OCR Mechanics 4 January 2004

1

$\omega_1^{\,2} = \omega_0^{\,2} + 2\dot{\omega}\theta$ (equivalent of the linear $v^2 = u^2 + 2as$)

$8^2 = 5^2 + 2\times {}^{-}0\cdot 3\times\theta$

$\theta = \mathbf{65}$ rad [2]

For the final complete revolution …

$\theta = \omega_1 t - \frac{1}{2}at^2$ ("$s = vt - \frac{1}{2}at^2$")

$2\pi = 0 - \frac{1}{2}\times {}^{-}0\cdot 3\times t^2$

$t^2 = {}^{4\pi}\!/_{0\cdot 3}$

$t = 6\cdot 4720... = \mathbf{6\cdot 47\ s}$ [3]

2

mass of rod $= \int_0^2 0\cdot 7 - 0\cdot 3x \;\mathrm{d}x = \left[0\cdot 7x - 0\cdot 15x^2\right]_0^2 = 0\cdot 8$

v

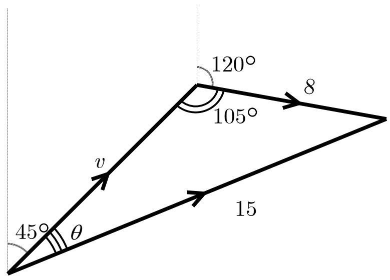
15

$0\cdot 8\bar{x} = \int_0^2 x\; 0\cdot 7 - 0\cdot 3x\;\mathrm{d}x = \left[0\cdot 35x^2 - 0\cdot 1x^3\right]_0^2 = 0\cdot 6$

$\bar{x} = \mathbf{0\cdot 75}$ [5]

3

${}_{\text{speedboat}}\mathbf{v}_{\text{ship}} + {}_{\text{ship}}\mathbf{v}_{\text{sea}} = {}_{\text{speedboat}}\mathbf{v}_{\text{sea}}$



Sine Rule …

$\frac{\sin\theta}{8} = \frac{\sin 105^\circ}{15}$

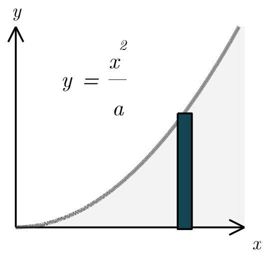
$\theta = 31\cdot 008...$

So the **bearing of the speedboat** $= 45^\circ + \theta = 076\cdot 008... = \mathbf{076\cdot 0}^\circ$ [4]

$\frac{v}{\sin\ 75^\circ - \theta} = \frac{15}{\sin\ 105^\circ}$

$v = 10\cdot 7858... = \mathbf{10\cdot 8\ ms^{-1}}$ [3]

4



area of lamina $= \int_0^a {}^{x^2}\!/_a\;\mathrm{d}x = \frac{1}{3}a^2$

mass of 'elemental strip' $= \rho y\delta x = \frac{3m}{a^2}\cdot\frac{x^2}{a}\cdot\delta x$

M.o.I $= \int_0^a \frac{4}{3}\ \frac{1}{2}y\ ^2\cdot\frac{3mx^2}{a^3}\mathrm{d}x = \frac{m}{a^5}\int_0^a x^6\,\mathrm{d}x = \frac{1}{7}\boldsymbol{ma^2}$ (show)

[7]

5

$$I_C = \tfrac{1}{2}mr^2 = \tfrac{1}{2}\times 4\times 0\cdot 6^2 = 0\cdot 72$$

parallel axes theorem …

$$I_A = I_c + m\times 0\cdot 4^2 = \mathbf{1\cdot 36\ \ kg\,m^2}$$ [2]

at the instant of release …

$$C = I\alpha$$

$$39\cdot 2\times 0\cdot 4\ -\ 4\cdot 8 = 1\cdot 36\times\alpha$$

$$\boldsymbol{\alpha} = \mathbf{8\cdot 00\ \ rad\,s^{-1}}$$

[3]

energy considerations …

gain in K.E. = loss in G.P.E. - work done by friction

$$\tfrac{1}{2}I\omega^2 = 4\times 9\cdot 8\times 0\cdot 4 - 4\cdot 8\times\tfrac{\pi}{2}$$

$$\omega^2 = 11\cdot 9708...$$

$$\omega = 3\cdot 45989... = \mathbf{3\cdot 46\ \ rad\,s^{-1}}$$

[4]

196

6

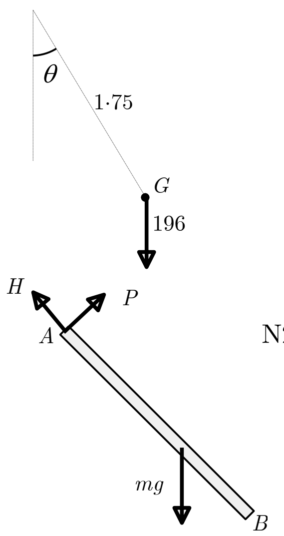
$M\,\overline{x} = \sum m_i\,x_i$ $\qquad 20\,\overline{x} = 15\times 1\cdot 4\ +\ 5\times 2\cdot 8$ $\qquad \boldsymbol{\overline{x} = 1\cdot 75}$

[2]

$$I = \tfrac{4}{3}ml^2 + 5\ \ 2l^{\ 2} = \tfrac{4}{3}\times 15\times 1\cdot 4^2\ +\ 20\times 1\cdot 4^2 = \mathbf{78\cdot 4\ \ kg\ m^2}$$

[2]

Hmg P



$$C = I\alpha$$

$$196\ \ 1\cdot 75\sin\theta\ = 78\cdot 4\times 3\cdot 5$$

$\boldsymbol{\sin\theta = 0\cdot 8}$ (show)

[3]

N2 radially BA $\qquad H - mg\cos\theta = m\omega^2 r$

$$H = 20\times 9\cdot 8\times 0\cdot 6 + 20\times 1\cdot 2^2\times 1\cdot 75 = \mathbf{168\ \ N}$$

N2 normal $\qquad mg\sin\theta - P = mr\dot{\omega}$

$$P = 20\times 9\cdot 8\times 0\cdot 8 - 20\times 1\cdot 75\times 3\cdot 5 = \mathbf{34\cdot 3}$$

[6]

7 Potential Energy Function (relative to position with $\theta = 0$)

[2]

$$V = mga\;\; 1 - \cos\theta\;\; + \tfrac{1}{2}\cdot \tfrac{mg}{2a}\cdot \;\; a\cos\theta\;\;^{2} - a^{2}$$

$$= \tfrac{1}{4}mga\;\; \cos^{2}\theta - 4\cos\theta + 3 \qquad\qquad {}^{-}\tfrac{\pi}{2} \le \theta \le \tfrac{\pi}{2}$$

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = \tfrac{1}{2}mga\;\; {}^{-}\cos\theta\sin\theta + 2\sin\theta\;\; = \tfrac{1}{2}mga\sin\theta\;\; 2 - \cos\theta$$

$$\left.\frac{\mathrm{d}V}{\mathrm{d}\theta}\right|_{\theta=0} = 0$$ and therefore $\theta = 0$ is an equilibrium position.

$$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = \tfrac{1}{2}mga\;\; 2\cos\theta + \sin^{2}\theta - \cos^{2}\theta \qquad\qquad \left.\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}\right|_{\theta=0} = \tfrac{1}{2}mga > 0$$

So the potential energy function has a local **minimum** at $\theta = 0$ and therfore we have a position of **stable equilibrium**. [7]

conservation of mechanical energy …

$$\tfrac{1}{4}mga\;\; \cos^{2}\theta - 4\cos\theta + 3\;\; + \tfrac{1}{2}m\;\; a\dot{\theta}\;\;^{2} = \text{constant}$$

differentiating …

$$\tfrac{1}{2}mga\dot{\theta}\;\; 2\sin\theta - \sin\theta\cos\theta\;\; + ma^{2}\dot{\theta}\ddot{\theta} = 0$$

$$\ddot{\theta} = {}^{-}\frac{g}{2a}\sin\theta\;\; 2 - \cos\theta$$

for small oscillations …… $\sin\theta \approx \theta$ and $\cos\theta \approx 1$

and so …. $\ddot{\theta} \approx {}^{-}\left(\frac{g}{2a}\right)\theta\,.$

The motion is therefore approximately SHM with period $T = \dfrac{2\pi}{\sqrt{g/2a}} = \boldsymbol{2\pi\sqrt{\dfrac{2a}{g}}}$

[7]