OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS — 2640

Mechanics 4

Monday **21 JANUARY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\ \text{m s}^{-2}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

 Registered Charity Number: 1066969 [Turn over

1 A wheel rotating about a fixed axis is slowing down with constant angular deceleration. Initially the angular speed is 24 rad s^{-1}. In the first 5 seconds the wheel turns through 96 radians.

(i) Find the angular deceleration. [2]

(ii) Find the total angle the wheel turns through before coming to rest. [2]

2 A uniform solid of revolution is formed by rotating the region bounded by the x-axis, the line $x = 1$ and the curve $y = x^2$ for $0 \leqslant x \leqslant 1$, about the x-axis. The units are metres, and the density of the solid is 5400 kg m^{-3}. Find the moment of inertia of this solid about the x-axis. [5]

3 A uniform rectangular lamina $ABCD$ of mass 0.6 kg has sides $AB = 0.4$ m and $AD = 0.3$ m. The lamina is free to rotate about a fixed horizontal axis which passes through A and is perpendicular to the lamina.

(i) Find the moment of inertia of the lamina about the axis. [3]

(ii) Find the approximate period of small oscillations in a vertical plane. [3]

4 A uniform circular disc has mass m, radius a and centre C. The disc is free to rotate in a vertical plane about a fixed horizontal axis passing through a point A on the disc, where $CA = \frac{1}{3}a$.

(i) Find the moment of inertia of the disc about this axis. [2]

The disc is released from rest with CA horizontal.

(ii) Find the initial angular acceleration of the disc. [2]

(iii) State the direction of the force acting on the disc at A immediately after release, and find its magnitude. [4]

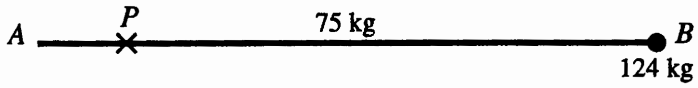
5 The region bounded by the x-axis, the y-axis, the line $x = \ln 5$ and the curve $y = e^x$ for $0 \leqslant x \leqslant \ln 5$, is occupied by a uniform lamina.

(i) Show that the centre of mass of this lamina has x-coordinate

$$\frac{5}{4}\ln 5 - 1.$$ [5]

(ii) Find the y-coordinate of the centre of mass. [3]

6



An arm on a fairground ride is modelled as a uniform rod AB, of mass 75 kg and length 7.2 m, with a particle of mass 124 kg attached at B. The arm can rotate about a fixed horizontal axis perpendicular to the rod and passing through the point P on the rod, where $AP = 1.2$ m.

(i) Show that the moment of inertia of the arm about the axis is $5220\,\text{kg m}^2$. [3]

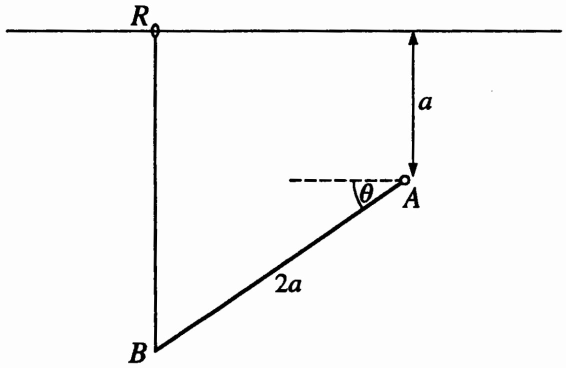
(ii) The arm is released from rest with AB horizontal, and a frictional couple of constant moment 850 N m opposes the motion. Find the angular speed of the arm when B is first vertically below P. [5]

7 At midnight, ship A is 70 km due north of ship B. Ship A travels with constant velocity $20\,\text{km h}^{-1}$ in the direction with bearing 140°. Ship B travels with constant velocity $15\,\text{km h}^{-1}$ in the direction with bearing 025°.

(i) Find the magnitude and direction of the velocity of A relative to B. [4]

(ii) Find the distance between the ships when they are at their closest, and find the time when this occurs. [5]

8



The diagram shows a uniform rod AB, of mass m and length $2a$, free to rotate in a vertical plane about a fixed horizontal axis through A. A light elastic string has natural length a and modulus of elasticity $\frac{1}{2}mg$. The string joins B to a light ring R which slides along a smooth horizontal wire fixed at a height a above A and in the same vertical plane as AB. The string BR remains vertical. The angle between AB and the horizontal is denoted by θ, where $0 < \theta < \pi$.

(i) Taking the reference level for gravitational potential energy to be the horizontal through A, show that the total potential energy of the system is

$$mga(\sin^2\theta - \sin\theta).$$ [3]

(ii) Find the three values of θ for which the system is in equilibrium. [5]

(iii) For each position of equilibrium, determine whether it is stable or unstable. [4]