15 rad s^{-1} to 25 rad s^{-1}. Find

(i) the angular acceleration of the propeller shaft, [2]

(ii) the time taken for this increase in angular speed. [2]

2



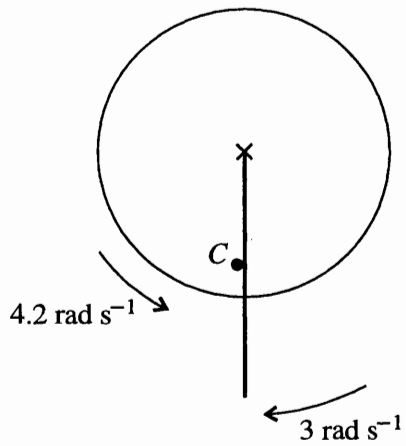
The diagram shows a uniform lamina $ABCDEF$ in which all the corners are right angles. The mass of the lamina is $3m$.

(i) Show that the moment of inertia of the lamina about AB is $3ma^2$. [3]

(ii) Find the moment of inertia of the lamina about an axis perpendicular to the lamina and passing through A. [2]

3 A uniform rod, of mass 0.75 kg and length 1.6 m, rotates in a vertical plane about a fixed horizontal axis through one end. A frictional couple of constant moment opposes the motion. The rod is released from rest in a horizontal position and, when the rod is first vertical, its angular speed is 3 rad s^{-1}.

(i) Find the magnitude of the frictional couple. [4]



A disc is rotating about the same axis. The moment of inertia of the disc about the axis is 0.56 kg m^2. When the rod is vertical, the disc has angular speed 4.2 rad s^{-1} in the opposite direction to that of the rod (see diagram). At this instant the rod hits a magnetic catch C on the disc and becomes attached to the disc.

(ii) Find the angular speed of the rod and disc immediately after they have become attached. [3]

4 A cruise ship C is sailing due north at a constant speed of $12\,\text{m s}^{-1}$. A boat B, initially 2000 m due west of C, sails with constant speed $11\,\text{m s}^{-1}$ on a straight line course which takes it as close as possible to C.

(i) Find the bearing of the direction in which B sails. [4]

(ii) Find the shortest distance between B and C in the subsequent motion. [4]

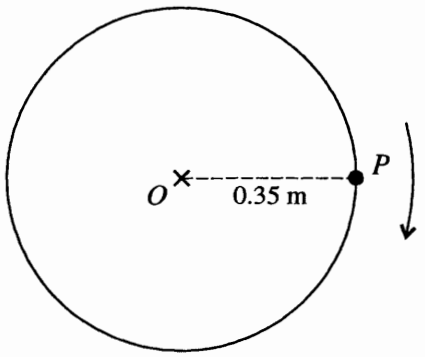
5 The region bounded by the x-axis, the line $x = 8$ and the curve $y = x^{\frac{1}{3}}$ for $0 \leqslant x \leqslant 8$, is rotated through 2π radians about the x-axis to form a uniform solid of revolution. The unit of length is the metre, and the density of the solid is $350\,\text{kg m}^{-3}$.

(i) Show that the mass of the solid is 6720π kg. [3]

(ii) Find the x-coordinate of the centre of mass of the solid. [3]

(iii) Find the moment of inertia of the solid about the x-axis. [4]

6



A wheel consists of a uniform circular disc, with centre O, mass 0.08 kg and radius 0.35 m, with a particle P of mass 0.24 kg attached to a point on the circumference. The wheel is rotating without resistance in a vertical plane about a fixed horizontal axis through O (see diagram).

(i) Find the moment of inertia of the wheel about the axis. [3]

(ii) Find the distance of the centre of mass of the wheel from the axis. [2]

At an instant when OP is horizontal and the angular speed of the wheel is $5\,\text{rad s}^{-1}$, find

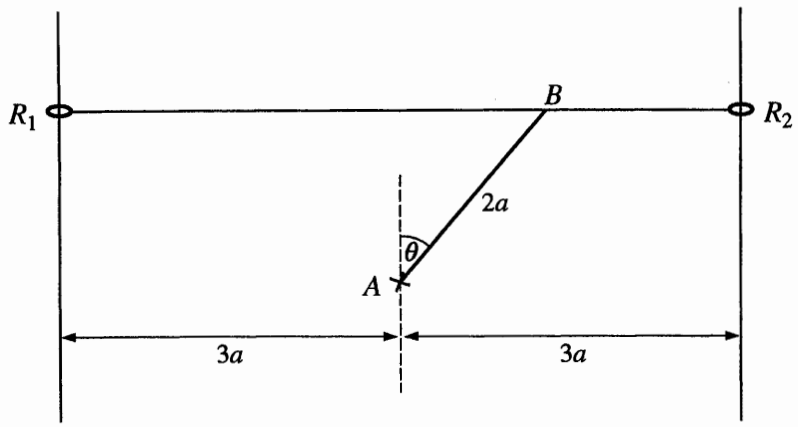
(iii) the angular acceleration of the wheel, [2]

(iv) the magnitude of the force acting on the wheel at O. [6]

[Question 7 is printed overleaf.]

[Turn over

7



A uniform rod AB, of mass m and length $2a$, is pivoted to a fixed point at A and is free to rotate in a vertical plane. Two fixed vertical wires in this plane are a distance $6a$ apart and the point A is half-way between the two wires. Light smooth rings R_1 and R_2 slide on the wires and are connected to B by light elastic strings, each of natural length a and modulus of elasticity $\frac{1}{4}mg$. The strings BR_1 and BR_2 are always horizontal and the angle between AB and the upward vertical is θ, where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ (see diagram).

(i) Taking A as the reference level for gravitational potential energy, show that the total potential energy of the system is

$$mga(1 + \cos\theta + \sin^2\theta).$$ [5]

(ii) Given that $\theta = 0$ is a position of stable equilibrium, find the approximate period of small oscillations about this position. [8]